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Zhang, Zhengping; Mouritsen, Ole G.; Zuckermann, Martin J.

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Magnetization direction in the Heisenberg model exhibiting fractional Brownian motion

Zhengping Zhang
Centre for the Physics of Materials, Department of Physics, McGill University, Rutherford Building, 3600 University Street, Montréal, Québec, Canada H3A 2T8

Ole G. Mouritsen
Canadian Institute of Advanced Research and Department of Physical Chemistry, The Technical University of Denmark, Building 206, DK-2800 Lyngby, Denmark

Martin J. Zuckermann
Centre for the Physics of Materials, Department of Physics, McGill University, Rutherford Building, 3600 University Street, Montréal, Québec, Canada H3A 2T8

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The temporal magnetization-direction fluctuations in the three-dimensional classical ferromagnetic Heisenberg model have been generated by Monte Carlo simulation and analyzed by the rescaled-range method to yield the Hurst exponent H. A value of $H \simeq 1$ has been found to apply in the ferromagnetic phase characterizing fractional Brownian motion, whereas a value $H \simeq 0.5$, reflecting ordinary Brownian motion, applies in the paramagnetic phase. A field-induced crossover from fractional to ordinary Brownian motion has been observed in the ferromagnetic phase.

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In a recent combined theoretical and experimental study of the director fluctuations in nematic liquid crystals [1,2] it was found from time-series analyses that the accumulated director fluctuations, $X(t)$, in the nematic phase exhibit a scaling property corresponding to fractional Brownian motion characterized by a Hurst exponent [3–6] with a value $H \simeq 1$, which indicates deviation from ordinary Brownian motion ($H = \frac{1}{2}$) [4]. The director in the nematic phase is subject to a continuous symmetry, and it is the presence of power correlations in the director motions that causes fractional Brownian motion to occur. Other systems that also possess continuous symmetry and a Goldstone mode in the ordered phase were suggested as also displaying fractional Brownian motion [1].

In the present paper we present the results of a Monte Carlo simulation study of the temporal fluctuations in the magnetization direction of the three-dimensional classical ferromagnetic Heisenberg model. Our main result is that the continuous symmetry in the magnetic system, in the same way as in the liquid crystal [1], leads to fractional Brownian motion, and that it is described by the same value of the Hurst exponent, $H \simeq 1$, as in the liquid-crystal system. Furthermore, a crossover to ordinary Brownian motion and $H = \frac{1}{2}$ can be induced by a uniform magnetic field which breaks the continuous symmetry of the magnetic order.

The three-dimensional ferromagnetic Heisenberg model is defined by the Hamiltonian

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i S_{zi} , \tag{1}$$

where $\mathbf{S}_i=(S_{xi},S_{yi},S_{zi})$ is a classical spin vector of unit length, h is a uniform magnetic field applied in the z direction, and $J>0$. The Heisenberg model in zero field has a second-order phase transition from a ferromagnetic phase to a paramagnetic phase at $k_B T_c/J=1.44$. The order parameter in the ferromagnetic phase is a macroscopic magnetic moment specified by a length (the magnetic order parameter) and a direction in space. The magnetization direction is not coupled to the lattice. Hence the ferromagnetic order is of continuous symmetry, O(3).

We have calculated equilibrium time series of the macroscopic magnetization direction, $\hat{\mathbf{e}}(t)=\sum_i \mathbf{S}_i/|\sum_i \mathbf{S}_i|$ given by its components $\hat{e}_\alpha(t)$, $\alpha=x,y,z$, as well as the time series of the magnetic order parameter $m(t)=L^{-3}|\sum_i \mathbf{S}_i|$. The calculations are carried out by conventional Monte Carlo simulation on a simple cubic lattice with periodic boundary conditions and $L^3=28^3$ lattice sites. This lattice size was shown to represent the thermodynamic limit by performing test calculations on other lattice sizes. The calculations are performed using single-site Glauber dynamics involving attempts to rotate the direction of the individual spins through a random solid angle. The time is measured in units of Monte Carlo steps per lattice site (MCS/S). The Glauber dynamics, which corresponds to the overdamped regime of the model dynamics, does not conserve the magnetic order parameter. This dynamics is not the true dynamics of an isolated spin system described by angular momentum operators for which the magnetization as well as the magnetization direction are conserved quantities. For a system quenched out of equilibrium, the true dynamics is, however, determined by the details in the coupling between the spin system and the environment.

