Performance Analysis of a Decoding Algorithm for Algebraic Geometry Codes

Jensen, Helge Elbrønd; Nielsen, Rasmus Refslund; Høholdt, Tom

Published in:
Information Theory, 1998. Proceedings. 1998 IEEE International Symposium on

Link to article, DOI:
[10.1109/ISIT.1998.708983](https://doi.org/10.1109/ISIT.1998.708983)

Publication date:
1998

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Jensen, H. E., Nielsen, R. R., & Høholdt, T. (1998). Performance Analysis of a Decoding Algorithm for Algebraic Geometry Codes. In *Information Theory, 1998. Proceedings. 1998 IEEE International Symposium on IEEE*.
<https://doi.org/10.1109/ISIT.1998.708983>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Performance Analysis of a Decoding Algorithm for Algebraic Geometry Codes

H.Elbrønd Jensen Dept. of Mathematics Technical University of Denmark Bldg.303 DK-2800 Lyngby Denmark Email h.elbrond.jensen@mat.dtu.dk	R.Refslund Nielsen Dept. of Mathematics Technical University of Denmark Bldg.303 DK-2800 Lyngby Denmark Email stud-rrn@mat.dtu.dk	T.Høholdt Dept. of Mathematics Technical University of Denmark Bldg.303 DK-2800 Lyngby Denmark Email tom@mat.dtu.dk
--	--	--

Abstract — We analyse the known decoding algorithms for algebraic geometry codes in the case where the number of errors is greater than or equal to $\lfloor (d_{FR} - 1)/2 \rfloor + 1$, where d_{FR} is the Feng-Rao distance.

I. INTRODUCTION

The fast decoding algorithm for one-point algebraic geometry codes of Sakata, Elbrønd Jensen, and Høholdt [1] decodes any error pattern of weight up to $\lfloor (d_{FR} - 1)/2 \rfloor$ where d_{FR} is the so-called Feng-Rao distance of the code. In this paper we analyse the performance of the decoding algorithm, when the number of errors is greater than or equal to $\lfloor (d_{FR} - 1)/2 \rfloor + 1$.

II. THE CODES AND THE DECODING ALGORITHM

Let $P_1, P_2, \dots, P_n, Q$ be F_q -rational points on a nonsingular absolutely irreducible curve χ of genus g defined over F_q . We consider an algebraic geometry code C_m of type $C_L(D, G)^\perp = C_\Omega(D, G)$, where $D = P_1 + P_2 + \dots + P_n$ and $G = mQ$.

If $f \in R$ and $\underline{y} \in F_q^n$ we define the syndrome $S_{\underline{y}}(f)$ to be

$$S_{\underline{y}}(f) = \sum_{i=1}^n y_i f(P_i)$$

so we have $\underline{y} \in C \iff S_{\underline{y}}(f) = 0$ for all f such that $\rho(f) \leq m$.

In the decoding situation we receive a vector $\underline{y}$ which is the sum of a codeword $\underline{c}$ and an error vector $\underline{e}$. We have $S_{\underline{y}}(f) = S_{\underline{e}}(f)$ if $\rho(f) \leq m$, so the syndromes $S_{\underline{e}}(f)$ can be calculated directly from the received word if $\rho(f) \leq m$.

If τ is the Hamming weight of $\underline{e}$ then it is well known e.g. [1] or [2] that if one knows the syndromes $S_{\underline{e}}(f)$ where $\rho(f) \leq 2(\tau + 2g) - 1$ then the error vector can be easily found. The objective of the decoder is therefore to determine the syndromes $S_{\underline{e}}(f)$ where $m < \rho(f) \leq 2(\tau + 2g) - 1$.

The decoding algorithm is a version of Sakata's generalization of the Berlekamp-Massey algorithm.

This algorithm indeed solves the decoding problem when $\tau \leq \lfloor (d_{FR} - 1)/2 \rfloor$ (with τ being the number of errors). See [2] or [1].

III. THE RESULTS

Let $P_1, \dots, P_\tau$ be the error points. We call these *independent*, if they give independent conditions on a function passing through these points, or equivalently that

$$L(\rho Q - (P_1 + \dots + P_\tau)) = 0 \text{ for } \rho \leq \rho_\tau$$

Theorem 1 If $m \geq 4g - 2$, $\tau > \lfloor (d_{FR} - 1)/2 \rfloor$, and the error points are independent then the algorithm fails.

The algorithm can fail by either giving no answer or a wrong answer, and indeed both cases can occur.

When $m < 4g - 2$ the situation is different. We have developed a fairly simple procedure to determine the performance of the decoding algorithm in this case also. We mention that for the Hermitian curve over F_{r^2} given by the equation

$$x^{r+1} + y^r + y = 0$$

which has genus $g = \frac{r(r-1)}{2}$ and r^3 F_{r^2} -rational points we can often do much better than predicted by the Feng-Rao bound.

If $r = 4$ we can get a $(64, 57, 4)$ -code over F_{16} , but two independent errors are always decoded correctly.

If $r = 8$ we get a $(512, 476, 9)$ -code over F_{64} , but here one can always decode 10 independent errors correctly. By similar considerations we can explain the results presented by O'Sullivan in [3].

The error points can fail to be independent in different ways. If we look at the case where $\tau = \lfloor (d_{FR} - 1)/2 \rfloor + 1$ and

$$L(\rho Q - (P_1 + \dots + P_\tau)) = 0 \text{ for } \rho < \rho_\tau$$

but $L(\rho_\tau Q - (P_1 + \dots + P_\tau)) \neq 0$, we have the following two theorems:

Theorem 2 The function in F_M with lowest poleorder ρ at Q is an element of $L(\rho Q - (P_1 + \dots + P_\tau))$ for at least $(q-1)^{\tau-1}$ of the $(q-1)^\tau$ possible choices of the error values.

Theorem 3 The algorithm corrects $\tau = \lfloor (d_{FR} - 1)/2 \rfloor + 1$ dependent errors correctly in almost all cases.

The question whether a random selected set of points on a curve are independent or not seems difficult. We have some numerical evidence for conjecturing that (at least on a Hermitian curve) that the probability of getting independent points is $1 - \frac{1}{q}$.

REFERENCES

- [1] S. Sakata, H. Elbrønd Jensen and T. Høholdt: "Generalized Berlekamp-Massey Decoding of Algebraic-Geometric Codes up to half the Feng-Rao Bound" *IEEE Trans. Inform. Theory*, vol 41, no. 6, pp. 1762-1768, nov. 1995.
- [2] M.E. O'Sullivan: "Decoding of Codes Defined by a single Point on a Curve" *IEEE Trans. Inform. Theory*, vol. 41, no. 6, pp. 1709-1719, nov. 1995.
- [3] M.E. O'Sullivan: "Decoding Hermitian Codes beyond $(d_{\min} - 1)/2$ " *Proceedings of the 1997 IEEE International Symposium on Information Theory*, June 20-July 4, 1997, Ulm, Germany.