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Directional Velocity Estimation Using a Spatio-Temporal Encoding Technique Based on Frequency Division for Synthetic Transmit Aperture Ultrasound

Fredrik Gran and Jørgen Arendt Jensen, Senior Member, IEEE

Abstract—This paper investigates the possibility of flow estimation using spatio-temporal encoding of the transmissions in synthetic transmit aperture imaging (STA). The spatial encoding is based on a frequency division approach. In STA, a major disadvantage is that only a single transmitter (denoting single transducer element or a virtual source) is used in every transmission.

The transmitted acoustic energy will be low compared to a conventional focused transmission in which a large part of the aperture is used. By using several transmitters simultaneously, the total transmitted energy can be increased. However, to focus the data properly, the signals originating from the different transmitters must be separated. To do so, the pass band of the transducer is divided into a number of subbands with disjoint spectral support. At every transmission, each transmitter is assigned one of the subbands. In receive, the signals are separated using a simple filtering operation. To attain high axial resolution, broadband spectra must be synthesized for each of the transmitters. By multiplexing the different waveforms on different transmitters over a number of transmissions, this can be accomplished. To further increase the transmitted energy, the waveforms are designed as linear frequency modulated signals. Therefore, the full excitation amplitude can be used during most of the transmission.

The method has been evaluated for blood velocity estimation for several different velocities and incident angles. The program Field II was used. A 128-element transducer with a center frequency of 7 MHz was simulated. The 64 transmitting elements were used as the transmitting aperture and 128 elements were used as the receiving aperture. Four virtual sources were created in every transmission. By beamforming lines in the flow direction, directional data were extracted and correlated. Hereby, the velocity of the blood was estimated. The pulse repetition frequency was 16 kHz. Three different setups were investigated with flow angles of 45, 60, and 75 degrees with respect to the acoustic axis. Four different velocities were simulated for each angle at 0.10, 0.25, 0.50, and 1.00 m/s. The mean relative bias with respect to the peak flow for the three angles was less than 2%, 2%, and 4%, respectively.

I. Introduction

Synthetic transmit aperture (STA) can be used for velocity estimation in ultrasound imaging [1]–[5]. The method is based on correlating data acquired with exactly the same emission sequence. To synthesize a large transmit aperture, several transmissions have to be carried out from different locations on the transmitting aperture. Beamforming the received signals from one transmission results in a low resolution image with poor resolution and contrast. The synthesis of a larger transmit aperture is achieved by summing data from different transmissions yielding a high-resolution image. This improves the resolution and contrast as well as the image quality.

If the object under investigation is moving very rapidly only a small number of transmissions is affordable. Having a large number of transmissions to generate a high resolution image would yield poor performance of the velocity estimator because the movement results in a smearing of the point spread function (PSF). When the number of transmissions per data set is small, the signal-to-noise-ratio (SNR), however, will drop. Virtual sources have been suggested to partly compensate for this [6]–[9]. Temporal encoding in the form of a linear frequency modulated signal also has been suggested to increase the SNR [10]–[17].

Spatial encoding has been suggested as a means of increasing SNR for STA when the number of transmissions per high-resolution image is low. Chiao et al. [18] suggested that a Hadamard space-time encoding scheme could be used to encode the transmitters. The method relies on exciting the transmitters simultaneously with the same excitation waveforms. The encoding is based on premultiplying the waveform on each transmitter by a row (or column) of the Hadamard matrix of the same dimension as the number of transmitters (2^k). The next transmission is premultiplied by the next row, and this procedure is repeated until all rows have been covered. Under the assumption of static target geometry, the echoes originating from the different transmitters can be extracted by summing and subtracting the received signals from the different transmissions. The target geometry has to be static from transmission to transmission, for the decoding to work properly. In flow estimation the target is not static, and it has been argued that this would degrade the method, making it unsuitable for flow imaging.

The general term transmitter will be used to represent a virtual source or a single transducer element. In [18] this was called phase center.
Chiao and Thomas suggested in [19] to use complementary codes (orthogonal Golay sequences) to encode the transmitters. Here the transmitters are assigned a specific binary code sequence for each transmitter, and correlation receivers are applied to separate the signals. From one transmission the intercode cross correlation is not zero, but by summing the result from different transmission using different code sequences, the inter-code, cross correlation can be made to vanish. Using the same procedure, perfect auto-correlation properties can be achieved for the combined auto-correlation functions. This approach also assumes static target geometry as data from different transmissions have to be combined.

