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Technical Notes and Correspondence

Identification of a Class of Nonlinear State-Space Models Using RPE Techniques

WEI-WU ZHOU AND MOGENS BLANKE

Abstract—The recursive prediction error methods in state-space form have been efficiently used as parameter identifiers for linear systems; and Ljung's innovations filter using a Newton search direction has especially proved to be quite ideal. In this note, the RPE method in state-space form is developed in the nonlinear case and extended to include the exact form of a nonlinearity, thus enabling structure preservation for certain classes of nonlinear systems. Both the discrete and the continuous-discrete versions of the algorithm in an innovations model are investigated, and a nonlinear simulation example shows a quite convincing performance of the filter as combined parameter and state estimator.

I. INTRODUCTION

In this note we present two parameter identifiers for nonlinear-discrete and continuous-discrete state-space models. These algorithms are investigated by using the linear recursive prediction error (RPE) method, Ljung and Söderström [9], in combination with nonlinear second-order filtering theory Jazwinski [7], and Maybeck [13]. See also Zhou [15] and [16].

II. MODEL AND ALGORITHM IN DISCRETE VERSION

We assume a nonlinear discrete state-space model of the following form:

$$x(t+1) = f(\theta, u; t, x(t)) + v(t) \qquad (1\text{-a})$$
$$y(t) = h(\theta; t, x(t)) + e(t) \qquad (1\text{-b})$$

where $f(\ )$ and $h(\ )$ are nonlinear functions of the state, $v(t)$ is white process noise, and $e(t)$ is uncorrelated measurement noise with statistics

$$Ev(t) = Ee(t) = 0$$

$$Ev(t)v^T(\tau) = Q_1(\theta)\delta_{t\tau}$$

$$Ee(t)e^T(\tau) = Q_2(\theta)\delta_{t\tau}$$

$$E_v(t)e^T(\tau) = Q_{12}(\theta)\delta_{t\tau}. \qquad (1\text{-c})$$

The initial value of the state $x(o)$ has the properties

$$Ex(0) = x_0(\theta)$$
$$E[x(0) - x_0(\theta)][x(0) - x_0(\theta)]^T = \Pi_0(\theta). \qquad (1\text{-d})$$

From the nonlinear filtering theory [13] it is known that an attractive and applicable nonlinear filter is the first-order filter with bias correction term (FOFBC), which is based on using first-order covariance and gain computations, but with the second-order terms in state expectation and prediction error equations. In this study we use the FOFBC method for identification of the nonlinear model (1-a), (1-b). When a fixed value θ is

given, the predictor corresponding to (1-a), (1-b) will be

$$\hat{x}(t+1, \theta) = f(\theta, u; t, \hat{x}(t, \theta)) + B_x(t)$$
$$+ K(t)[y(t) - h(\theta; t, \hat{x}(t, \theta)) - B_y(t)] \qquad (2\text{-a})$$

$$\hat{y}(t|\theta) = h(\theta; t, \hat{x}(t, \theta)) \qquad (2\text{-b})$$

where the second-order bias correction term $B_x(\theta; t)$ is an n_x-vector. With B_{xk} and f_k denoting the kth component of B_x and f vectors, respectively [7]

$$B_{xk}(t) = \frac{1}{2} \operatorname{tr} \left\{ \frac{\partial^2 f_k(\theta, u; t, \hat{x}(t, \theta))}{\partial x^2} P(t) \right\} \qquad (2\text{-c})$$

and $B_y(t)$ is similarly the n_y-vector with kth component

$$B_{yk}(t) = \frac{1}{2} \operatorname{tr} \left\{ \frac{\partial^2 h_k(\theta; t, \hat{x}(t, \theta))}{\partial x^2} P(t) \right\}. \qquad (2\text{-d})$$