Time series for $\hat{\mathbf{e}}(t)$ and $m(t)$ have been determined for different temperatures both in the ordered ferromagnetic phase and in the disordered paramagnetic phase. Results have been obtained in zero magnetic field, $h=0$, as well as for a series of finite field values. The time series have been analyzed by the rescaled-range (R/S) method [1–6] as well as via the power spectrum

$$P(f)=\left|\int X(t)\exp(-i2\pi f t)dt\right|^2 \tag{2}$$

of the accumulated fluctuations $X(t)$. For a time series $u(t)$, we determine the accumulated fluctuations over the time range τ as

$$X(t,\tau)=\sum_{t'=1}^{t}[u(t')-\langle u\rangle_\tau] , \tag{3}$$

where the average of the stochastic variable $u(t)$ over the same time range is given by

$$\langle u\rangle_\tau=\frac{1}{\tau}\sum_{t=1}^{\tau}u(t) . \tag{4}$$

The R/S analysis is based on a range

$$R=\max[X(t,\tau)]-\min[X(t,\tau)],\quad 0\le t\le\tau \tag{5}$$

and a standard deviation

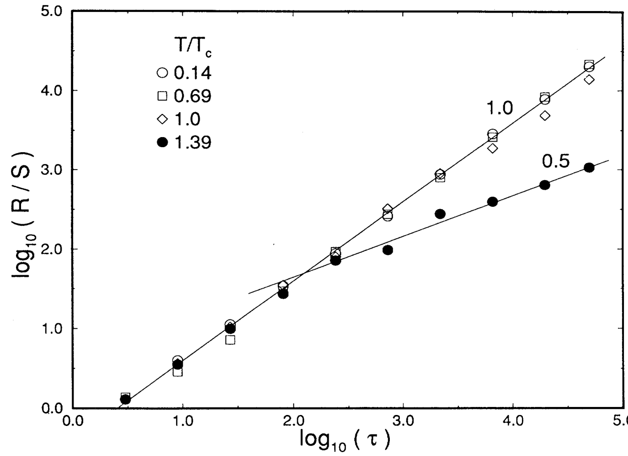

FIG. 1. Log-log plot of the R/S value vs time range τ for the zero-field magnetization direction fluctuations at three different temperatures in the ferromagnetic phase: $T/T_c=0.14$ (○), $T/T_c=0.69$ (□), $T/T_c=1.00$(◇); and one temperature in the paramagnetic phase: $T/T_c=1.39$(●). The best linear fits to the data sets are given by the solid lines, $R/S\sim\tau^H$, with slopes H as indicated.

$$S=\left\{\frac{1}{\tau}\sum_{t=1}^{\tau}[u(t')-\langle u\rangle_\tau]^2\right\}^{1/2} . \tag{6}$$

In terms of these quantities, the Hurst exponent describes the scaling properties of the accumulated fluctuations as [6]

$$R/S\sim\tau^H . \tag{7}$$

Provided scaling holds [6], the scaling power β of the power spectrum in Eq. (2), $P(f)\sim f^{-\beta}$, is related to the Hurst exponent as $\beta=2H+1$. For a time series of statistical independent events $H=\frac{1}{2}$, which corresponds to ordinary Brownian motion. For $H\neq\frac{1}{2}$, the correlation function has power-law decay and infinitely long correlations. This latter case has been termed fractional Brownian motion by Mandelbrot [4].