In this paper a method for spatial encoding developed in [20]–[22] is evaluated for blood flow estimation as suggested in [23]. To focus the data properly, the information originating from different transmitters must be separated. To do so, the pass band of the transducer is divided into a number of subbands with disjoint spectral support. At every transmission, each transmitter is assigned one of the subbands. In receive, the signals are separated using a simple filtering operation. The decoding can be performed instantaneously at the receiver, making the decoding process insensitive to motion. To attain high-axial resolution, the synthesized broadband time signal for one transmitter can be seen in Fig. 2. The majority of the transmitters are designed as linear phase finite impulse response (FIR) filters. The two signal sets can be seen in Fig. 1. The filters with odd index and the second set of the signals with even index. This makes it possible to encode four transmitters simultaneously, but it requires eight transmissions before a resolution. To do so, the pass band of the transducer is divided into a number of subbands with disjoint spectral support. At every transmission, each transmitter is assigned one of the subbands. In receive, the signals are separated using a simple filtering operation. The decoding can be performed instantaneously at the receiver, making the decoding process insensitive to motion. To attain high-axial resolution, each transmitter produces a smooth broadband spectrum. In this paper, the synthesized broadband time signal for one transmitter is defined as:

\[
m_i(t) = s_i(-t) * h_i(t),
\]

where \( * \) denotes the convolution operator and \( h_i(t) \) is an additional filter applied to shape the synthesized broadband spectrum. Consider the filtered \( i \)-th waveform:

\[
r_i(t) = s_i(t) * m_i(t) = s_i(t) * s_i(-t) * h_i(t),
\]

with the Fourier transform

\[
F\{r_i(t)\} = |S_i(f)|^2 H_i(f),
\]

where \( S_i(f) \) is the Fourier transform of \( s_i(t) \) and \( H_i(f) \) is the Fourier transform of \( h_i(t) \). The spectrum \( H_i(f) \) is chosen such that the combined spectrum for the different signals produce a smooth broadband spectrum. In this paper, \( H_i(f) \) was chosen to be:

\[
H_i(f) = \begin{cases} 
\frac{1}{2} \left( 1 + \cos \left( 2\pi \frac{T - f_i}{B} \right) \right) & \text{if } 0 \leq f \leq \frac{B}{2}, \\
0 & \text{otherwise},
\end{cases}
\]

as these spectra combined, \( \sum_i H_i(f) \) give a broadband spectrum with a smooth passband. The filters \( h_i(t) \) were designed as linear-phase finite impulse response (FIR) filters in which the filter coefficients were optimized using least-squares error minimization. In this paper, eight waveforms were generated. The duration was 25 \( \mu s \), the bandwidth was 2 MHz, and the function \( b(t) \) was chosen to be a Tukey-window with 35% tapering. The Tukey window is defined as:

\[
b(t) = \begin{cases} 
\frac{1}{2} \left( 1 + \cos \left( 2\pi \frac{t - \frac{\alpha T}{2}}{\alpha T} \right) \right) & 0 \leq t \leq \frac{\alpha T}{2}, \\
1 & \frac{\alpha T}{2} \leq t \leq \left( 1 - \frac{\alpha}{2} \right) T, \\
\frac{1}{2} \left( 1 + \cos \left( 2\pi \frac{t - \left( 1 - \frac{\alpha}{2} \right) T}{\alpha T} \right) \right) & \left( 1 - \frac{\alpha}{2} \right) T \leq t \leq T
\end{cases}
\]

where \( T \) is the duration of the window and \( \alpha = 0.35 \) is the amount of tapering applied. The center frequencies can be seen in Table I. To reduce intersignal interference caused by overlapping frequencies, the waveforms were grouped in two sets. The first signal set consisted of the signals with odd index and the second set of the signals with even index. This makes it possible to encode four transmitters simultaneously, but it requires eight transmissions before a broadband spectrum can be synthesized for each transmitter. The two signal sets can be seen in Fig. 1. The filters \( h_i(t) \) were specified to be FIR filters with a duration of 18.75 \( \mu s \). The synthesized broadband time signal for one transmitter can be seen in Fig. 2. The majority of the

2This is only an approximation. The waveforms will not have disjoint spectral support in practice as this would require that infinite code sequences were used.

Fig. 1. The spectra for the two sets of excitation signals can be seen as the top figure. The corresponding separation filters are given as the two bottom figures. Note how the FIR filters \( h_i(t) \) affect the frequency characteristic of the matched filters. The FIR filters reduce the inter-transmitter interference and shapes the filtered spectrum of each transmitter.

