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Tarnow, Viggo

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Viggo Tarnow

Department of Mechanical Engineering, Technical University of Denmark, Bygning 358, DK 2800 Lyngby, Denmark

The air flow resistivity of glass wool has been measured in different directions. The glass wool was delivered from the manufacturer as slabs measuring $100\times600\times900$ mm^3, where the surface 600×900 mm^2 was parallel with the conveyor belt used in the manufacturing. Directions in the glass wool are described by a coordinate system with the X axis perpendicular to the conveyor belt, the Z axis in the direction the conveyor belt moves, and the Y axis perpendicular to the two other axes. It was found that the resistivities in the Y and Z directions were equal in all cases. For density 14 kg/m^3 the mean resistivity in the X direction was 5.88 kPa s m^{-2} and in the Y direction 2.94 kPa s m^{-2}. For density 30 kg/m^3 the mean resistivity in the X direction was 15.5 kPa s m^{-2} and in the Y direction 7.75 kPa s m^{-2}. A formula for prediction of resistivity for other densities is given. By comparing measured values of sound attenuation with results calculated from resistivity data, it is demonstrated that the measured attenuation can be predicted in a simple manner. © *2002 Acoustical Society of America.* [DOI: 10.1121/1.1476686]

I. INTRODUCTION

Many materials used for acoustic isolation and attenuation are open so air can flow through them. It has been known for many years that air flow resistivity can be used to predict the acoustical properties of acoustical materials. The resistivity is measured by sending an air flow with constant velocity through a sample of the material, and measuring the air velocity and pressure drop over the sample.

Delany and Bazley[1] have given very popular formulas for the acoustical properties of fiber materials where the only parameters are air flow resistivity, air density, and sound velocity in air. The researchers of the school of porous materials in Le Mans, France, have studied the acoustical properties by using resistivity as one of several parameters. A review has been presented by Allard.[2] In parallel with this work, Keith Attenborough in England worked on theories for predicting acoustical properties of fiber materials.[3]

Bies and Hansen[4] measured the resistivity of many sorts of fiber materials, and they plotted the resistivity versus the density of the material. In the following, data for air flow resistivity of glass wool as a function of bulk density will be presented.

Many different measuring devices have been used. Stinson and Daigle[5] reviewed many earlier setups and presented a new one. Iannace *et al.*[6] described a method that uses water to displace air in a tube with the same diameter as the sample. The measurement reported in the following uses an existing device that produces a constant air flow that can be measured with error less than 1%. The pressure drop over a sample is measured by an electronic nanometer with a resolution of 0.01 Pa.

Measurements by the author[7] showed that the sound attenuation of glass wool is anisotropic, and it is interesting to measure the resistivity of the glass wool used for the measurements of attenuation (and phase velocity) to see whether the usual methods used to predict the attenuation of acoustic sinusoidal waves works for the attenuation in different directions. This paper presents measurements of resistivity in three main directions in glass wool, and investigates whether these resistivities can be used to predict the attenuation of sound waves at audio frequencies.

II. MODEL FOR COMPUTING THE RESISTIVITY FROM FIBER DIAMETERS AND MASS DENSITY

The arrangement of glass fibers in glass wool is very complicated and one knows only a little about it. Great simplifications of the actual geometry are necessary to get a model that yields acoustic attenuation. In the classical theory of porous materials, one assumes that the material is solid with circular holes. This model is very far from the actual geometry of glass wool. Alternatively, one could assume that the fibers all have the same diameter and are parallel but placed randomly. In this case it is shown in a paper by the author[8] that the resistivity $R_\perp$ for flow perpendicular to fibers is

$$R_\perp = \frac{4\pi\eta}{b^2[-0.640\cdot\ln d - 0.737 + d]}, \qquad (1)$$

where η is the viscosity of air, b^2 is the mean area per fiber, and d is the volume density of fibers, which equals fiber volume divided by total volume. The resistivity for flow parallel with the fiber $R_\parallel$ was shown in Ref. 8 to be half the value for flow perpendicular to fibers, $R_\parallel = R_\perp/2$. This model has one direction with low resistivity, and perpendicular directions have high resistivity. This is not found in glass wool, where there is one direction with high resistivity and perpendicular directions have low resistivity.

