Integration of two-dimensional complex functions

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Published in:
IEEE Transactions on Antennas and Propagation

Publication date:
1976

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
a horizontal antenna can be removed if it is situated at least 
\(0.7/\sqrt{\varepsilon_f})\) away from the ground plane. For the worst case 
\(\phi(x,r) = 0\), it has been found that the reflection-coefficient 
method yields a solution within 10 percent of the exact Sommerfeld 
formulation both in the real and imaginary parts of impedance 
elements even when two horizontal currents are as much as 1000\(\delta\) apart. All of the above restrictions are valid even for low 
values of the dielectric constant of the ground plane (\(\varepsilon_f \approx 2\)). However, for parallel vertical wires over an imperfect ground 
plane the reflection coefficient method yields a result accurate 
to within 10 percent of the exact Sommerfeld formulation in 
both the real and imaginary parts of the impedance elements 
under all conditions of the ground as long as they are 
\(0.7/\sqrt{\varepsilon_f})\) away from the surface of the ground plane.

Input data required for this program include the conductivity 
and the dielectric constant of the ground, the operating frequency, 
the total number of elements in the linear array, the radii and 
feed voltages of the wires, the lengths and spacings of individual 
elements, and the angular steps at which the specified field 
pattern is to be computed.

Computer output consists of all input data together with the 
current distribution of each wire, input impedances correspond-
respectively.

Acknowledgment

Thanks are due to Dr. Bradley J. Strait for valuable guidance and advice, and to Dr. R. F. Harrington for helpful suggestions.

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where $\varepsilon$ is a user specified small positive number. The corresponding stop criterion in the Simpson integration is

$$\left| \frac{T_n^{(1)} - T_{n-1}^{(1)}}{T_n^{(1)}} \right| < \varepsilon, \quad r > 2. \quad (7)$$

The programs QATR3 and SIMP3 make use of a two-dimensional trapezoidal algorithm [3] to approximate (1) directly:

$$T_r^{(0)} = \frac{1}{2} T_r^{(0)} - \frac{\Delta x \Delta y}{2^{2r}} \left[ \frac{1}{2} \sum_j F_j^{(0)} + \sum_k F_k^{(0)} \right]$$

$$T_0^{(0)} = \Delta x \Delta y \sum_{i=1}^{N} \frac{1}{4} F_i^{(0)}$$

$$\Delta x = x_u - x_l, \quad \Delta y = y_u - y_l \quad (8)$$

where the $i$-summation includes the integrand sample points in the four corners of the rectangular integration interval. It is understood from (8) that the $r$th trapezoidal approximation $T_r^{(0)}$ to the integrand is defined by $T_r^{(0)}$ and two summations over new sample points not involved in the calculation of $T_r^{(0)}$. The new sample points are homogeneously distributed over the integration interval by halving the mesh size (see Fig. 2). In (8) the superscript $bo$ indicates new sample points at the boundary of the integration interval whereas superscript $in$ indicates interior points. Subsequently (5) is used for the Romberg extrapolations (QATR3) and with $m = 1$ for the Simpson method (SIMP3). The convergence criteria used are as defined in (6) and (7). The program structure is shown in Fig. 1(b).

The program listings contain extensive information on input and output parameters. An example is shown in Fig. 3 for QATR2.

The programs described above have been used for the calculation of two-dimensional phase integrals and numerical results are given in [4].

**REFERENCES**