Axial sidelobes are below \(-60\) dB, with exception for the near sidelobes that are at \(\sim -50\) dB. This is acceptable in a STA system used for flow estimation because the short transmission sequence used to synthesize the transmitting aperture will cause the lateral sidelobes to have a level of \(\sim -40\) dB.

### III. Beamforming

This section describes the beamforming that has been applied to properly focus the data. When a broadband excitation signal has been synthesized for every transmitter on the aperture, it is possible to beamform the synthesized broadband data as if only one transmitter was active in every transmission. This allows the synthetic aperture focusing technique described in [24] to be applied.

Because the system has access to defocused transmissions from all transmitters on the aperture, it is possible to focus the acoustic energy on any arbitrarily chosen point of interest. The amplitude in a point \( \vec{r}_p \) in the image is given by:

\[
   H(\vec{r}_p) = \sum_{k=1}^{K} \sum_{q=1}^{Q} a_q(t_{pkq}) r_{kq}(t_{pkq}) ,
\]

where \( a_q(t_{pkq}) \) is a weighting function (apodization) over the receiving aperture, which ideally would be changing with spatial coordinates to keep a constant F-number. However, in this paper the apodization is constant over space and is chosen as a Hanning window over all 128 receiving elements. The time signal \( r_{kq}(t) \) is the received synthesized broadband echo on the \( q \):th receiver, originating from the \( k \):th transmitter. Here \( t_{pkq} \) is the time corresponding to the geometrical travel distance from the \( k \):th transmitter to the point \( \vec{r}_p \) and back to the \( q \):th receiver:

\[
   t_{pkq} = \frac{||\vec{r}_p - \vec{r}_{xmt,k}|| + ||\vec{r}_{cv,q} - \vec{r}_p||}{c} ,
\]

where \( c \) is the speed of sound, and \( \vec{r}_p - \vec{r}_{xmt,k} \) is the vector from the transmitter to the point \( \vec{r}_p \), and \( \vec{r}_{cv,q} - \vec{r}_p \) is the vector from the point \( \vec{r}_p \) to the receiver as indicated in Fig. 3. The beamformed data resulting from one transmission is denoted low-resolution image, and the result of

---

**TABLE I**

The Center Frequencies of the Different Waveforms.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( f_i [\text{MHz}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.3000</td>
</tr>
<tr>
<td>2</td>
<td>4.3512</td>
</tr>
<tr>
<td>3</td>
<td>5.4024</td>
</tr>
<tr>
<td>4</td>
<td>6.4536</td>
</tr>
<tr>
<td>5</td>
<td>7.5048</td>
</tr>
<tr>
<td>6</td>
<td>8.5560</td>
</tr>
<tr>
<td>7</td>
<td>9.6072</td>
</tr>
<tr>
<td>8</td>
<td>10.6584</td>
</tr>
</tbody>
</table>

---

Fig. 2. The time response of a synthesized broadband pulse. After eight transmissions, broadband pulses can be synthesized for each transmitter. The axial sidelobe level is below \(-60\) dB except for the near sidelobes which have a level of \(\sim -50\) dB. The top figure displays the log compressed envelope of the time signal. The bottom figure displays the actual time response on a linear scale.
summing a number of low-resolution images to synthesize
a larger aperture results in a high-resolution image. Sum-
mimg contributions from all transmissions for all receiving
elements gives both dynamic transmit and receive focusing.

To obtain directional data suitable for finding the velocity
of the moving particles, lines were beamformed in the
direction of the flow [25], [26]. The principle can be seen
in Fig. 4. Correlating successive lines, the spatial shift be-
tween the data can be found and, thus, the velocity. The
spatial distance between data points in the directional data
were beamformed. The direction of the flow was assumed to be
known.

IV. VELOCITY ESTIMATOR

The velocity of the moving blood scatterers was found
using a cross-correlation velocity estimator [28], [29]. No
stationary signal was present; therefore, stationary echo
canceling was ignored. However, it should be noted that,
if a stationary echo-canceling filter would be applied, the
results may degrade. This is going to be a subject for fur-
ther investigation.

