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Iterative Controller Tuning for Processes with Fold Bifurcations

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Abstract

Processes involving fold bifurcation are notoriously difficult to control in the vicinity of the fold where most often optimal productivity is achieved . In cases with limited process insight a model based control synthesis is not possible. This paper uses a data driven approach with an improved version of iterative feedback tuning to optimizing a closed loop performance criterion, as a systematic tool for tuning process with fold bifurcations.

Keywords: Iterative feedback tuning, PID tuning, Fold bifurcations

1. Introduction

Optimal process control requires a controller that is synthesized based on a performance criterion. In order to minimize such a criterion a model for the process is normally required. For a number of processes it is difficult at best to obtain an accurate model that approximates the static and dynamic behavior, especially if the process exhibits nonlinear behaviors such as a bifurcation or other forms of inherent limitations. Optimizing productivity in continuous processes with fold bifurcations often lead to a desired operating point near a fold. A classic example is cultivation of Saccharomyces cerevisiae for biomass production where the optimal production rate of biomass is close to the critical dilution rate. This process is described by a complex process model [1] as often is the case in biotechnology. Operation in the vicinity of the fold requires a tight control in order to prevent a significant loss in productivity.

The problem of tuning controllers to operation close to a fold bifurcation, based on limited a priory knowledge of the process model, is being investigated in this paper. Closed loop identification techniques are used in order gain process knowledge on the actual loop behavior rather that the process, since it is the performance of the loop that is subject to optimization. Closed loop controller tuning is an iterative procedure where successive steps of closed loop experiments and model estimation/controller parameter updates is conducted since the control in a given loop will affect the quality of the measurements collected on the loop [2]. When tuning a loop on a process exhibiting a fold, it can be necessary to initiate the optimization far from the fold and then move stepwise closer as performance increases. In case a process model is not known then the first controller can be synthesized based on a model estimated from an open loop experiment which inherently has to take place far from the fold. This paper provides an improved methodology for iterative feedback tuning where perturbations are used to increase the convergence rate.

2. Iterative Feedback Tuning (IFT)

The basic idea of the iterative performance enhancement method, is to formulate a cost function and use an optimization algorithm to minimize this cost function with respect to the controller parameters using a Gauss-Newton algorithm. Evaluations of the partial derivatives of the cost function with respect to the controller parameters, $\boldsymbol{\rho}$, are based on measurements taken form the closed loop system. The basic methodology was presented in [3] and have since been extended and tested in a number of papers [4].

Given a classic quadratic cost function with penalty on deviations of the controlled variable from a desired trajectory and on the manipulated variable or its increment, the condition for a local minimizer requires the gradient of the cost function with respect to the controller parameters to be zero. The gradient of the cost function can be evaluated given knowledge of the in- and outputs and there gradients with respect to the controller parameters. For a two degree of freedom control loop where C_r works on the reference and C_y acts on the feedback the gradients of the in- and outputs are given as

$$\begin{aligned} \frac{\partial y}{\partial \rho} &= \frac{1}{C_r(\rho)}\frac{\partial C_r}{\partial \rho}T(\rho)r - \frac{1}{C_r(\rho)}\frac{\partial C_y}{\partial \rho}T(\rho)y \\ \frac{\partial u}{\partial \rho} &= \frac{\partial C_r}{\partial \rho}S(\rho)r - \frac{\partial C_y}{\partial \rho}S(\rho)y \end{aligned} \tag{1}$$

By performing the following three experiments, estimates of the gradients of the input and output can be achieved [3].

- $\boldsymbol{r}^1 = \boldsymbol{r}$ i.e. the reference in the first experiment is the same as for normal operation of the process.
- $\boldsymbol{r}^2 = \boldsymbol{y}^1$ i.e. the reference in the second experiment is the output from the first experiment
- $\boldsymbol{r}^3 = \boldsymbol{r}$ i.e. the reference in the third experiment is the same as for normal operation of the process just as in the first experiment.

A convenient property of the IFT method is that only the noise in the second and third experiment act as a nuisance, while the noise in the first experiment plays an active part in the minimization [3]. In cases where IFT is used to tune for noise rejection it is only the noise in the first experiment that drives the optimization. This can lead to very slow convergence. In such cases it can be advantageous to add an external perturbation signal on the reference in the first experiment. On a nonlinear application the IFT method will produce a first order approximation of the gradients and has therefore reasonably good properties close to the point of linearization [4].

