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Decoding Xing-Ling codes

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Abstract — This paper describes an efficient decoding method for a recent construction of good linear codes as well as an extension to the construction. Furthermore, asymptotic properties and list decoding of the codes are discussed.

I. XING-LING CODES

In [1] Xing and Ling describes a new construction of a class of linear codes (here referred to as Xing-Ling codes) which resulted in several improvements to Brouwer's table [2] of good linear codes.

A Xing-Ling code is a subfield subcode of a Reed-Solomon code over $\mathbb{F}_{q^2}$. While a Reed-Solomon code of dimension K is obtained by evaluating elements of $\mathbb{F}_{q^2}$ in all polynomials of degree at most $K - 1$, a Xing-Ling code is obtained by evaluating certain elements of $\mathbb{F}_{q^2}$ in certain polynomials of degree at most $K - 1$. The elements and polynomials are chosen in such a way that the result is a code over $\mathbb{F}_q$.

For any integer, K , let V_K denote the $\mathbb{F}_q$ -vector space spanned by all monomials and monic binomials of degree less than K which only give values in $\mathbb{F}_q$ when evaluated in elements from $\mathbb{F}_{q^2}$.

We then have the following definition of Xing-Ling codes:

Definition 1 Let $A \subseteq \mathbb{F}_q$ and $B \subseteq \mathbb{F}_{q^2} \setminus \mathbb{F}_q$ be given such that $\beta^q \notin B$ for all $\beta \in B$ and let K be given such that $V_K \neq V_{K-1}$. Then the following set is a Xing-Ling code:

$$XL(A, B, K) := \{f(A, B) \mid f \in V_K\}$$

The main parameters of Xing-Ling codes are summarized in the following theorem:

Theorem 2 ([1], Theorem 2.5, 2.6, 2.9, and 2.10) The code $XL(A, B, K)$ satisfies the following:

1. The code is a linear code over $\mathbb{F}_q$.
2. Let the number of elements of A and B be denoted by

$$n_A := |A| \quad \text{and} \quad n_B := |B|.$$

The length of the code is then $n = n_A + n_B$ and if $K - 1 = qr + s$ where $0 \leq s \leq q$ then the dimension is $k = (r(r + 1))/2 + s + 1$.

3. Let

$$z := \begin{cases} \max\{2(r - 1), r + s\} & \text{if } q \text{ is odd} \\ \max\{r - 1, s\} & \text{if } q \text{ is even.} \end{cases} \quad (1)$$

Then the minimum distance, d , satisfies $d \geq d^*$ where

$$d^* := n - \left\lfloor \frac{K - 1 + \max\{\min\{z, n_A\}, 2n_A - \delta q\}}{2} \right\rfloor$$

with $\delta = 2$ for q odd and $\delta = 1$ for q even. Notice that for q odd the first term of the max-expression is always largest since $n_A \leq q$.

¹This work was done at Department of Mathematics, DTU.

In the definition of Xing-Ling codes we have the constraints $n_A \leq q$ and $n_B \leq (q^2 - q)/2$ so the length of a Xing-Ling code is at most $q(q + 1)/2$. However, the paper [3] describes how to extend the code with one position by evaluating in the point at infinity on the projective line. This gives a few improvements to Brouwer's table.

II. DECODING

Suppose that a word $r \in \mathbb{F}_q^n$ is received. The goal is to find the polynomial $\hat{f} \in V_K$ such that $\hat{f}$ corresponds to the Xing-Ling codeword closest to r . The paper describes an efficient method that calculates $\hat{f}$ if the corresponding codeword has distance less than half the designed minimum distance from the received word. The method is sketched below.

The word r is decomposed into two blocks, $r = (r_A, r_B)$ where r_A are the received values on the A -positions — the positions corresponding to the set A — and similarly r_B are the received values on the B -positions.

If $n_A \leq z$ (with z defined in Eqn. (1)) then it turns out that it suffices to decode the word (r_A, r_B, r_B) with a suitable Reed-Solomon code over $\mathbb{F}_{q^2}$.

If $n_A > z$ then this approach fails if too many errors occurred on the B -positions. In that case the word r_A is decoded with a Reed-Solomon code over $\mathbb{F}_q$. This results in an estimate, u , of $\hat{f}(\mathbb{F}_q)$. If q is even then decoding (u, r_B, r_B) with a Reed-Solomon code gives the result. If q is odd then the decoding is done with a so-called generalized Reed-Solomon m -code which is defined in the paper.

III. ASYMPTOTIC RESULTS

Let an infinite sequence of Xing-Ling codes be constructed for alphabet sizes tending to infinity such that for each alphabet size, q , we have $n_A = 0$ and $n_B = n = (q^2 - q)/2$ and such that the information rate (k/n) tends to a constant, κ .

For $q \rightarrow \infty$ it is then shown that the designed minimum distance, d^* , satisfies

$$d^*/n \rightarrow 1 - \sqrt{\kappa}$$

and that a fraction of errors, $t/n \rightarrow \tau$, can be efficiently list decoded whenever

$$\tau < 1 - \sqrt{\sqrt{\kappa}}.$$

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