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Robust $\mathcal{H}_2$ Performance for Sampled-data Systems

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Abstract

Robust $\mathcal{H}_2$ performance conditions under structured uncertainty, analogous to well known methods for $\mathcal{H}_\infty$ performance, have recently emerged in both discrete and continuous-time. This paper considers the extension into uncertain sampled-data (SD) systems, taking into account inter-sample behavior. Convex conditions for robust $\mathcal{H}_2$ performance are derived for different uncertainty sets.

1 Introduction & Background

In Dullerud [Du95], a thorough study of robustness analysis for sampled-data systems was undertaken. In the configuration of Figure 1, $G$ is a continuous time linear time-invariant (LTI) system, controlled by a discrete-time LTI controller $K_d$ by means of ideal sample and hold devices with synchronized period $h$. This makes the nominal closed loop map $M$ (from $(p, u)$ to $(q, z)$) periodically time varying (PTV), instead of LTI as is usual in robust control. The system is affected by dynamic uncertainty $\Delta$, which has spatial structure and can be LTI, PTV, or arbitrarily time-varying (LTV). Methods for robust stability and $\mathcal{H}_2$ performance evaluation were studied in [Du95], extending the standard theory for continuous or discrete time systems.

In this paper we consider the question of robust $\mathcal{H}_2$ performance for sampled data systems, following recent results in [Pag96a, Pag96b] in the standard case, which closely resemble the robust $\mathcal{H}_\infty$ theory. For sampled data systems, we extend these conditions for both PTV and LTV perturbations.

1.1 Lifting

For a general introduction see [CF95] and references therein. The Laplace, Lift and the $\Lambda$ (or $Z$) transforms between the signal spaces are related as seen below, we use the same accents (from [Du95]) for operators mapping within the domains.

$$\begin{align*}
\hat{f}(\lambda) & \leftarrow \Lambda \hat{f}(k) \in \mathcal{L}_2 \triangleq \mathcal{L}_2[0; h]
\end{align*}$$

The lifting technique converts the PTV operator $M$ to LTI in the lifted domain. In the $\Lambda$-domain, it amounts to the operator $f(\lambda) \mapsto \hat{M}(\lambda)f(\lambda)$, where at each $\lambda$ in the unit disk, $\hat{M}(\lambda)$ is an operator on $\mathcal{L}_2[0; h]$.

1.2 $\mathcal{H}_2$ Performance for TV systems

For LTI systems the $\mathcal{H}_2$ norm is given by

$$
||T||_{\mathcal{H}_2}^2 \triangleq \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{trace}(\hat{T}(j\omega)^* \hat{T}(j\omega))d\omega = \sum_{i=1}^{n} ||T\delta e_i||_{\mathcal{L}_2}^2
$$

where $T\delta e_i$ is the impulse response for the $i$-th input channel. For TV systems, this impulse response varies in time; one possible generalization of the $\mathcal{H}_2$ norm used in [BJ92] (a different one is given in [Pag96b]) is to average over time. For PTV systems, we average over the period:

$$
||T||_{\mathcal{H}_2} = \frac{1}{h} \int_{0}^{h} \sum_{i=1}^{n} ||T\delta e_i||_{\mathcal{L}_2}^2 d\tau
$$
Going to the $A$ domain, this is equivalent to the form given in (3), where $||A||_{HS}$ is the Hilbert-Schmidt norm of an operator on $L^2[0; h]$, see [BJ92, CF95].

$$||\hat{T}||^2_{HS} = \frac{1}{2\pi} \int_0^{2\pi} ||(e^{i\theta})||^2_{HS} d\theta$$  \hspace{1cm} (3)

For arbitrary LTV systems (2) may be generalized by taking the limit as $h \to \infty$.

## 2 Robust $H_2$ Performance SD for TV uncertainty

The sets of full block structured LTV and PTV perturbations (of period $h$) are given by

$$\Delta_{LTV} = \{ \Delta = \text{diag}(\Delta_1, \ldots, \Delta_F) : \Delta_k \in \mathfrak{L}(L^2[0; h]) \}$$

$$\Delta_{PTV} = \{ \Delta \in \Delta_{LTV} : D_h \Delta = \Delta D_h \}$$

$T_{zw}(\Delta)$ denotes the map from $w$ to $z$ (see Fig. 1).

### 2.1 PTV perturbation case

Let $\Delta \in \Delta_{PTV}$. At each $\lambda = e^{i\theta}$, we introduce a scaling which commutes with $\Delta(e^{i\theta})$, $X(\theta) \in X \triangleq \{ X = \text{diag}(x_1, x_2, \ldots), x_k \geq 0 \}$. We define a matrix multiplication operator on $\mathfrak{L}(L^2[0; h])$.

**Condition 1** There exists functions $X(\theta) \in X$ and $Y(\theta) \in \mathfrak{L}(L^2[0; h])$, such that

$$\tilde{M}(e^{i\theta}) \begin{bmatrix} X(\theta) & 0 \\ 0 & 0 \end{bmatrix} \tilde{M}(e^{i\theta}) - \begin{bmatrix} X(\theta) & 0 \\ 0 & Y(\theta) \end{bmatrix} < 0$$  \hspace{1cm} (4)

for all $\theta \in [0; 2\pi]$ and

$$\int_0^{2\pi} \text{tr} Y(\theta) d\theta = \int_0^{2\pi} \int_0^h \text{tr} Y(t, t) dt d\theta < 2\pi \int_0^h \text{tr} Y(t, t) dt$$  \hspace{1cm} (5)

**Remark 1** In (5) we use the trace of an operator $Y \in \mathfrak{L}(L^2[0; h])$, this is defined as

$$\text{tr} Y = \sum_{i=1}^{\infty} \langle Y b_i, b_i \rangle = \int_0^h \text{tr} Y(t, t) dt$$  \hspace{1cm} (6)

where $b_i$ is any orthonormal basis of $L^2[0; h]$, and $Y(t, t)$ is the kernel representation of $Y$.

**Proposition 1** If Condition 1 holds and $\Delta \in B_{\Delta_{PTV}}$, then the system is robustly stable and

$$\sup_{\Delta \in B_{\Delta_{PTV}}} \| T_{zw}(\Delta) \|_{H_2} < 1.$$  \hspace{1cm} (7)

### 2.2 LTV perturbation case

**Proposition 2** If Condition 1 holds for a constant function $X(\theta) \equiv X \in X$, and $\Delta \in B_{\Delta_{LTV}}$, then the uncertain system is robustly stable and

$$\sup_{\Delta \in B_{\Delta_{LTV}}} \| T_{zw}(\Delta) \|_{H_2} < 1.$$  \hspace{1cm} (8)

## 3 Conclusion and further directions

Conditions for robust $H_2$ performance for sampled-data systems have been derived under time-varying uncertainty (PTV or arbitrary LTV). Only sufficiency was shown; it is expected that necessity results will follow if one adopts the notion of $H_2$ performance in [Pag96a, Pag96b], and replaces PTV uncertainty by a “quasi-PTV” notion (see [Du95]). Further work includes state-space computations for these conditions, and more refined conditions for the case of purely LTI uncertainty.

## References


