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Robust $\mathcal{H}_2$ Performance for Sampled-data Systems

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Abstract

Robust $\mathcal{H}_2$ performance conditions under structured uncertainty, analogous to well known methods for $\mathcal{H}_\infty$ performance, have recently emerged in both discrete and continuous-time. This paper considers the extension into uncertain sampled-data (SD) systems, taking into account inter-sample behavior. Convex conditions for robust $\mathcal{H}_2$ performance are derived for different uncertainty sets.

1 Introduction & Background

In Dullerud [Du195], a thorough study of robustness analysis for sampled-data systems was undertaken. In the configuration of Figure 1, $G$ is a continuous time linear time-invariant (LTI) system, controlled by a discrete-time LTI controller $K_d$ by means of ideal sample and hold devices with synchronized period $h$. This makes the nominal closed loop map $M$ (from $(p, u)$ to $(q, z)$) periodically time varying (PTV), instead of LTI as is usual in robust control. The system is affected by dynamic uncertainty $\Delta$, which has spatial structure and can be LTI, PTV, or arbitrarily time-varying (LTV). Methods for robust stability and $\mathcal{H}_\infty$ performance evaluation were studied in [Du195], extending the standard theory for continuous or discrete-time systems.

In this paper we consider the question of robust $\mathcal{H}_2$ performance for sampled data systems, following recent results in [Pag96a, Pag96b] in the standard case, which closely resemble the robust $\mathcal{H}_\infty$ theory. For sampled data systems, we extend these conditions for both PTV and LTV perturbations.

1.1 Lifting

For a general introduction see [CF95] and references therein. The Laplace, Lift and the $\Lambda$ (or $Z$) trans-
Going to the \( \Lambda \) domain, this is equivalent to the form given in (3), where \( \| \cdot \|_{HS} \) is the Hilbert-Schmidt norm of an operator on \( L_2[0; h] \), see [BJ92, CF95].

\[
\| \hat{T} \|_{H_2} = \frac{1}{2\pi} \int_0^{2\pi} \| \hat{T}(e^{i\theta}) \|_{HS}^2 d\theta \quad (3)
\]

For arbitrary LTV systems (2) may be generalized by taking the limit as \( h \to \infty \).

2 Robust \( H_2 \) Performance SD for TV uncertainty

The sets of full block structured LTV and PTV perturbations (of period \( h \)) are given by

\[
\begin{align*}
\Delta_{LTV} & \triangleq \{ \Delta = \text{diag}(\Delta_1, \ldots, \Delta_F) : \Delta_k \in \mathcal{L}(L_2^m) \} \\
\Delta_{PTV} & \triangleq \{ \Delta \in \Delta_{LTV} : D_h \Delta = \Delta D_h \} \\
T_{zw}(\Delta) & \text{ denotes the map from } w \text{ to } z \text{ (see Fig. 1).}
\end{align*}
\]

2.1 PTV perturbation case

Let \( \Delta \in \Delta_{PTV} \). At each \( \lambda = e^{i\theta} \), we introduce a scaling which commutes with \( \Delta(e^{i\theta}) \), \( X(\theta) \in \mathcal{X} \triangleq \{ X = \text{diag}(x_1 I_{m_1}, \ldots, x_F I_{m_F}) \} \) (constant matrix multiplication operator on \( L_2[0; h] \)).

Condition 1 There exists functions \( X(\theta) \in \mathcal{X} \) and \( Y(\theta) \in \mathcal{L}(L_2[0; h]) \), such that

\[
M(e^{i\theta})^* \begin{bmatrix} X(\theta) & 0 \\ 0 & Y(\theta) \end{bmatrix} M(e^{i\theta}) - \begin{bmatrix} X(\theta) & 0 \\ 0 & Y(\theta) \end{bmatrix} < 0 \quad (4)
\]

for all \( \theta \in [0; 2\pi] \) and

\[
\int_0^{2\pi} \text{trace} \ Y(\theta) \frac{d\theta}{2\pi} = \int_0^{2\pi} \int_0^h \text{trace} \ Y_h(t, t) dt d\theta \quad < 1. \quad (5)
\]

Remark 1 In (5) we use the trace of an operator \( Y \in \mathcal{L}(L_2[0; h]) \); this is defined as

\[
\text{tr} \ Y \triangleq \sum_{i=1}^{\infty} \langle Y b_i, b_i \rangle = \int_0^h \text{trace} \ Y(t, t) dt,
\]

where \( b_i \) is any orthonormal basis of \( L_2[0; h] \), and \( Y(t, t) \) is the kernel representation of \( Y \).

Proposition 1 If Condition 1 holds and \( \Delta \in B_{\Delta_{PTV}} \), then the system is robustly stable and

\[
\sup_{\Delta \in B_{\Delta_{PTV}}} \| T_{zw}(\Delta) \|_{H_2} < 1. \quad (7)
\]

Proof: See [RP97].

Remark 2 This sufficient condition is convex in the unknowns \( X(\theta), Y(\theta) \). The "frequency" and "time" dependence of \( M \) is reflected in \( Y_h(t, t) \in \mathbb{C}^{n \times m} \). A finite dimensional approximation can be obtained by gridding. Clearly, this condition also holds for LTI perturbations, however, the LTI behaviour can be further exploited (see [RP97]).

2.2 LTV perturbation case

Proposition 2 If Condition 1 holds for a constant function \( X(\theta) \equiv X \in \mathcal{X} \), and \( \Delta \in B_{\Delta_{LTV}} \), then the uncertain system is robustly stable and

\[
\sup_{\Delta \in B_{\Delta_{LTV}}} \| T_{zw}(\Delta) \|_{H_2} < 1. \quad (8)
\]

3 Conclusion and further directions

Conditions for robust \( H_2 \) performance for sampled-data systems have been derived under time-varying uncertainty (PTV or arbitrary LTV). Only sufficiency was shown; it is expected that necessity results will follow if one adopts the notion of \( H_2 \) performance in [Pag96a, Pag96b], and replaces PTV uncertainty by a "quasi-PTV" notion (see [Du95]). Further work includes state-space computations for these conditions, and more refined conditions for the case of purely LTI uncertainty.

References


[Pag96a] F. Paganini. Robust \( H_2 \) performance for continuous time systems. LIDS-P-2342, 1996.