A selection of the data for the R/S values, as a function of time range τ, obtained for the magnetization direction fluctuations is given in Fig. 1. The magnetization direction fluctuations calculated are recorded as the fluctuations in one of the components of $\hat{\mathbf{e}}(t)$. All three components are found to behave statistically similarly. The data in Fig. 1 for three different temperatures in the ferromagnetic phase show that the fluctuations display a remarkably clear scaling behavior over a wide range of τ characterized by a Hurst exponent, $H\simeq 1$, which implies fractional Brownian motion. The value of H is independent of temperature and remains at the value $H\simeq 1$ up to the critical point. In contrast, the data for the magnetization direction fluctuations in the paramagnetic phase shown in Fig. 1 demonstrate that ordinary Brownian motion, $H\simeq\frac{1}{2}$, applies over long time ranges. The corresponding data for the power spectrum, $P(f)$ in Eq. (2), shown in Fig. 2 supports this picture and indicates that the scaling relation $\beta=2H+1$ holds, at least at low fre-

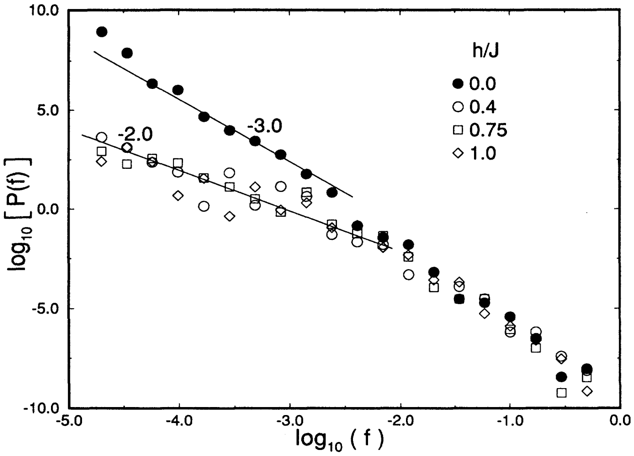

FIG. 2. Log-log plot of the power spectrum, $P(f)$ in Eq. (2), vs frequency f (in units of reciprocal MCS/S) for the magnetization direction fluctuations at a temperature $T/T_c=0.14$ in the ferromagnetic phase. The data correspond to four different values of the applied magnetic field: $h/J=0$ (●), 0.4 (○), 0.75 (□), and 1.0 (◇). The solid lines have slopes H, as indicated.

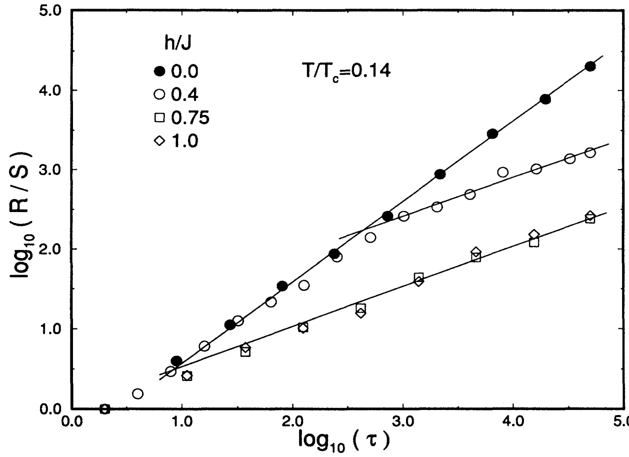

FIG. 3. Log-log plot of the R/S value vs time range τ for the magnetization direction fluctuations at a temperature $T/T_c=0.14$ in the ferromagnetic phase. The data correspond to four different values of the applied magnetic field: $h/J=0$ (●), 0.4 (○), 0.75 (□), and 1.0 (◇). The best linear fits to the data sets over long time ranges are given by the solid lines, $R/S \sim \tau^H$, with $H \simeq 1$ for zero field and $H \simeq 0.5$ for finite fields.

quencies. This is in close accordance with our previous findings for the liquid-crystal model [1], where the scaling power β, obtained from the power spectrum, was close to -3 for low frequencies and displayed a crossover to a smaller exponent ~ -4 for higher frequencies. Our data for the power spectrum are not accurate enough to assess whether this behavior may suggest a breakdown of the scaling relation, $\beta=2H+1$.

If the continuous symmetry of the ferromagnetic order is broken by an ordering field, the fractional Brownian motion in the ferromagnetic phase is destroyed, as seen in Fig. 3, and there is a crossover to ordinary Brownian motion over long time ranges corresponding to a Hurst exponent value of $H \simeq 0.5$. The crossover occurs at shorter time ranges, the stronger the field is. A related crossover is also seen in the power-spectrum data in Fig. 2.