3. Case study

In this case study a simple model for a continuous fermentation with Haldane kinetics exhibiting fold bifurcations is used to illustrate the advantages of the proposed methodology. The process is given by the following state space system

$$\begin{aligned} \frac{dX}{dt} &= \mu_{\max} \frac{XS}{k_2 S^2 + S + k_1} - X \frac{F_{sub} + F_{water}}{V} \\ \frac{dS}{dt} &= -\frac{\mu_{\max}}{Y} \frac{XS}{k_2 S^2 + S + k_1} + S_F \frac{F_{sub}}{V} - S \frac{F_{sub} + F_{water}}{V} \end{aligned} \qquad (2)$$

Where X and S is the biomass and substrate concentration respectively. The reactor volume, V, is 5 L, the concentration of substrate, S_F, in the substrate feed flow, F_{sub}, is 20 g/L. The process is operated with a constant dilution rate = F_{tot}/V. The kinetic model parameters are μ_{max} = 1 h^{-1}, Y = 0.5, k_1 = 0.03 g/L and k_2 = 0.5 L/g. The substrate feed concentration is disturbed by low pass filtered noise with variance of 0.68 and step changes of with a magnitude of 1 g/L. The cut off frequency of the second order filter is one fourth of the sample time. The process is controlled by adjusting the fraction of the substrate flow rate with respect to the total flow rate. Given the manipulated variable, u, the two flow rates of the substrate and water are given by

$$F_{sub} = \frac{F_{tot}}{1 + \exp(-0.25u)}, \quad F_{water} = F_{tot} - F_{sub} \qquad (3)$$

For u=0 the steady state behavior for the process has been investigated and the solutions for the biomass are shown in figure 1 where the fold are located at a dilution rate of 0.8032 h^{-1}.

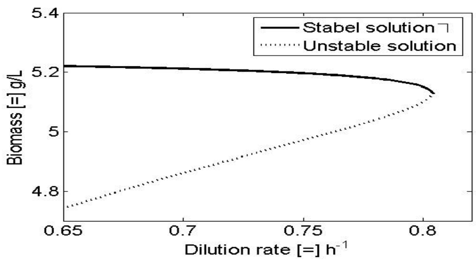


Figure 1 Steady state solution for the continuous fermentation model near optimal biomass productivity.

It is desired to operate the process at maximum productivity which is at a dilution rate of 0.8019 h^{-1}. Here a dilution rate of 0.795 h^{-1} is the aim, which will require a controller that can reject both the noise and occasionally step disturbances. For a dilution rate at 0.7 h^{-1} it is possible to operate the system in open loop and an ARMAX model of the process has been estimated. Given this model a PID controller has been designed using Internal Model Control tuning rules. The controller is implemented as a two degree of freedom controller with PID action in the feedback loop with derivative filter and PI action on the reference. Further more the parameters in a first order filter have been optimized such that the determinant of the Hessian of the cost function with respect to the controller parameters are maximized under the constraint that the cost may not exceed 10^{-4} for the following cost function

$$Cost = \frac{1}{2N} \mathrm{E}\left[\sum_{t=2}^{N} (\mathrm{y}_t(\rho) - y_{d,t})^2 + 0.01 \sum_{t=2}^{\mathrm{N}} (u_t(\rho) - u_{t-1}(\rho))^2 \right] \tag{4}$$

The system is simulated for 24 hours in closed loop, sampled every five minutes. With this initial control 100 Monte Carlo simulations shows that the process can be kept around the upper branch in the bifurcation diagram for a dilution rate up till 0.792 h^{-1} with a Cost = 0.000226. At higher values the combination of a negative step and noise disturbances can make the system shift to the lower branch corresponding to wash out of the biomass.

An initial pre tuning at a dilution rate of 0.7 h^{-1} is performed using the IFT method using the estimated filter to generate a signal that perturbs the process rendering faster convergence. The results are presented in table 1 as the controller parameters and the value of the cost function of one experiment of the system under normal operational conditions. A rapid decrease in the cost is observed and the pre tuning is stopped after the fourth iteration, were it is apparent that no further improvement is occurring. Simulations with the initial and the last controller are shown in figure 2(a). It was observed that pre tuning without perturbing the process, requires approximately three times as many iterations in order to achieve the same performance improvement.