Because four transmitters were used, eight transmis-
sions had to be carried out before broad-band spectra
could be synthesized for each transmitter. The velocity
was found by correlating lines that had been beamformed
in the direction of the flow. To find the velocity, lines ob-
tained using the same transmission were correlated. A line in the k:th high-resolution image is denoted $h_l(n)$, and
the corresponding line in the next high-resolution image
obtained with the same transmission sequence is denoted
$h_{l+1}(n)$. The cross correlation between the lines is:

$$ R(m) = \frac{1}{N} \sum_{n=0}^{N-1} h_l(n) h_{l+1}(n + m), $$  \hspace{1cm} (9)

where $N$ is the length of the lines in samples. The spa-
tial shift between two high-resolution lines was found by
finding the peak in the cross-correlation function:

$$ m_{\max} = \arg \max_m R(m). $$  \hspace{1cm} (10)

To get a more precise result, quadratic interpolation was
performed using the samples $(m_{\max} - 1, m_{\max}, m_{\max} +
1)$. The maximum lag$^4$ resulting from the interpolation is
given by [29], [30]:

$$ m_v = m_{\max} - \frac{R(m_{\max} + 1) - R(m_{\max} - 1)}{2(R(m_{\max} + 1) - 2R(m_{\max}) + R(m_{\max} - 1))}. $$  \hspace{1cm} (11)

The velocity was found:

$$ v_{\max} = \frac{m_v \Delta x}{T_h}, $$  \hspace{1cm} (12)

where $\Delta x$ is the spatial distance between samples in the
high-resolution lines, and $T_h$ is the time period between
two successive, high-resolution images obtained with the
same transmission sequence$^5$.

V. SIMULATION SETUP

The method has been evaluated using the simulation
tool Field II [31], [32]. A 128-element, linear-array trans-
ducer was used with the parameters shown in Table II. The
central 64 transducer elements were used as the transmit-
ting aperture, and 128 elements were active as the receiv-
ing aperture. Four virtual sources were created [6]–[9], [33].
A virtual source was created by focusing 16 transducer el-
ements 5 mm behind the face of the transducer. The pur-
pose was to emulate a high-energy virtual spherical source.
The basic principle can be seen in Fig. 5. The different

$^4$May be fractional.
$^5$In this paper $T_h = 8/f_{pr}$ because eight emissions were used to
create a high-resolution image.
TABLE II
DIFFERENT PARAMETERS USED IN THE SIMULATION.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>f&lt;sub&gt;pr&lt;/sub&gt;</td>
<td>16</td>
<td>kHz</td>
</tr>
<tr>
<td>f&lt;sub&gt;0&lt;/sub&gt;</td>
<td>7</td>
<td>MHz</td>
</tr>
<tr>
<td>λ</td>
<td>0.2200</td>
<td>mm</td>
</tr>
<tr>
<td>pitch</td>
<td>0.2080</td>
<td>mm</td>
</tr>
<tr>
<td>height</td>
<td>4.5</td>
<td>mm</td>
</tr>
<tr>
<td>No xmt elem.</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>No rcv elem.</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>No virt. sources</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Focal depth</td>
<td>−5</td>
<td>mm</td>
</tr>
<tr>
<td>Δx</td>
<td>1/10</td>
<td>λ</td>
</tr>
<tr>
<td>Line length</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Vessel radius</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>Depth of vessel</td>
<td>30</td>
<td>mm</td>
</tr>
</tbody>
</table>

TABLE III
THE TRANSMISSION SEQUENCE USED FOR THE FLOW ESTIMATION.
XMT No. Denotes the Transmitter of Interest and Em. No. is the Emission Number.

<table>
<thead>
<tr>
<th>XMT No.</th>
<th>Em. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

VI. Motion Effects Broadband Synthesis

To evaluate the SNR, consider the following very simple model of the received signal:

\[ r(t) = p(t) \ast h(t) + v(t), \]

where \( p(t) \) is the combined filtered waveforms influenced by motion as depicted in Fig. 6, and \( h(t) \) is a function representing the interaction between the wave and the interrogated tissue. The function \( h(t) \) is modeled as a Gaussian distributed, white process with zero mean. The variable \( v(t) \) is the filtered measurement noise and is assumed to be independent of the signal. The SNR then is given as:

\[ \text{SNR} = \frac{E[p(t) \ast h(t)]^2}{E|v(t)|^2}, \]

where \( E[\cdot] \) is the expectancy value of \( \cdot \). Now, consider the autocorrelation of \( p(t) \ast h(t) \):

\[ R_{ph}(\tau) = R_p(\tau) \ast R_h(\tau), \]

where \( R_p(\tau) \) and \( R_h(\tau) \) are the autocorrelation functions of \( p(t) \) and \( h(t) \), respectively. By using the fact that \( h(t) \) is modeled as a white Gaussian distributed process with zero mean, the autocorrelation function of \( h(t) \) can be written:

\[ R_h(\tau) = S_h \delta(\tau), \]

where \( S_h \) is the power spectral density of \( h(t) \). Combining (16) and (17) gives:

\[ R_{ph}(\tau) = S_h R_p(\tau). \]

The numerator in (15) is (18) evaluated at \( \tau = 0 \); (18) evaluated at \( \tau = 0 \) gives:

\[ R_{ph}(0) = S_h R_p(0) = S_h \cdot E_p, \]

A resolution cell was assumed to be \( \lambda \times \lambda \times \lambda \).