In the expressions for B_{xk} and B_{yk}, $P(t)$ is the prediction error covariance. One finds that use of the recursive prediction error method by Ljung and Söderström [9], directly on the nonlinear predictor model (2-a), (2-b) is hardly feasible, due to computational complexity. If a linear measurement equation is chosen instead, however, complexity of the algorithm is reduced significantly. Then the predictor has the following form:

$$\hat{x}(t+1, \theta) = f(\theta, u; t, \hat{x}(t, \theta)) + B_x(t)$$
$$+ K(t)[y(t) - H(\theta)\hat{x}(t, \theta)] \qquad (3\text{-a})$$

$$\hat{y}(t|\theta) = H(\theta)\hat{x}(t, \theta). \qquad (3\text{-b})$$

The assumption of a linear measurement is valid in a wide class of practical applications. Then the recursive prediction error method using a Newton search direction for parameter updating can be applied to the model (3-a), (3-b). The derivation for the linear case can be found in Ljung and Söderström [9]. Details for the nonlinear extension are given in Zhou [15] and [16]. The algorithm will consist of the following set of recursive equations:

$$\epsilon(t) = y(t) - \hat{y}(t) \qquad (4\text{-a})$$

$$R(t) = R(t-1) + \alpha(t)[\Psi(t)S^{-1}(t)\Psi^T(t) - R(t-1)] \qquad (4\text{-b})$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \alpha(t)R^{-1}(t)\Psi(t)S^{-1}(t)\epsilon(t) \qquad (4\text{-c})$$

$$K(t) = [F_t P(t) H_t^T + Q_{12}(t)]S^{-1}(t) \qquad (4\text{-d})$$

$$P(t+1) = F_t P(t) F_t^T + Q_1(t) - K(t)S(t)K^T(t) \qquad (4\text{-e})$$

$$S(t) = H_t P(t) H_t^T + Q_2(t) \qquad (4\text{f})$$

$$\hat{x}(t+1) = f(\theta, u; t, x(t, \theta)) + B_x(t) + K(t)\epsilon(t) \qquad (4\text{-g})$$

$$\hat{y}(t+1) = H_t \hat{x}(t+1) \qquad (4\text{-h})$$

$$W(t+1, \hat{\theta}) = \bar{W}_x(t, \hat{\theta}) + \bar{M}_\theta(t) + \bar{K}_t \epsilon(t) - K(t)D_t \qquad (4\text{-i})$$

$$\Psi^T(t+1) = H_t W(t+1, \hat{\theta}) + D_t. \qquad (4\text{-j})$$

When deriving the gradient $\Psi^T(t+1)$, we differentiate (3-b) with respect to θ and have introduced the notation below for the sake of brevity. The equivalent linear expressions are explained in Ljung and Söderström [9]. In this nonlinear case, the explicit structure of the nonlinearity is reflected

in the gradient through the matrices (5-c) and (5-d) below. The effect on convergence properties from these terms are demonstrated in the example. The terms (5-c), (5-d), and (2-c) need to be calculated for the particular nonlinear structure in each case. However, the results obtainable make this effort worthwhile. The notation is

$$F_t = \frac{\partial f(\theta, u; t, \hat{x}(t, \theta))}{\partial x} \Bigg|_{\theta = \hat{\theta}(t)} \quad \text{(5-a)}$$

$$H_t = H(\hat{\theta}(t)) \quad \text{(5-b)}$$

$$\bar{W}_x(t, \theta) = \frac{\partial}{\partial \theta} \{f(\theta, u; t, \hat{x}(t, \theta)) + B_x(t, x(t, \theta))\}$$

$$- K(t)H(\theta) \frac{\partial}{\partial \theta} \hat{x}(t, \theta)|_{\theta = \hat{\theta}(t)} \quad \text{(5-c)}$$

where the derivative is taken of x only, i.e., θ is considered fixed in $f(\theta, \cdots)$

$$\bar{M}_\theta(t) = \frac{\partial}{\partial \theta} \{f(\theta, u; t, \hat{x}) + B_x(\theta, t)\}|_{\theta = \hat{\theta}(t)} \quad \text{(5-d)}$$

where the derivative is taken of the parameter matrices of $f + B_x$, not implicitly of the x term. Note that the first part of (5-c) and (5-d) together give the derivative of $f + B_x$ with respect to θ

$$\bar{K}_t = \frac{\partial}{\partial \theta} K(t)|_{\theta = \hat{\theta}(t)} \quad \text{(5-e)}$$

$$D_t = \frac{\partial}{\partial \theta} (H(\theta)x)|_{\theta = \hat{\theta}(t)} \quad \text{(5-f)}$$