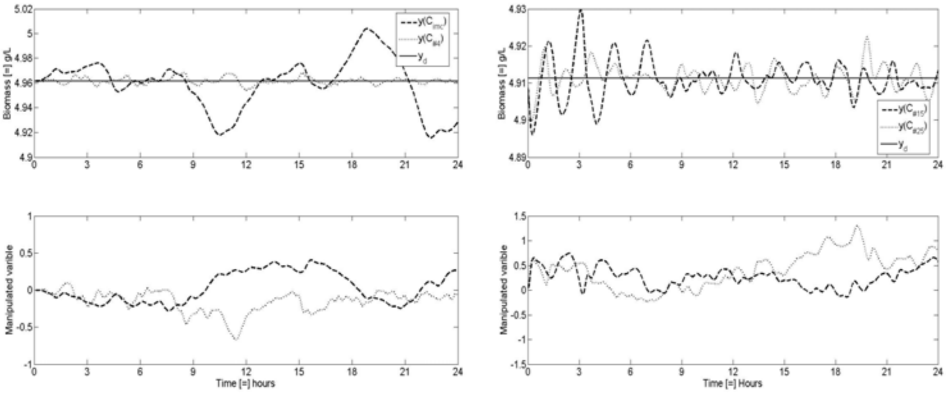
Table 1 Controller parameters for four iterations with the IFT method at dilution rate 0.7 h^{-1} and the cost for each controller for an experiment with out external perturbation.

Controller	Kc	τ_I	τ_D	$Cost \cdot 10^5$
Initial (C_{imc})	2.22	0.367	2.06	11.1
IFT $C_{\#1}$	2.255	0.2138	3.412	5.26
IFT $C_{\#2}$	2.532	0.0633	4.520	1.00
IFT $C_{\#3}$	2.260	0.0332	5.590	0.635
IFT $C_{\#4}$	2.753	0.0317	3.725	0.738

In order to tune the process for operation closer to the fold a new set of iterations are conducted at a dilution rate of 0.79. With the enhanced controller it is now possible to perform experiments with external perturbation at this high dilution rate if the gain in the filter is reduced to one fourth of the original value. Having reduced perturbations of the system makes the convergence much slower. How many iterations that has to be conducted at this point of operation is more complicated to judge. The results from up to 20 iterations are shown in table 3. 20 iterations correspond to 60 experiments which is naturally an undesired high number. Two set of 100 Monte Carlo simulations are conducted for the controllers at dilution rate 0.795 which corresponds to the desired point of operation. The first series is only affected by noise but in the second a negative step disturbances is included. The results given in table 2 shows that the performance of the loop is enhanced trough out the iterations and that it requires 15 or more iterations in this specific case to obtain a controller which is robust with respect to both the process noise and a step disturbance when the system is operated at the desired dilution rate. Simulations of the step and noise disturbance for the 15'th and 20'th controller are shown in figure 2(b).

Table 2 Controller parameters for every fifth of 20 iterations with the IFT method at dilution rate 0.79 h^{-1} and the cost for each controller for an experiment without external perturbation. The average cost for two sets of 100 Monte Carlo simulations at dilution rate 0.795 h^{-1} with noise and noise plus a negative step change respectively are presented.

Controller	Kc	τ_I	τ_D	$Cost \cdot 10^5$	$MC_1 \cdot 10^5$	$MC_2 \cdot 10^5$
Initial	2.753	0.0317	3.725	1.374	2.39	-
IFT $C_{\#5}$	3.801	0.0394	1.934	1.366	1.35	-
IFT $C_{\#10}$	5.508	0.0543	1.069	0.917	1.16	-
IFT $C_{\#15}$	6.845	0.0727	0.955	0.930	0.930	1.51
IFT $C_{\#20}$	11.04	0.0970	0.552	0.646	0.804	1.37



(a) Noisy simulation of the closed loop system at dilution rate 0.7 h^{-1} with the initial IMC controller and the fourth iterate from the IFT tuning method.

(b) Noisy simulation with an initial negative step disturbance to the closed loop system at dilution rate 0.795 h^{-1} with the iterate no. 15 and 25 from the IFT tuning method.

Figure 2 Closed loop simulations of the system with controllers from the IFT tuning method. The measured biomass and the manipulated variable are shown for the simulations together with the step point y_d.

4. Conclusions

In absence of a reliable process model IFT tuning have been shown useful for tuning controllers for a nonlinear system containing a fold bifurcation. The method utilize that a sluggish controller is sufficient far from the critical point and moves closer to the optimal operation point as the loop performance is enhanced. The number of necessary iterations with the IFT method will depend on the process, the desired operation point and the desired performance and whether external perturbation of the system is possible.

5. References

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