This assumes that the movement is uniform over the target function and purely axial to the transmitter and the receiver of interest.
Fig. 6. The synthesized broadband beamformed response as a function of velocity. The target is a single point scatterer located at a depth of 30 mm on the central axis of the transducer. The synthesized pulse is valid for the second virtual source. The graphs to the left represent the axial response, and the graphs to the right represent the lateral response.

where $E_p$ is the energy of $p(t)$. The decrease in SNR for a moving target with the synthesized broadband pulse having energy $E_{mov}$, compared to a stationary target with a synthesized broadband pulse with energy $E_{stat}$ is (in logarithmic scale):

$$10 \log_{10} \left( \frac{E_{mov}}{E_{stat}} \right),$$

(20)

because the variance of the filtered noise is assumed to be the same for the two situations as well as the power spectral density of $h(t)$. Eq. (20) is evaluated as a function of velocity for the synthesized broadband pulses in Fig. 7.

VII. Simulation Results

The directional lines were beamformed and extracted. Succeeding high-resolution lines acquired using the same transmission sequence were correlated. To improve the estimates, 10 successive estimates for the cross-correlation function were averaged, so that 80 transmissions were used to generate one velocity estimate. The peak of the averaged cross-correlation function was found, and thereby the velocity was estimated. Table IV shows the angles and velocities that were investigated. To evaluate performance, bias and standard deviation were calculated for the estimates of the velocity for the different simulation setups.

### TABLE IV

<table>
<thead>
<tr>
<th>Angle</th>
<th>Velocities</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>0.10, 0.25, 0.50, 1.00 m/s</td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td>0.10, 0.25, 0.50, 1.00 m/s</td>
<td></td>
</tr>
<tr>
<td>75°</td>
<td>0.10, 0.25, 0.50, 1.00 m/s</td>
<td></td>
</tr>
</tbody>
</table>
For each setup, 22 full velocity estimates were used to calculate the bias and standard deviation. The mean of the velocity estimates is given by:

$$\overline{v}(z) = \frac{1}{M} \sum_{m=0}^{M-1} \hat{v}_m(z),$$  \hspace{1cm} (21)

where $\hat{v}_m(z)$ is the $m$:th estimate for the velocity. The total number of estimates is given by $M$. The bias for the estimates is given by (as a function of depth):

$$b(z) = \overline{v}(z) - v(z),$$  \hspace{1cm} (22)

where $\overline{v}(z)$ is the estimate for the velocity at the depth $z$ and $v(z)$ is the true velocity for this depth. The standard deviation for the estimates is given by:

$$\sigma(z) = \sqrt{\frac{1}{M} \sum_{m=0}^{M-1} [\hat{v}_m(z) - \overline{v}(z)]^2}. $$  \hspace{1cm} (23)

The mean relative bias and the mean relative standard deviation are given by the mean bias and standard deviation relative to the peak velocity in the vessel. Mean relative bias (in percent):

$$\overline{b}_{rel} = \frac{1}{v_{max}} \left( \frac{1}{Z} \sum_{z=0}^{Z-1} b(z) \right) \cdot 100\%,$$  \hspace{1cm} (24)

where the samples $z = 0, \ldots, Z - 1$ are the samples within the vessel. Mean relative standard deviation (in percent):

$$\sigma_{rel} = \frac{1}{v_{max}} \sqrt{\left( \frac{1}{Z} \sum_{z=0}^{Z-1} \sigma^2(z) \right)} \cdot 100\%, $$  \hspace{1cm} (25)

where $z$ is the sample index indicating where in the vessel the standard deviation is evaluated, and $z = 0, \ldots, Z - 1$ are the samples within the vessel. The results for the velocity estimation for 45° for the four different velocities can be seen in Figs. 9–12.

