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Brief Announcement: Local Advice and Local Decompression

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ABSTRACT

In this work we study *local computation with advice*: the goal is to solve a graph problem Π with a distributed algorithm in $f(\Delta)$ communication rounds, for some function f that only depends on the maximum degree Δ of the graph, and the key question is how many bits of advice per node are needed. Our main results are:

- (1) Any *locally checkable labeling problem* (LCL) can be solved in graphs with *sub-exponential growth* with only 1 bit of advice per node. Moreover, we can make the set of nodes that carry advice bits arbitrarily sparse.
- (2) The assumption of sub-exponential growth is necessary: assuming the *Exponential-Time Hypothesis* (ETH), there are LCLs that cannot be solved in general with any constant number of bits per node.
- (3) In any graph we can find an *almost-balanced orientation* (indegrees and outdegrees differ by at most one) with 1 bit of advice per node, and again we can make the advice arbitrarily sparse.
- (4) As a corollary, we can also *compress an arbitrary subset of edges* so that a node of degree d stores only $d/2 + 2$ bits, and we can *decompress* it locally, in $f(\Delta)$ rounds.
- (5) In any graph of maximum degree Δ , we can find a Δ -coloring (if it exists) with 1 bit of advice per node, and again, we can make the advice arbitrarily sparse.
- (6) In any 3-colorable graph, we can find a 3-coloring with 1 bit of advice per node. Here, it remains open whether we can make the advice arbitrarily sparse.

CCS CONCEPTS

• **Theory of computation** → **Distributed computing models; Distributed algorithms.**

KEYWORDS

distributed advice, distributed decompression, locality

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1 INTRODUCTION

Our work explores *what can and cannot be computed locally with the help of advice*. Our main focus is understanding advice in the context of classic local graph problems, such as vertex coloring.

While computation with different forms of advice has been explored in a wide range of distributed settings [10, 11, 16–20, 24, 26–28, 35–37, 39], there is hardly any prior work on solving classic local graph problems. A rare example of prior work is [14] from 2007, which studied the question of how much advice is necessary to break Linial's [33] lower bound for coloring cycles.

We initiate a systematic study of exactly how much advice is needed in the context of a wide range of graph problems. Let us first formalize the setting we study:

A graph problem Π can be *solved with β bits of advice* if there exists a $T(\Delta)$ -round distributed algorithm \mathcal{A} , such that for any graph G that admits a solution to Π , there is an assignment of β -bit labels on vertices, such that the output of \mathcal{A} on the labeled graph is a solution to Π .

We will work in the usual LOCAL model of distributed computing. In an n -node graph, the nodes are labeled with unique identifiers from $\{1, 2, \dots, \text{poly}(n)\}$. We emphasize that the advice may depend



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on the assignment of identifiers, and algorithm \mathcal{A} can freely make use of both the advice and the identifiers.

Now we seek to understand this question:

What is the smallest β such that Π can be solved with β bits of advice?

Note that if Π is, for example, the 3-coloring problem, it is trivial to solve with $\beta = 2$ bits of advice per node, as we can directly encode the solution. The key question is how much better we can do.

Our work shows that for many problems the key threshold is not whether we can achieve, say, 1 bit of advice per node, but whether we can make the advice *arbitrarily sparse*, that is, make the ratio between 1s and 0s assigned to the nodes of the graph to be an arbitrarily small constant. This is particularly useful, as it enables us to *compose* multiple sparse advice schemes, so that it suffices to use just one bit per node in total. Hence, a large part of this work addresses the following question:

Which problems admit arbitrarily sparse advice?

We show that some problems can be solved with arbitrarily sparse advice. On the other hand, we also show that assuming the Exponential-Time Hypothesis, for any constant c , there exist problems that cannot be solved with c bits of advice. Finally, there are some problems like 3-coloring that can be solved with 1 bit of advice, but where it is not clear whether it can be solved with arbitrarily sparse advice. We discuss all of those points in what follows; we refer to the full version of this work [1] for more details.