$$\alpha(t) = 1/(1 + t). \quad \text{(5-g)}$$

The gain factor α is a convenient choice, chosen from experience. This version of the filter (4-a)-(4-j) includes a calculation of the Kalman gains in (4-d)-(4-f) and K_t is calculated from (4-d)-(4-f). The noise covariances Q_1, Q_2, Q_{12} need to be known in this case. Following Ljung [12], the predictor can assume an innovations model of the form:

$$\hat{x}(t+1, \theta) = f(\theta, u; t, \hat{x}(t, \theta)) + B_x(t) + K(\theta)\epsilon(t) \quad \text{(6-a)}$$
$$y(t) = H(\theta)\hat{x}(t, \theta) + \epsilon(t) \quad \text{(6-b)}$$

where $\epsilon(t)$ is the innovation due to measurement t, and $K(\theta)$ is a set of (as yet undetermined) steady-state Kalman gains, which is treated as parameters and will be identified directly along with the system parameters. This gives less complex computations, and the algorithm corresponding to (6-a)-(6-b) will then be as follows:

$$\epsilon(t) = y(t) - \hat{y}(t) \quad \text{(7-a)}$$

$$\hat{\Lambda}(t) = \hat{\Lambda}(t-1) + \alpha(t)[\epsilon(t)\epsilon^T(t) - \hat{\Lambda}(t-1)] \quad \text{(7-b)}$$

$$R(t) = R(t-1) + \alpha(t)[\Psi(t)\hat{\Lambda}^{-1}(t)\Psi^T(t) - R(t-1)] \quad \text{(7-c)}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \alpha(t)R^{-1}(t)\Psi(t)\hat{\Lambda}^{-1}(t)\epsilon(t) \quad \text{(7-d)}$$

$$\hat{x}(t+1) = f(\hat{\theta}, u; t, x(t, \hat{\theta})) + B_x(t) + K_t\epsilon(t) \quad \text{(7-e)}$$

$$\hat{y}(t+1) = H_t\hat{x}(t+1) \quad \text{(7-f)}$$

$$W(t+1, \hat{\theta}) = \bar{W}_x(t, \hat{\theta}) + \overline{M_\theta^*}(t) - K_t D_t \quad \text{(7-g)}$$

$$\Psi^T(t+1) = H_t W(t+1, \hat{\theta}) + D_t \quad \text{(7-h)}$$

where

$$K_t = K(\hat{\theta}(t))$$

$$H_t = H(\hat{\theta}(t)) \quad \text{(7-i)}$$

$$\overline{M_\theta^*} = \frac{\partial}{\partial \theta} \{f(\theta, u; t, \hat{x}) + B_x(t) + K(\theta)\epsilon(t)\}|_{\theta = \theta(t_i)}. \quad \text{(7-j)}$$

$W_x(t, \theta)$, D_t are defined in (5-c)-(5-f), respectively, and $B_x(t)$ is defined

in (2-c). It is noted that in version (7-a)-(7-h) one has to use (4-e) and (4-f) in order to obtain the covariance matrix $P(t)$ in $B_x(t)$. Hence, the covariances Q_1 and Q_2 need to be known to provide the bias correction in the nonlinear system, while Q_1 and Q_2 would not be required in the linear case if the measurement vector $y(t)$ has the same dimension as the state x, and the matrix H is an identity matrix, then the covariance matrix is

$$P(t) = E\{(x(t) - \bar{x}(t))(x(t) - \bar{x}(t))^T\}$$

$$= E\{\epsilon(t)\epsilon^T(t)\}.$$

Since $y(t) = H_t x(t) = x(t)$. Consequently, the matrix $P(t)$ can be replaced by $\Lambda(t)$ in this case, and $P(t)$ no longer needs to be calculated. The covariances Q_1 and Q_2 need not be known for bias correction calculation either in this case.