The corresponding mean relative bias for the different velocities is depicted in Fig. 16 as the solid line. The bias is below 2% in all four simulations. The standard deviation depicted in Fig. 17 as the solid line is significantly higher for the simulation with a peak velocity of 1 m/s. This can be explained by examining Fig. 12, which displays several false peaks in the velocity estimation. The result for the velocity estimation for 60° for the simulation with a peak velocity of 1 m/s can be seen in Fig. 13.

The mean relative bias for the four different velocities at 60° is shown in Fig. 16 as the dashed line. The bias is below 2% in all four simulations. The mean relative standard deviation given in Fig. 17 tends to drop for higher velocities. It should, however, be noted that the absolute standard deviation is higher for the higher velocities. The result for the velocity estimation for 75° for the simulation with peak velocity of 1 m/s can be seen in Fig. 15. The mean relative bias for the different velocities is given in Fig. 16 as the dotted line. The bias is below 4% in all four simulations. The mean relative standard deviation given in Fig. 17 tends again to drop for higher velocities. However, once again the absolute standard deviation is higher for the higher velocities.

The influence of applying a stationary echo canceling filter was investigated for the setup with 60° flow angle
Fig. 9. The estimated velocity as a function of depth. The angle of the flow relative to the central axis of the transducer was 45°. The flow had a parabolic velocity distribution over the vessel with peak velocity 0.1 m/s.

TABLE V

<table>
<thead>
<tr>
<th></th>
<th>Without echo-canc.</th>
<th>With echo-canc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rel. bias</td>
<td>1.32%</td>
<td>1.26%</td>
</tr>
<tr>
<td>Rel. std.</td>
<td>0.407%</td>
<td>0.399%</td>
</tr>
</tbody>
</table>

and peak velocity 1 m/s. The echo-canceling filter was a running mean subtraction filter:

\[ h_{ec}^c(n) = h_l(n) - \frac{1}{K} \sum_{k=0}^{K-1} h_{l-k}(n), \quad (26) \]

where \( K \) is the duration of the filter in slow time and \( h_{ec}^c(n) \) is the \( l \)th echo-cancelled, high-resolution line. The amplitude spectrum of this filter can be seen in Fig. 8. In this investigation \( K = 10 \). The velocity estimation was repeated, and the result from this simulation can be seen in Fig. 14. It can be seen that the estimated profile does not change drastically. When comparing the mean relative bias and standard deviation, it was found that both were actually improved after echo-canceling, as can be seen in Table V.

VIII. Conclusions

This paper has investigated the feasibility to use spatial encoding of the transmits by means of frequency division for velocity estimation. A simulation study was carried out where a vessel with a radius of 5 mm was scanned at a depth of 30 mm. Several different velocities were tested, 0.1, 0.25, 0.50, and 1.00 m/s. Three angles were scanned per velocity 45°, 60°, and 75°. The mean bias relative to the peak velocity was for 45° less than 2%, for 60° less than 2% and for 75° less than 4%. Therefore, it is concluded that it is possible to estimate blood velocities using spatial encoding of the transmitters based on frequency division. This makes it possible to exploit the benefits of using spatial encoding in blood flow estimation, which would not be possible with, e.g., Hadamard encoding.

REFERENCES

Fig. 11. The estimated velocity as a function of depth. The angle of the flow relative to the central axis of the transducer was 45°. The flow had a parabolic velocity distribution over the vessel with peak velocity 0.50 m/s.

Fig. 12. The estimated velocity as a function of depth. The angle of the flow relative to the central axis of the transducer was 45°. The flow had a parabolic velocity distribution over the vessel with peak velocity 1.00 m/s.

Fig. 13. The estimated velocity as a function of depth. The angle of the flow relative to the central axis of the transducer was 60°. The flow had a parabolic velocity distribution over the vessel with peak velocity 1.00 m/s.

Fig. 14. The estimated velocity as a function of depth for the echo-cancelled data. The angle of the flow relative to the central axis of the transducer was 60°. The flow had a parabolic velocity distribution over the vessel with peak velocity 1.00 m/s.
Fig. 15. The estimated velocity as a function of depth. The angle of the flow relative to the central axis of the transducer was 75°. The flow had a parabolic velocity distribution over the vessel with peak velocity 1.00 m/s.

Fig. 16. The mean relative bias as a function of velocity. The solid line represents the results for 45°, the dashed line 60°, and the dotted line 75°.

Fig. 17. The mean relative standard deviation as a function of velocity. The solid line represents the results for 45°, the dashed line 60°, and the dotted line 75°.


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