2 LCLS IN BOUNDED-GROWTH GRAPHS

Locally checkable labeling problems (LCL), first introduced by Naor and Stockmeyer [38] in the 1990s, are one of the most extensively studied families of problems in the theory of distributed graph algorithms [2–5, 7–9, 13, 21, 23, 41]. These are graph problems that can be specified by giving a *finite* set of valid local neighborhoods. Many key problems such as vertex coloring, edge coloring, maximal independent set, maximal matching, sinkless orientation, and many other splitting and orientation problems are examples of LCLs, at least when restricted to bounded-degree graphs.

We show how to solve *any* LCL problem with just one bit of advice in graphs with a sub-exponential growth (the number of nodes in a radius- r neighborhood is sub-exponential in r):

Any LCL problem can be solved with 1 bit of advice per node in graphs with sub-exponential growth.

We also show that the encoding can be made arbitrarily sparse.

One prominent application of this result is its connection with *distributed proofs* [12, 29–32], and in particular with *locally checkable proofs* [25]. Consider any LCL Π . Assume that our task is to prepare a distributed proof that shows that in a graph G there exists a feasible solution of Π (for example, if Π is the task of 10-coloring, then the task is to certify that the chromatic number of G is at most 10). Now if G has sub-exponential growth, we can use our result to prepare a 1-bit advice that enables the algorithm to find a

solution of Π . Our advice is the proof: to verify it, we simply try to recover a solution with the help of the advice, and then check that the output is feasible in all local neighborhoods. We obtain the following corollary:

Any LCL problem admits a locally checkable proof with 1 bit per node in graphs with sub-exponential growth.

3 LCLS IN GENERAL GRAPHS

At this point a natural question is whether the assumption about bounded growth is necessary. Could we solve all LCL problems in all graphs with 1 bit of advice? We show that the answer is likely to be no:

Fix any β . If all LCL problems can be solved locally with at most β bits of advice, then the Exponential-Time Hypothesis (ETH) is false.

The intuition here is that if some LCL problem Π can be solved with, say, 1 bit of advice per node with some local algorithm \mathcal{A} , then we could solve it with a centralized sequential algorithm as follows: check all 2^n possible assignments of advice, apply \mathcal{A} to decode the advice, and see if the solution is feasible. The total running time (from the centralized sequential perspective) would be $2^n \cdot n \cdot s(n)$, where $s(n)$ is the time we need to simulate \mathcal{A} at one node. Then we need to show that assuming the Exponential-Time Hypothesis, this is too fast for some LCL problem Π . However, the key obstacle is that it may be computationally expensive to simulate \mathcal{A} , as it might perform arbitrarily complicated calculations that depend on the numerical values of the unique identifiers.

Hence, we need to show that \mathcal{A} can be made cheap to simulate. The key ingredient is the following technical result, which we prove using a Ramsey-type argument that is inspired by the proof of Naor and Stockmeyer [38]:

Assume that problem Π can be solved with β bits of advice per node, using some algorithm \mathcal{A} . Then the same problem can be also solved with β bits of advice using an *order-invariant* algorithm \mathcal{A}' , whose output does not depend on the numerical values of the identifiers but only on their relative order.

Now \mathcal{A}' can be represented as a finite lookup table; hence the simulation of \mathcal{A}' is cheap, and we can make a formal connection to the Exponential-Time Hypothesis.

4 BALANCED ORIENTATIONS

We now move on to a specific graph problem: we study the task of finding balanced and almost-balanced orientations. The goal is to orient the edges so that for each node indegree and outdegree differ by at most 1. This is a hard problem to solve in a distributed setting, while slightly more relaxed versions of the problem admit efficient (but not constant-time) algorithms [22].

Here it is good to note that if we could place our advice on *edges*, then trivially one bit of advice per edge would suffice (simply use the single bit to encode whether the edge is oriented from lower to

higher identifier). However, we are here placing advice on *nodes*, and encoding the orientation of each incident edge would require a number of bits proportional to the maximum degree. Surprisingly, we can do it, in any graph:

We can find almost-balanced orientations with 1 bit of advice per node.

Again, we can make the advice arbitrarily sparse.

5 DISTRIBUTED DECOMPRESSION

Equipped with the advice schema for solving almost-balanced orientations, we can now make a formal connection to what we call *distributed decompression*. Here the task is to encode some graph labeling so that it can be decompressed locally (in $T(\Delta)$ rounds).