III. MODEL AND ALGORITHM IN CONTINUOUS-DISCRETE VERSION

In most applications involving the identification of parameters of a physical continuous-time system, it is generally preferable to use a continuous-discrete algorithm. The reasons are primarily structure preservation of known parts of the system and the possibility to include bounds on parameter estimates of physical parameters whose constraints are known. The latter is a practical way to overcome part of the difficulties with possible local minima when identifying parameters of nonlinear systems. As in the presentation in Section II the discrete measurement equation will be chosen in its linear version, and an innovations model is employed. We, hence, assume the nonlinear continuous-discrete state-space model of the form:

$$\frac{d}{dt} x(t|t_i) = f(\theta, u; t, x(t|t_i)) + v(t|t_i) \quad \text{(8-a)}$$

$$y(t_{i+1}) = H(\theta)x(t_{i+1}) + e(t_{i+1}) \quad \text{(8-b)}$$

where $f(\ )$ is the nonlinear function of state. $v(t|t_i)$ is white process noise, $e(t_i)$ is uncorrelated measurement noise with statistics,

$$Ev(t) = Ee(t_i) = 0$$

$$Ev(t)v^T(\tau) = Q_1(\theta)\delta(t - \tau)$$

$$Ee(t_i)e^T(t_j) = Q_2(\theta)\delta_{ij}.$$

The second-order predictor using an innovations model will be

$$\frac{d}{dt} \hat{x}(t|t_i, \theta) = f(\theta, u; t, \hat{x}(t|t_i, \theta)) + B_x(t|t_i) \quad \text{(9-a)}$$

$$\epsilon(t_{i+1}) = y(t_{i+1}) - H(\theta)\hat{x}(t_{i+1}^-, \theta) \quad \text{(9-b)}$$

$$\hat{x}(t_{i+1}^+, \theta) = \hat{x}(t_{i+1}^-, \theta) + K(t_{i+1}, \theta)\epsilon(t_{i+1}) \quad \text{(9-c)}$$

where $\epsilon(t_i + 1)$ is the innovation due to measurement $t_i + 1$, and $K(t_{i+1}, \theta)$ comprise parameterized steady-state Kalman gains. The algorithm corresponding to (9-a)-(9-c) will be as follows:

$$\frac{d}{dt} \hat{x}(t|t_i^+) = f(\hat{\theta}, u; t, \hat{x}(t|t_i^+, \hat{\theta})) + B_x(t|t_i) \quad \text{(10-a)}$$

$$\frac{d}{dt} P(t|t_i^+) = F_i P(t|t_i^+) + P(t|t_i^+)F_i^T + Q_1(t_i) \quad \text{(10-b)}$$

$$\frac{d}{dt} W(t|t_i^+, \theta) = \bar{W}_X^*(t|t_i^+, \hat{\theta}) + \bar{M}_\theta(t_i). \quad \text{(10-c)}$$

The linear version of the continuous-discrete problem was derived by Gavel and Azevedo [4]. For the nonlinear version, we get by integration of (10-a)-(10-c) $x(t_{i+1}-)$, $P(t_{i+1}-)$, $W(t_{i+1}-, \hat{\theta})$ as

$$W(t_{i+1}^+, \hat{\theta}) = [I - K(t_i, \hat{\theta})H_i] W(t_{i+1}^-, \hat{\theta})$$

$$+ N(t_{i+1}, \hat{\theta}) - K(t_i, \hat{\theta})D(\hat{\theta}, x(t_{i+1}^-)) \quad \text{(10-d)}$$
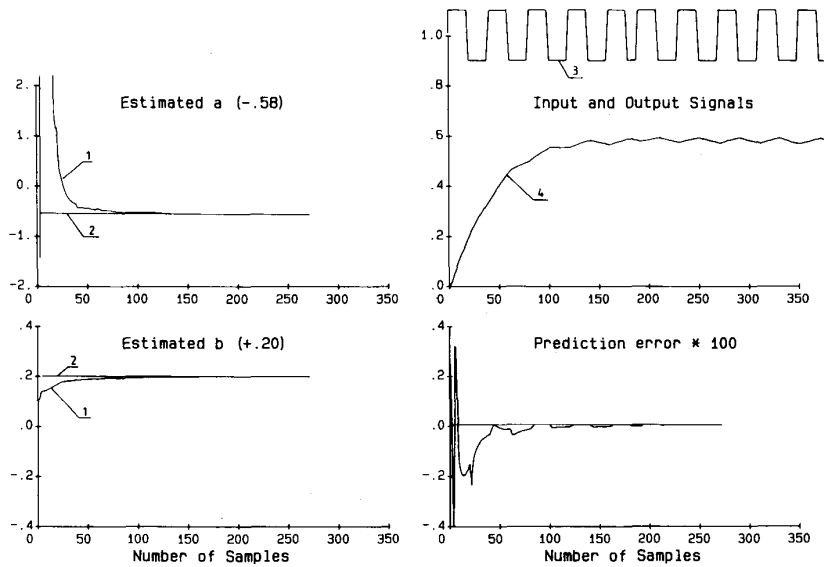
Fig. 1.   Identification of parameters a and b of (11) with square wave perturbation on the input signal. The nonlinear estimator is used. (1) is estimate, (2) is true value, (3) is input, and (4) is output signal.

