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Robustness of Linear Systems towards Multi-Dissipative Perturbations

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Abstract

We consider the question of robust stability of a linear time invariant plant subject to dynamic perturbations, which are dissipative in the sense of Willems with respect to *several* quadratic supply rates. For instance, parasitic dynamics are often both small gain *and* passive.

We reduce several robustness analysis questions to linear matrix inequalities: Robust stability, robust $\mathcal{H}_2$ performance and finally robust performance in presence of disturbances with Finite Signal-to-Noise Ratios.

1 Introduction

It is popular to deal with dynamic perturbations in control systems using the framework of dissipativity, originally introduced in [5]. Passivity and small gain are examples of dissipativity properties.

This paper considers perturbations which are dissipative with respect to *several* supply rates. Examples are parasitic dynamics, which (with proper choice of inputs and outputs) are both passive and small gain, and block diagonal perturbations, which inherit the dissipativity properties of each of the diagonal elements.[1]

For systems with multi-dissipative perturbations, we derive conditions in terms of linear matrix inequalities [1] which guarantee robust stability, robust $\mathcal{H}_2$ performance and performance in the presence of Finite-Signal-to-Noise Ratio disturbances.

Proofs, examples and discussion of the statements in this paper can be found in [4], which also contains extensions to state feedback design and some hints about extensions to non-linear systems.

2 Preliminaries

We deal with systems Σ with state-space representations:

$$\Sigma: \quad \begin{array}{rcl} \dot{x}(t) &=& Ax(t)+Bw(t) \\ z(t) &=& Cx(t)+Dw(t) \end{array} \qquad (1)$$

All signals are real finite-dimensional vector signals.

Our notion of dissipativity is identical to the original one of Willems [5], specialized to LQ systems:

[1] Similar motivation has led to the Integral Quadratic Constraint framework of Megretski and Rantzer, Savkin and Petersen, and others. See [4] for references and a comparative discussion.

Definition 1: A system Σ is said to be *dissipative* with respect to the supply rate $s(w,z) = (x'\ w')Q(x'\ w')'$ if there exist a *storage* function $V(x) = x'Px$ where $P = P' \geq 0$, such that the differential dissipation inequality $\dot{V}(x(t)) \leq s(w(t), z(t))$ holds, or equivalently:

$$\begin{bmatrix} PA + A'P & PB \\ B'P & 0 \end{bmatrix} - Q \leq 0$$

□

The following stronger notion of dissipativity is useful for robust stability analysis of LQ systems:

Definition 2: The system Σ is said to be *strongly dissipative* with respect to the supply rate s, if for some $\epsilon > 0$ it is dissipative with respect to the supply rate $s(w,z) - \epsilon\|z\|^2$. Let V be a corresponding storage function, then V is said to be a *strong storage function* for Σ w.r.t. s. □

This definition fits together with the problem of robustness towards unknown perturbations:

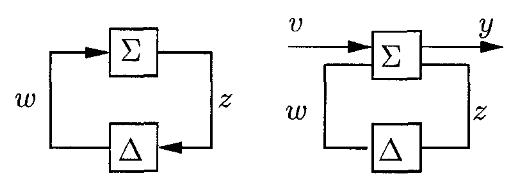

Figure 1: Setup for Robust Stability and Performance

Definition 3: Let a nominal system Σ be connected in feedback with a perturbation $\Delta \in \mathbf{\Delta}$ as in figure 1 (left). The configuration $(\Sigma, \mathbf{\Delta})$ is said to be *robustly stable*, if for any initial conditions x_0 and ξ_0 (of Σ and Δ, resp.) the signals $w(\cdot)$ and $z(\cdot)$ are well defined and $z \in \mathcal{L}_2$. □

Our stability condition then differs slightly from [5]:

Lemma 4: Let every $\Delta \in \mathbf{\Delta}$ be dissipative w.r.t. $-s$ and let Σ be strongly dissipative w.r.t. s. Then the feed-back configuration $(\Sigma, \mathbf{\Delta})$ is robustly stable. □

In this paper we consider systems Σ connected in feedback as in figure 1 with a perturbation Δ, which is dissipative w.r.t. the p supply rates $-s_i = -(x'\ w')Q_i(x'\ w')'$, $i \in \{1,\ldots,p\}$. We use the symbol $\mathbf{\Delta}$ to denote this particular class of *multi-dissipative* perturbations, i.e.

$$\mathbf{\Delta} = \{\Delta :\ \text{dissipative w.r.t.}\ -s_i;\ i \in \{1,\ldots,p\}\} \quad (2)$$

To each perturbation $\Delta \in \mathbf{\Delta}$ and each supply rate $-s_i$ corresponds a storage function W_i defined on the state space of Δ.