We need a model with high resistivity in one direction and low resistivity in all directions perpendicular to this di-

rection. In glass wool the fibers are mainly placed in dense layers separated by less dense layers. One could imagine that in a volume of perhaps 1 cm^3 there are 20 parallel layers, and in each layer the fibers are parallel and perpendicular to the X axis of a rectangular XYZ coordinate system, but the directions of the fibers vary from layer to layer. The fiber directions are evenly distributed in the YZ plane, and to get the macroscopic resistivity, an average is done over the directions. In this model the resistivity in the X direction is high, and the resistivity is low for directions in the YZ plane. The resistivity R_x in the X direction is equal to the resistivity of flow perpendicular to parallel fibers, because all fibers in the model are perpendicular to the X direction and interaction between layers is neglected, $R_x = R_\perp$. In this model the resistivities in the Y and Z directions are equal, which is also observed experimentally. Other arrangements of fibers could be imagined. One could assume that in a layer the fibers are locally parallel but the direction of fibers change from place to place. This model would also require an average over the directions of fibers in the YZ plane to get the macroscopic resistivity.

To compute the resistivity in the Y direction it is convenient to use the conductivity tensor G_{ij}, where i and j can take the values x, y, z. The conductivity tensor is defined by the following equation:

$$u_i = \sum_{j=x,y,z} G_{ij} \frac{\partial p}{\partial x_j}, \qquad (2)$$

where u_i is the macroscopic velocity and p is the macroscopic pressure. We compute the components of the tensor in case all the fibers are parallel to each other. If they are perpendicular to the X direction, and the angle from the Z axis to the fibers is ν, then the conductivity tensor has the components

$$G = \begin{bmatrix} G_\perp & 0 & 0 \\ 0 & G_\perp \cos^2 \nu + G_\| \sin^2 \nu & (G_\| - G_\perp) \sin \nu \cdot \cos \nu \\ 0 & (G_\| - G_\perp) \sin \nu \cdot \cos \nu & G_\perp \sin^2 \nu + G_\| \cos^2 \nu \end{bmatrix}, \qquad (3)$$

where $G_\perp = 1/R_\perp$ and $G_\| = 1/R_\|$. It is physically reasonable to assume that a pressure gradient is given in the glass wool, in this case, from Eq. (2) it is seen that one must make an average over the conductivity tensor. We average over the angle and take all angles for equally probable. This gives the average tensor $\langle G \rangle$, which equals

$$\langle G \rangle = \begin{bmatrix} G_\perp & 0 & 0 \\ 0 & G_\perp/2 + G_\|/2 & 0 \\ 0 & 0 & G_\perp/2 + G_\|/2 \end{bmatrix}. \qquad (4)$$

Because $G_\|/G_\perp = R_\perp/R_\| = 2$, the resistivity tensor becomes

$$R = R_\perp \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}. \qquad (5)$$

III. MEASUREMENT SETUP

The setup is shown in Fig. 1. The samples are cut from glass wool slabs with a tube, which has a sharp edge. The diameters of the samples are 76 mm and the length 100 mm. After the samples have been cut, the tube, with the sample inside, is mounted in the setup shown in Fig. 1. The air stream through the sample comes from the cylinder-shaped bell, which is a little heavier than the contra weight to the right. The fall velocity of the bell was measured with a stop watch. To find the air velocity through the sample, the fall velocity of the bell was multiplied by the ratio 6.27 between the area of the bell and cross section area of the sample. The air velocity through the samples ranged from 11 to 6 mm/s. The pressure drop over the sample was measured by a digital manometer with a resolution of 0.01 Pa.

The resistivity R is found from the present reading p, the length of the sample l, and the measured air velocity u:

$$R = \frac{p}{lu}. \qquad (6)$$

After the pressure and fall velocity of the bell were measured, the sample was removed from the sample holder and weighed. From the weight and the volume of the sample, the mass density of the sample was calculated.
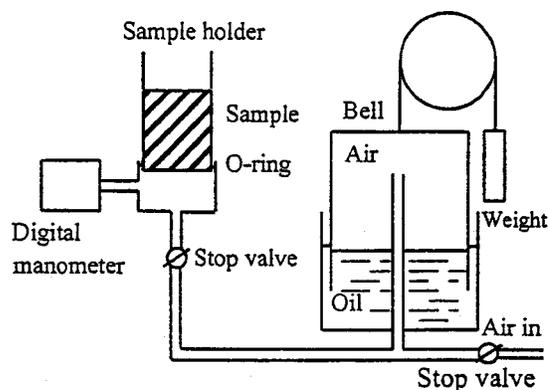
FIG. 1. The setup for measuring air flow resistivity of glass wool. Air is let into the movable bell, which is sealed by an oil reservoir below. When the bell is full, the inlet valve is closed and the stop valve before the sample holder is opened. The bell falls as air flows through the sample. The fall velocity of the bell is measured, and the pressure drop over the sample is measured. From these measurements and the dimensions of the sample and the area of the bell, the resistivity is found.

