On the maximum backscattering cross section of passive linear arrays

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puted as

$$\xi(x) = \begin{cases} (k/2a^2)^{1/3}x, & \text{convex parabola, } a > 0 \\ (k/2|a|)^{1/3} \exp(i2\pi/3)x, & \text{concave parabola, } a < 0. \end{cases} \quad (16)$$

III. CONCLUDING REMARKS

By using the concept of a negative radius of curvature, an analytic continuation of a parameter in the well known expression for the Fock currents on a convex parabola leads to a residue series representation for the fields on the surface of a concave parabola. The residue series result is applied to the calculation of the fields on the surface of a specific concave parabola, and the resulting fields are in excellent agreement with the fields calculated using an integral equation solution.

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On the Maximum Backscattering Cross Section of Passive Linear Arrays

L. SOLYMAR AND J. APPEL-HANSEN

Abstract—The maximum backscattering cross section of an equispaced linear array connected to a reactive network and consisting of isotropic radiators is calculated for $n = 2, 3$, and 4 elements as a function of the incident angle and of the distance between the elements. On the basis of the results obtained, some conjectures pertaining to the general n -element array are presented.

A general formula for the relative scattering cross section (scattering cross section divided by the scattering cross section of the individual element) of an n -element linear array connected to an n -port matching network was derived in [1]. The approach was made specific in [2], where both theoretical and experimental results were presented for two parallel dipoles. In the present paper we shall give contour plots for the maximum back-scattering cross

section of linear arrays consisting of 2, 3, and 4 equispaced isotropic elements. Based on the results some conjectures are presented for an n -element array.

The relative backscattering cross section may be expressed as follows [1], [2]:

$$A_b(\theta) = |f(\theta)(Z_a + Z_m)^{-1}f(\theta)|^2 \quad (1)$$

where

$$f(\theta) = [\exp jkd \cos \theta, \exp j2kd \cos \theta, \dots, \exp jnkd \cos \theta] \quad (2)$$

and $Z_a = R_a + jX_a$ is the impedance matrix of the array, $Z_m = R_m + jX_m$ is the impedance matrix of the matching network, k is the free space wavenumber, d is the distance between the elements, θ is the incident angle, and n is the number of elements. It was assumed that the self-reactance of the elements was zero.

Backscattering for $\theta = 90^\circ$ is a special case of specular scattering, so we may quote the general result of [2] that the maximum occurs when

$$(Z_m)_{ii} = 0 \quad (Z_m)_{ik} = -j(X_a)_{ik} \quad (3)$$

and the value of the maximum is

$$A_b(90^\circ) = G_{\max}^2(90^\circ) \quad (4)$$

where $G_{\max}(90^\circ)$ is the maximum gain an array can have in the broadside direction. No such simple relation exists for other angles, and we are faced with the general problem of finding the $n(n+1)/2$ unknown elements of the matrix Z_m (assumed to be purely reactive).

The maximum backscattering cross section is found partly analytically (when the expressions are not too complex) and partly numerically, with the aid of a computer. Plots of $[A_b(\theta)]_{\max} = \text{constant}$ are shown in Fig. 1 for $n = 2, 3$, and 4 as functions of the incident angle θ and the spacing in wavelengths d/λ . In view of (4), the $\theta = 90^\circ$ sections of the diagrams should (and do) agree with the curves of Tai [3], calculated for maximum broadside gain.

The general shape of all three diagrams is the same. We feel justified, therefore, to extend our observations to n elements in general (they can be taken as having been proved for $n \leq 4$, and as conjectures for $n > 4$).

i) The optimum matching network (not necessarily unique—there may be other solutions as well) satisfies the following conditions:

$$(X_m)_{ii} = -(X_m)_{n-i+1, n-i+1} \quad (X_m)_{ik} = -(X_a)_{ik} \quad i \neq k \quad (5)$$

and

$$(X_m)_{(n+1)/2, (n+1)/2} = 0, \quad \text{for } n \text{ odd.} \quad (6)$$

ii) For $d/\lambda = 0.5$ and 1, $[A_b(\theta)]_{\max} = n^2$ in the directions $\theta = 0^\circ$ and 90° , and does not exceed n^2 in any other direction.

iii) For $d/\lambda < 0.5$, the maximum of $[A_b(\theta)]_{\max}$ as a function of θ occurs at $\theta = 0^\circ$.

iv) For $0.5 < d/\lambda < 1$ the maximum of $[A_b(\theta)]_{\max}$ as a function of θ occurs at $\theta = 90^\circ$.

v) The absolute maximum of $[A_b(\theta)]_{\max}$ for a given value of n occurs at $\theta = 0^\circ$ when $d/\lambda \rightarrow 0$ and its value is $\frac{1}{4}(n + n^2)^2$.

The last conclusion is particularly interesting because it shows the "supergain" effect ($[A_b(\theta)]_{\max}$ is proportional to n^4 when $n \gg 1$) and states, at the same time, that the absolute maximum of backscattering is always below the absolute maximum of forward scattering

$$\frac{1}{4}(n + n^2)^2 < n^4, \quad \text{for } n > 1 \quad (7)$$

as may be easily proven.

Taking real elements, the conclusions are not expected to alter appreciably. We found, for example, that the main change for half-wave dipoles in parallel is that the relative backscattering cross section is somewhat increased in the direction $\theta = 90^\circ$ and reduced for $\theta = 0^\circ$.

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Fig. 1. Contour plots of $[A_b(\theta)]_{\max}$. (a) $n = 2$. (b) $n = 3$. (c) $n = 4$.

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Backscattering by Cylinders Enclosed in Wire Cages

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Abstract—Calculations of the backscattering width of various metal cylinders enclosed in wire cages are compared with measured values obtained from a standing-wave-ratio method. The calculations are done by assuming that the wire cage simulates a thin continuous current sheath with suitable boundary conditions. The

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