3 Robust stability

It was noted already in [5] that the storage functions for a dynamical system with respect to a single supply rate form a convex set. For a multi-dissipative system Δ a stronger statement holds: The set of those $(W, -s)$ for which W is a storage function for Δ w.r.t. the supply rate $-s$ is a convex cone. In particular, $\Delta \in \mathbf{\Delta}$ is dissipative w.r.t. any supply rate in the convex cone generated by $-s_i$, $i = 1, \ldots, p$. Combining with lemma 4 yields:

Theorem 5: Given $(\Sigma, \mathbf{\Delta})$ as in (1) and (2). If the following LMI has a solution $P \geq 0, d \geq 0, \epsilon > 0$:

$$\begin{bmatrix} PA + A'P & PB \\ B'P & 0 \end{bmatrix} - \sum_{i=1}^{p} d_i Q_i + \epsilon \begin{bmatrix} C' \\ D' \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \leq 0$$

then the configuration $(\Sigma, \mathbf{\Delta})$ is robustly stable. □

4 Guaranteed $\mathcal{H}_2$ Performance

We now expand the system with a noise input $v(t)$ and a performance output $y(t)$, c.f. figure 1:

$$\Sigma : \begin{array}{rcl} \dot{x} & = & Ax + Bw + Gv \\ z & = & Cx + Dw \\ y & = & Hx + Jw \end{array}$$

We seek a bound on the $\mathcal{H}_2$-norm[2] of the unknown closed-loop system $\mathcal{F}_l(\Sigma, \Delta)$ from v to y:

Theorem 6: The following bound holds:

$$||\mathcal{F}_l(\Sigma, \Delta)||^2_{\mathcal{H}_2} \leq \inf_{V, d_i, \epsilon} \text{trace } G'VG$$

where the infimization is subject to

$$\begin{bmatrix} VA + A'V & VB \\ B'V & 0 \end{bmatrix} - \sum_{i=1}^{p} d_i Q_i$$

$$+ \begin{bmatrix} H' \\ J' \end{bmatrix} \begin{bmatrix} H & J \end{bmatrix} + \epsilon \begin{bmatrix} C' \\ D' \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \leq 0$$

as well as $V = V' \geq 0,\ d_i \geq 0,\ \epsilon > 0$. □

5 Finite Signal-to-Noise Ratios

In this section we examine the FSN system[3] in figure 2 where $\Delta \in \mathbf{\Delta}$ and v_1 is an exogenous white noise signal with intensity 1. The new component Δ_F is a *noise generator* with a signal-to-noise ratio of 1, i.e. v_2 is a white noise signal with intensity equal to the variance of y_2. The system model is

$$\Sigma : \begin{array}{rcl} \dot{x} & = & Ax + Bw + G_1 v_1 + G_2 v_2 \\ z & = & Cx + Dw \\ y_i & = & H_i x + J_i w \end{array}$$

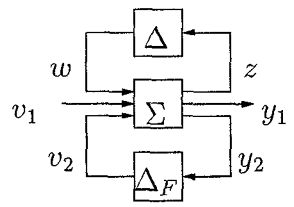

Figure 2: Including an FSN noise generator

We use $V(x) = x'Px$ as a candidate for a stochastic storage function and obtain:

Theorem 7: The bound on the variance of y_1 holds:

$$\mathcal{E}\{\frac{1}{T}\int_0^T |y_1|^2 \, dt | 0\} \leq \text{tr}(G_1' P G_1)$$

provided $P = P' \geq 0$, $d \geq 0$ and $\epsilon > 0$ satisfy

$$\begin{bmatrix} PA + A'P & PB \\ B'P & 0 \end{bmatrix} - \sum_i d_i Q_i + \epsilon \begin{bmatrix} C' \\ D' \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix}$$

$$+ \text{tr}(G_2' P G_2) \begin{bmatrix} C_2' \\ D_2' \end{bmatrix} \begin{bmatrix} C_2 & D_2 \end{bmatrix} + \begin{bmatrix} C_1' \\ D_1' \end{bmatrix} \begin{bmatrix} C_1 & D_1 \end{bmatrix} \leq 0$$

Minimization of this bound is a linear matrix inequality problem. □

Generalizations to systems with several FSN noise generators are immediate and presented in [4].

[2] When Δ is not linear time invariant, the result holds [4] whether we interpret $\mathcal{H}_2$ norm as a bound on the impulse response or on the steady-state variance. A related discussion can be found in [3].

[3] See [2] for an introduction to FSN systems. A related subject is systems with multiplicative noise, see [1].

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