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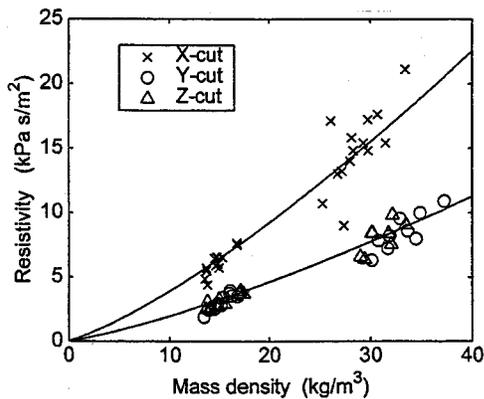FIG. 2. Resistivity versus mass density for two types of glass wool with mean density 14 and 30 kg/m^3. The resistivity was measured in three perpendicular directions X, Y, and Z. The X direction is perpendicular to fibers. The upper curve is the resistivity in the X direction, calculated from Eq. (1), and the lower curve is the resistivity for the Y and Z directions found by dividing the resistivities in the X direction by 2.

The accuracy of the pressure measurement was limited by fluctuations of the pressure reading about 0.05 Pa. The velocity was measured with an accuracy of 0.5%. The overall accuracy of the measurement of resistivity was better than 2%.

IV. RESULTS OF MEASUREMENTS

The glass wool came from a major producer. It was delivered in slabs measuring $100 \times 600 \times 900$ mm^3. A rectangular coordinate system is used to describe the directions in the glass wool. The X axis is perpendicular to the large surface, 600×900 mm^2. This surface is parallel to the conveyor belt used in the manufacture. The Z axis is in the direction of the movement of the conveyor belt, and the Y axis is perpendicular to the two other axes. Samples were cut from different places and directions in the glass wool slabs, covering the whole slab evenly. No dependence on location was seen in the resistivity and density, but the variation of the data showed that the glass wool is inhomogeneous in both resistivity and density. The results are shown in Fig. 2. One can see that the resistivity for a given density is highest in the X direction and the resistivities in the Y and Z directions are equal within the scatter of the data. In the paper by Castagnede et al.,[9] it was assumed that the resistivity is proportional to the mass density, but the direct measurement in Fig. 2 shows this is not the case.

In Fig. 2 the curve for the X direction was computed from Eq. (1). The density d was found from $d = \rho_w / \rho_f$, where ρ_w is the density of glass wool and $\rho_f = 2550$ kg/m^3 is the density of glass in the fibers. The area per fiber b^2 was computed from $b^2 = \pi a^2 / d$, where a is the radius of the fibers. To fit the curve to the data, a was adjusted to 5.15 μm. Measurement with a microscope gave a fiber radius of 3.4 μm with a standard deviation of 1.3 μm. The fitted radius is somewhat higher than the directly measured radius. Therefore, the glass wool is more open on a microscopic scale than one would expect with a completely random distribution of parallel fibers. A possible explanation could be that the fibers

have a tendency to cling together in pairs. This is supported by microscopic inspection of glass wool samples.

The resistivity curve for the Y and Z direction is 0.50 times the one for the X direction. From acoustical measurements Allard et al.[10] deduced the ratio $R_y / R_x = 0.6$ for glass wool with $R_x = 25\,000$ Pa s m^{-2}. The ratio 0.50 found here is not in accordance with the calculation above, which gave the ratio 0.667. The reason for the discrepancy could be the layered structure of the glass wool. A cut perpendicular to the Y or Z axis of glass wool of density 15 kg/m^3 shows an irregular layered structure with dense layers about 0.3 mm thick and approximately parallel with the YZ plane. The distances between the dense layers are about 0.6 mm. This structure is consistent with the large difference in the elastic moduli for the different directions. The elastic modulus for compression in the X direction is approximately 50 times smaller than the one for compression in the Y and Z directions.[7] Obviously the less dense layers have a very low stiffness. The resistivity in the Y direction is smaller than for a homogeneous model because when the air flows through the glass wool in the Y direction, most air will flow between the dense layer, where the local resistivity is smallest. This means that the overall resistivity measured will be smaller than the resistivity for a homogeneous distribution of fibers. Therefore, we measure the ratio 0.50 instead of 0.667.