In contrast to the fluctuations in the magnetization direction (in the absence of a field) the fluctuations in the order parameter (magnetization) shown in Fig. 4 for $T \neq T_c$ exhibit ordinary Brownian motion ($H=\frac{1}{2}$) in either phase. It should be noted that the order parameter in a finite system assumes a finite value above T_c. The order-parameter fluctuations exhibit normal Brownian motion independent of temperature and independent of the strength of the applied magnetic field. At the critical point, the order-parameter fluctuations (in zero magnetic field) should exhibit power-law correlations and a Hurst exponent different from $\frac{1}{2}$. It is difficult to sample the order-parameter fluctuations at the critical point in a finite system over long time ranges. Furthermore, the accumulated fluctuations over long ranges may be sensitive to the precise value used for the transition temperature. The data shown in Fig. 4 for the order-parameter fluctuations at the critical point $T=T_c$ for the infinite system suggest that the Hurst exponent value differs from $\frac{1}{2}$ at intermediate time ranges and increases towards $H=1$. At large time ranges the effective exponent value is in between $\frac{1}{2}$ and 1, but the system size is too small to reliably determine the order-parameter fluctuations over long time ranges.

The physical interpretation of the numerical simulation results of the magnetization and magnetization direction fluctuations in the Heisenberg model is as follows: In the ordered ferromagnetic phase up to the critical point, the magnetization direction is subject to a con-

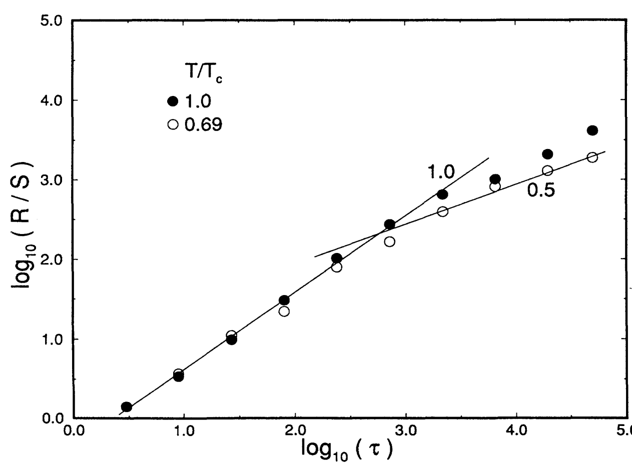

FIG. 4. Log-log plot of the R/S value vs time range τ for the zero-field magnetization fluctuations for a temperature in the ferromagnetic phase, $T/T_c=0.69$, (○), and a temperature close to the critical point, $T/T_c=1$ (●). The best linear fits to the different data sets over long time ranges are given by the solid lines, which have slopes H, as indicated.

tinuous symmetry since its direction is not coupled to the lattice and there is no activation barrier for directional rotation corresponding to the presence of a Goldstone mode. Hence the magnetization direction is subject to critical fluctuations for all temperatures within the ferromagnetic phase. This leads to fractional Brownian motion of the magnetization direction, as observed. In contrast, the order parameter, i.e., the magnetization, is not a critical mode, except at T_c, where it exhibits fractional Brownian motion. At all other temperatures, in both the ferromagnetic and the paramagnetic phases, the order-parameter fluctuations are of short range and are associated with ordinary Brownian motion. The continuous symmetry of the magnetization direction can be lifted by an ordering magnetic field, in which case the fluctuations become quenched and the mode is no longer critical. Ordinary Brownian motion then results over long time ranges, as observed. At short time ranges, however, the symmetry-breaking field is not capable of destroying the power-law correlations, and there is a crossover to ordinary Brownian motion only at longer ranges. This crossover occurs for shorter time ranges the stronger the field is.

Our findings of fractional Brownian motion in the magnetization direction fluctuations in the three-dimensional Heisenberg model, together with recent similar results for fractional Brownian motion of director fluctuations both in simulations and experiments on liquid crystals [1], all with the same value of the Hurst exponent, $H \simeq 1$, suggest that fractional Brownian motion may be a generic phenomenon in systems characterized by an order parameter of continuous symmetry. There is at present no theory for the value of H, and it is a challenge to construct such a theory and possibly relate the Hurst exponent to well-known dynamic critical exponents.

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