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Comments and Corrections

Comments on “Dynamical Noise in Single-Mode Distributed Feedback Fiber Lasers”

Søren Dyøe Agger and Jørn Hedegaard Povlsen

The comments given here refer to the two excellent and complete papers by Foster published in 2004 [1], [2]. After closely studying the papers, we have found a couple of issues that require comments in order to understand and use the model. Despite this, one should honor the achievements in the papers in which Foster establishes an accurate analytical model for a DFB fiber laser.

The comments relate to a gain parameter and the noise arising from pump power fluctuations and something that affects the noise equations, which we believe to be a small misprint.

I. INTENSITY AND AMPLITUDE GAIN

In the two articles, the results for the theory of the passive phase shifted grating are derived using the coupled-mode equations for the amplitude of the electric field. The gain equations for the active rare-earth medium may be derived from the rate-equations. This leads to a possible source of confusion of amplitude and intensity gain parameters, which we here try to sort out. Adopting the notation of [2], the amplitude gain Eq. (49) and (30) of [1] and [2] reads

$$g_1 = \int_{z_l}^{z_r} \gamma |e(z)|^2 dz \quad (1)$$

where γ is the amplitude gain coefficient.

From the rate-equations, the time-derivative of the amplitude gain-coefficient of Eq. (19) in [2] reads

$$\dot{\gamma} = \tau_g^{-1} \left[-\gamma \left(1 + \frac{|E(z,t)|^2}{P_s} \right) + \gamma_t \right] \quad (2)$$

which is then equated to zero and the steady-state gain $\hat{\gamma} = \gamma$ is subsequently substituted into (1). The threshold gain γ_t is found from the inversion and gain equation to be [2, eq. (21c)], which reads

$$\gamma_t = \frac{-a_l + \frac{g_l a_p - a_l g_p}{g_p + a_p} P_p / P_t}{1 + P_p / P_t}. \quad (3)$$

Here, we note that the absorption and gain parameters $a_{p,l}$ and $g_{p,l}$ are amplitude parameters and that the excited-state lifetime τ_{sp} in P_t is an amplitude lifetime, i.e., in the case of erbium $\tau_{sp} \approx 20$ ms. So far, all

equations are in amplitude coefficients for consistency. However, for this to be true, we find that the gain lifetime τ_g must be defined as

$$\tau_g = \frac{\tau_{sp}}{2 \left(1 + \frac{P_p}{P_t} \right)} \quad (4)$$

which is a factor 1/2 from [2, eq. (21b)]. This is found from the rate equations leading to (2) by identifying τ_g^{-1} as

$$\tau_g^{-1} = P_a + P_e + \frac{2}{\tau_{sp}} = \frac{2a_p P_p}{\hbar \omega_p \rho} + \frac{2g_p P_p}{\hbar \omega_p \rho} + \frac{2}{\tau_{sp}} \quad (5)$$

where P_a and P_e are the rates of pump absorption and emission, respectively, $a_p = (1/2)\sigma_a \rho \Gamma_p$, and $g_p = (1/2)\sigma_e \rho \Gamma_p$. Here, ρ is the density of rare-earth ions, Γ_p is the confinement factor, and $\sigma_{a,e}$ are the absorption and emission cross sections. It should also be noted here that, including pump emission, P_t should read

$$P_t = \frac{P_{sp} \omega_p}{(a_p + g_p) \omega_l}. \quad (6)$$

Also, in the equation for the pump power absorption defined in [2] as

$$P_p(z, t) = P_p(z_l) e^{-\int_{z_l}^{z_r} \alpha(z', t) dz'} \quad (7)$$

the power absorption coefficient should read

$$\alpha(z, t) = 2 \left(-g_p + \frac{(g_p + a_p)}{(g_l + a_l)} (g_l - \gamma(z, t)) \right) \quad (8)$$

using amplitude coefficients.

Gain and absorption parameters are often quoted in intensity parameters, which means that using $g = \sigma_e \Gamma_c \rho$ and $a = \sigma_a \Gamma_c \rho$ requires that the threshold amplitude gain is corrected to

$$\gamma_t = \frac{1 - a_l + \frac{g_l a_p - a_l g_p}{g_p + a_p} P_p / P_t}{1 + P_p / P_t} \quad (9)$$

where τ_{sp} in P_t is now the usual excited-state lifetime of approximately 10 ms for an erbium ion. Then, all other equations are correct including the gain-lifetime in [2, eq. (21b)]. However, it must be emphasized that, for the remainder of this text, all coefficients are amplitude coefficients.

With the above in mind, we have compared the analytical model for the steady-state DFB fiber laser to numerical simulations which also featured spatial hole burning [3]. The comparison is shown in Fig. 1 for a symmetric π -phase-shifted DFB fiber laser with low pump absorption. The similarity is convincing.