V. CONSTANT AIR FLOW RESISTIVITY AND SOUND ATTENUATION

For many years it has been assumed that the sound velocity and attenuation of sinusoidal sound waves in fiber materials could be calculated from the resistivity measured with steady air flow and the frequency. The formulas by Delany and Bazley[1] are very popular. However, they are empirical and not accurate at low frequencies. An alternative is to compute the wave number and characteristic impedance by formulas for complex mass density ρ_d and compressibility given by Wilson.[11] I prefer here the complex resistivity R, instead of the complex mass density, because the resistivity is real and constant at low frequencies. By definition, $R = -i\omega\rho_d$, where ω is the angular frequency, and we use the complex time factor $e^{-i\omega t}$. The complex resistivity R is

$$R = \frac{-i\omega\rho}{1 - (1 - i\omega\tau_R)^{-1/2}}, \qquad (7)$$

where $\rho = 1.20$ kg/m^3 is the mass density of air at 20 °C and 1 atm pressure, and τ_R is a time constant. The time constant is related to the constant air flow resistivity R_i, where i stands for x, y, or z. The low-frequency limit of Eq. (7) gives

$$\tau_{Ri} = \frac{2\rho}{R_i}. \qquad (8)$$

The compressibility of air between fibers depends on frequency, being isothermal at low frequencies and adiabatic at high frequencies. Wilson's formula for the complex compressibility C is

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$$C = \frac{1}{\gamma P}\left[1 + \frac{\gamma - 1}{(1 - i\omega\tau_C)^{1/2}}\right], \tag{9}$$

where $\gamma = 1.40$ for atmospheric air, P is the atmospheric air pressure, and τ_C is a time constant. To find a relation between constant flow resistivity and the compressibility time constant we assume the fibers are parallel. Stinson and Champoux[12] derived a relation between resistivity and compressibility. Their derivation assumed flow in tubes, but the differential equations and boundary conditions for local velocity and temperature are the same for flow in tubes and flow parallel to parallel fibers. Therefore their derivation is also valid for parallel fibers. Equation (8) in their paper reads $C = (1/\gamma P)\{\gamma - (\gamma - 1)[\rho/\rho_d(N\omega)]\}$, where N is Prandtl number, which for atmospheric air is $N = 0.71$. ($N = c_p\eta/\kappa$, where c_p is the air heat capacity at constant pressure, η is the air viscosity, and κ is the heat conductivity of air.) If $\rho_d(N\omega)$ is calculated from our Eq. (7) and set into Stinson and Champoux's equation for compressibility, one gets Eq. (9) with $\tau_C = N\tau_\parallel$, where $\tau_\parallel$ is the resistivity time constant for flow parallel with parallel fibers. In Ref. 8 it is shown that for parallel fibers the resistivity perpendicular to the fibers is two times the resistivity parallel with the fibers. We now apply Eqs. (7) and (8) to parallel fibers. It then follows from Eq. (8) that the time constant for flow parallel to fibers is two times the one for perpendicular flow. We know the perpendicular resistivity R_x from measurements on real glass wool, and from Eq. (8) one gets the time constant for perpendicular flow with $R_i = R_x$. This time constant is multiplied by 2 to get the time constant for parallel flow, and this is multiplied by N to give Eq. (10):

$$\tau_C = \frac{4N\rho}{R_x}. \tag{10}$$

We assume that the glass wool is restrained mechanically from moving under influence of the sound wave. The sound wave number k is then

$$k = \sqrt{i\omega RC}, \tag{11}$$

and the characteristic impedance is

$$Z = \sqrt{\frac{iR}{\omega C}}. \tag{12}$$

For plane harmonic waves in glass wool, the real part of the wave number k gives the phase shift per meter of the sound pressure amplitude, and the imaginary part of the wave number gives the relative attenuation of sound pressure per meter. The phase shift of the wave is mainly determined by the inertia of the air itself, and it is not very sensitive to parameters of a model, but the sound attenuation is sensitive to these parameters. Therefore, we compare a calculation of the attenuation from Eqs. (9)–(11) with measurements of attenuation. The measurements were done on a pile of glass wool in which an approximately plane wave could propagate without reflections from boundaries. The sound pressures on each side of the layer of glass wool 0.200 m thick were measured with microphones. The attenuation in decibels was divided by this thickness to give the attenuation per meter.
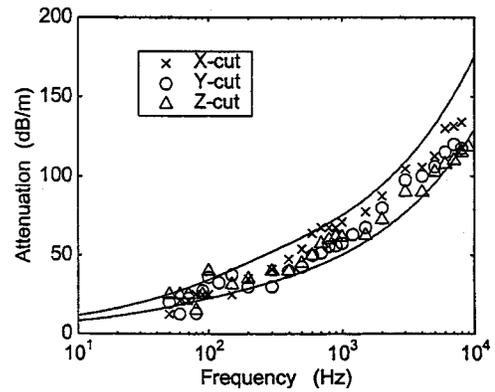