Local decompression is closely linked with local computation with advice. If we can compress some solution to Π with only β bits per node, and decompress it locally, then we can also solve Π with β bits of advice per node. Furthermore, if Π is a problem such that for any graph there is only one feasible solution, then the two notions coincide.

We will now show yet another connection between local decompression and local computation with advice. Consider the task of compressing an *arbitrary subset of edges* $X \subseteq E$. In a trivial encoding, we label each node v of degree d with a d -bit string that indicates which of the incident edges are present in X . On the other hand, we need a total of $|E|$ bits in order to distinguish all subsets of the edge-set E . For d -regular graphs, this means we need at least $d/2$ bits per node to recover an arbitrary subset of edges.

It turns out that once we can solve almost-balanced orientations, we can also compress a subset of edges efficiently. We simply use 1 bit of advice per node to encode an almost-balanced orientation. Now a node of degree d has outdegree $\delta \leq \lceil d/2 \rceil$, and it can simply store a δ -bit vector that indicates which of its outgoing edges are in X . Overall, we will need $\lceil d/2 \rceil + 1$, i.e. $\leq d/2 + 2$, bits per node:

We can encode an arbitrary set of edges $X \subseteq E$ so that a node of degree d only needs to store $\lceil d/2 \rceil + 1$ bits, and we can decompress X locally, in $T(\Delta)$ rounds.

6 VERTEX Δ -COLORING

Next we study the problem of finding a Δ -coloring in graphs of maximum degree Δ . Our main result there is:

In any graph of maximum degree Δ , we can find a Δ -coloring (if it exists) with 1 bit of advice per node.

Again, we can make the advice arbitrarily sparse.

Our schema for encoding Δ -colorings consists of three steps. First, we compute a vertex coloring with $O(\Delta^2)$ colors, with the help of advice. Then we reduce the number of colors down to $\Delta + 1$, using the algorithm by [6, 15, 34]. Finally, we follow the key idea of the algorithm by Panconesi and Srinivasan [40] to turn $(\Delta + 1)$ -coloring into a Δ -coloring, and again we will need some advice to make this part efficient.

7 VERTEX 3-COLORING

So far we have seen primarily results of two flavors: many problems can be solved with 1 bit of advice so that we can make the advice arbitrarily sparse, while there are also some problems that require arbitrarily many bits of advice.

We now turn our attention to a problem that seems to lie right at the boundary of what can be done with only 1 bit per node: vertex 3-coloring in any 3-colorable graph. Note that this is a problem that is hard to solve without advice not only in the distributed setting (it is a global problem) but also in the centralized setting (it is an NP-hard problem).

In the centralized setting, 1 bit of advice per node makes the problem easy: we can simply use the bit to indicate which nodes are of color 3. Then the rest of the graph has to be bipartite, and we can simply find a proper 2-coloring in polynomial time.

In the distributed setting, the trivial solution does not work: 2-coloring in bipartite graphs is still a global problem. Nevertheless, we show that 3-coloring is still doable with 1 bit of advice:

In any 3-colorable graph, we can find a 3-coloring with 1 bit of advice per node.

Our encoding essentially uses one bit to encode one of the color classes, but we adjust the encoding slightly so that throughout the graph there are *local hints* that help us to also choose the right parity for the region that we need to 2-color.

Here, our encoding genuinely needs one bit per node (it just barely suffices); we cannot make our advice arbitrarily sparse.

8 COMPOSABILITY FRAMEWORK

A key technique that we use in many of our algorithms is the framework of *composable schemas*. While the definition is a bit technical, it has two key properties:

- (1) As the name suggests, composable schemas can be easily composed, in the following sense: once we have (1) a composable schema for solving Π_1 and (2) a composable schema for solving Π_2 assuming an oracle for Π_1 , we can also compose them and obtain (3) a composable schema that solves Π_2 without the oracle. This way we can solve problems in a modular fashion, in essence using schemas as “subroutines.”
- (2) A composable schema can be then encoded with only 1 bit of advice per node, and we can make the advice arbitrarily sparse.

For example, our algorithms for finding almost-balanced orientations and Δ -coloring with advice are based on the framework of composable schemas. We refer to the full version of this work [1] for more details.

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