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Fig. 1. Output power as a function of grating strength for a background intensity attenuation of $a_0 = 0$ and 40 dB/km of a symmetric π -phase shifted DFB laser with low pump absorption. Solid curve is analytical model calculations and squares are numerical calculations.

II. PUMP-INDUCED NOISE

This takes the offset in [2, eqs. (67)–(69)], which we state here for convenience

$$\dot{\gamma} = \tau_g^{-1} \left[-\gamma \left(1 + \frac{|E|^2}{P_s} \right) + \gamma_t \right] + q \tau_g^{-1} \Delta_p \quad (10)$$

where

$$q = \frac{g_l - \hat{\gamma}}{(P_t + P_p)}. \quad (11)$$

Inserting this into the linearized gain equation [2, eq. (39b)] gives

$$\begin{aligned} \dot{\gamma} &= \frac{1}{\tau_g} \left[-(\hat{\gamma} + \eta) \left(1 + \frac{\hat{A}^2 (1 + 2\xi) |e|^2}{P_s} \right) + \gamma_t \right] + \frac{q \Delta_p}{\tau_g} \\ &= \frac{1}{\tau_g} \left[-\eta \left(1 + \frac{\hat{A}^2 |e|^2}{P_s} \right) - \frac{2 \hat{A}^2 \hat{\gamma} |e|^2 \xi}{P_s} \right] + \frac{q \Delta_p}{\tau_g} \end{aligned} \quad (12)$$

where the first term derives identically to the derivations made in [2, eqs. (39b) and (40)]. Concentrating on the last term of (12), it is also Fourier transformed, and η is isolated on the left side of the equation to become

$$\eta = \frac{q \Delta_p}{i \nu \tau_g + 1 + \frac{A^2 |e(z)|^2}{P_s}} \quad (\text{last term}) \quad (13)$$

where η signifies the last term of (12). Using partial fraction decomposition and, for simplicity, assuming that $P_p \gg P_t$ such that $\gamma_t = g_l$ and assuming that $g_p = 0$, it becomes

$$\eta = \frac{g_l \Delta_p}{P_p i \tau_g \nu} \left(\frac{1}{1 + \frac{Z}{1 + i \tau_g \nu}} - \frac{1}{1 + Z} \right) \quad (14)$$

where $Z = A^2 |e|^2 / P_s$. This can now be multiplied by $|e(z)|^2$ and integrated to give

$$\begin{aligned} h(\nu) &= \frac{g_l \Delta_p}{P_p i \nu \tau_g g_1} \left[G \left(\frac{\hat{X}}{i \nu \tau_g + 1} \right) - G(\hat{X}) \right] \\ &= \frac{\Delta_p}{P_p} \frac{\nu_0^2 \tau_c}{2 i \nu} \Gamma(\hat{X}) \end{aligned} \quad (15)$$

where $\nu_0^2 = 2c\gamma_t/\tau_g$ is the small-signal relaxation frequency, and the cavity lifetime is defined as $\tau_c = 1/cg_1$. The total matrix equation is then

$$\begin{bmatrix} i \nu & -\frac{1}{\tau_c} \\ \nu_0^2 \tau_c \Gamma & i \nu \end{bmatrix} \begin{bmatrix} \xi \\ h \end{bmatrix} = \frac{\Delta_p}{P_p} \frac{\nu_0^2 \tau_c}{2} \Gamma \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (16)$$

which is seen to differ from [2, eq. (70)]. The extra pole found in [2] at $\nu = i \tau_g^{-1}$ in [2, eqs. (73) and (74)] is thus not found. The power fluctuation spectrum then becomes

$$S_P(\nu) = \frac{|\Delta_p(\nu)|^2}{P_p^2} \frac{\nu_0^4 \Gamma^2(\nu)}{|\nu^2 - \nu_0^2 \Gamma(\nu)|^2} P_{\text{out}}^2 \quad (17)$$

from which we note that the dc asymptotic of the relative noise becomes

$$\lim_{\nu \rightarrow 0} \frac{\frac{S_P(\nu)}{P_{\text{out}}^2}}{\frac{|\Delta_p(\nu)|^2}{P_p^2}} = 1 \quad (18)$$

which is reasonable for a laser with a linear (P_p, P_{out}) characteristic above threshold.

Finally, we note in [2, eq. (56)] that the optical output power is defined as if A is the real electrical field amplitude. However, this is inconsistent with the previous equations in [2], where A^2 is implicitly given in dimensions of power. We believe that the single-sided output power in the symmetric cavity laser is given by

$$\begin{aligned} P_{\text{out}}(L/2) &= |E(L/2)|^2 = \frac{2}{L_c} e^{-\kappa L} \hat{A}^2 \\ &= \gamma_c \hat{A}^2 \end{aligned} \quad (19)$$

such that ϵ_0 in [2, eq. (56)] and everywhere else it appears should be replaced by $2/c\epsilon$.

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