FIG. 3. Measured attenuation per meter for plane sound waves in glass wool of mean density 14 kg/m^3, in three perpendicular directions X, Y, and Z. The X direction is perpendicular to the fibers. The upper curve is the attenuation in the X direction, calculated from the measured resistivities by Eqs. (8)–(11). The lower curve is the attenuation in the Y and Z directions calculated in the same way.

The glass wool layer was perpendicular to the propagation direction of the sound wave. Further details of the measurements were given in Ref. 7. For the light glass wool of mean density 14 kg/m^3, the results are shown in Fig. 3. The lines were calculated from a mean resistivity, which was calculated by Eq. (1) from the mean density in the same way as the curves in Fig. 2 were found. For mass density 14 kg/m^3, the mean resistivity was $R_x = 5880$ Pa s m^{-2} and $R_y = 2940$ Pa s m^{-2}. For the X direction the calculated curves are a little higher than the measured points in the 700–8000 Hz frequency range. In the 50–600 Hz frequency range, the measured points are below the curve, due to movements of the glass wool caused by the sound wave. The glass wool is dragged by the friction between fibers and moving air in the sound waves.[7,13] Calculation of the wave number by Eq. (11) assumes the fibers do not move. This may be the case in some applications of glass wool, if the movement of the glass wool is mechanically constrained. The measured attenuations per meter in the Y and Z directions are almost equal and a little higher than the curve calculated from the resistivity. The elastic moduli in the Y and Z direction are about 50 times the one for the X direction,[7] thus the glass wool has much higher mechanical stiffness in these directions. Therefore, there is little movement of fibers with sound waves in these directions, and most of the measured points are above the curve calculated for fixed fibers.

The attenuation of sound waves in glass wool of density 30 kg/m^3 is shown in Fig. 4. The mean resistivity for this density was computed in the same way as above, and the resistivities were $R_x = 15500$ Pa s m^{-2} and $R_y = 7750$ Pa s m^{-2}. Most points were obtained from attenuation measurements with a glass wool layer 0.200 m thick, but the data for X directions in the 1500–8000 Hz frequency range was measured with a layer 0.100 m thick. The data for the X direction are close to the curve in the 700–8000 Hz frequency range. They are below the curve in the 50–600 Hz frequency range, because the glass wool moves with the sound wave, which was shown in Refs. 7 and 13. The data for Y and Z directions follow the curve because the glass
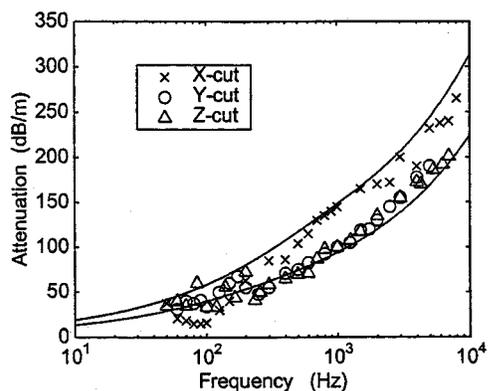
FIG. 4. Measured attenuation per meter for plane sound waves in glass wool of mean density 30 kg/m^3, in three perpendicular directions X, Y, and Z. The X direction is perpendicular to the fibers. The upper curve is the attenuation in the X direction, calculated from the measured resistivities by Eqs. (8)–(11). The lower curve is the attenuation in the Y and Z directions calculated in the same way.

wool does not move, due to the high mechanical stiffness of the glass wool in these directions.[7]

VI. CONCLUSION

The measured constant air flow resistivities of glass wool with density 14 and 30 kg/m^3 were used to compute the attenuation per meter of plane sound waves. The calculated attenuation was reasonably close to direct measured attenuation per meter. Given the constant air flow resistivity, one can compute the wave number from Eqs. (7)–(11), provided the fibers do not move. This procedure can be expected to be valid in the 0–10 000 Hz frequency range. It has the proper low-frequency limit and better theoretical foundation then the empirical equations of Delany and Bazley.[1] Even without direct experimental data for the characteristic impedance, it is reasonable to assume that the characteristic impedance could be predicted from constant air flow resistivity by Eq. (12) and Eqs. (7)–(10).

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