$$\Psi^T(t_{i+1}) = H_i W(t_{i+1}^+, \hat{\theta}) + D(\hat{\theta}, x(t_{i+1}^-)) \tag{10-e}$$

$$\hat{y}(t_{i+1}^-) = H_i x(t_{i+1}^-) \tag{10-f}$$

$$\epsilon(t_{i+1}) = y(t_{i+1}) - \hat{y}(t_{i+1}^-) \tag{10-g}$$

$$\hat{\Lambda}(t_{i+1}) = \hat{\Lambda}(t_i) + \alpha(t_{i+1})[\epsilon(t_{i+1})\epsilon^T(t_{i+1}) - \hat{\Lambda}(t_i)] \tag{10-h}$$

$$R(t_{i+1}) = R(t_i) + \alpha(t_{i+1})[\Psi(t_{i+1})\hat{\Lambda}^{-1}(t_{i+1})\Psi^T(t_{i+1}) - R(t_i)] \tag{10-i}$$

$$\hat{\theta}(t_{i+1}) = \hat{\theta}(t_i) + \alpha(t_{i+1})R^{-1}(t_{i+1})\Psi(t_{i+1})\hat{\Lambda}^{-1}(t_{i+1})\epsilon(t_{i+1}) \tag{10-j}$$

$$\hat{x}(t_{i+1}^+) = \hat{x}(t_{i+1}^-) + K(t_{i+1}, \hat{\theta})\epsilon(t_{i+1}) \tag{10-k}$$

$$P(t_{i+1}^+) = P(t_{i+1}^-) - K(t_{i+1}, \hat{\theta})H_i P(t_{i+1}^-) \tag{10-l}$$

where $B_x(t|t_i)$ again is an n_x-vector with the kth component

$$B_{xk}(t|t_i) = \frac{1}{2} \text{tr} \left\{ \frac{\partial^2 f_k(\theta, u; t, \hat{x}(t|t_i, \theta))}{\partial x^2} P(t) \right\} \Bigg|_{\theta = \hat{\theta}(t_i)} \tag{10-m}$$

and the following notation is used:

$$\bar{W}_x^*(t|t_i, \hat{\theta}) = \frac{\partial}{\partial \theta} \{f(\theta; t, \hat{x}(t|t_i, \theta)) + B_x(t|t_i)\}|_{\theta = \hat{\theta}(t_i)} \tag{10-n}$$

with the derivative being taken of x only.

$$\bar{M}_\theta(t_i) = \frac{\partial}{\partial \theta} \{f(\theta, u; t, \hat{x}) + B_x(t|t_i)\}|_{\theta = \hat{\theta}(t_i)} \tag{10-o}$$

with the derivative being taken of the parameter matrices of f and B_x, not of x.

$$D(\hat{\theta}, \hat{x}(t_{i+1}^-)) = \frac{\partial}{\partial \theta} \{H(\theta)\hat{x}\}|_{\theta = \hat{\theta}(t_i)}$$

$$N(t_{i+1}, \hat{\theta}) = \left[\frac{\partial}{\partial \theta} K(t_{i+1}, \theta) \right] \Bigg|_{\theta = \hat{\theta}(t_i)} \epsilon(t_{i+1})$$

$$H_i = H(\hat{\theta}(t_i))$$

$$F_i = \frac{\partial f(\theta, u; t, \hat{x}(t|t_i, \theta))}{\partial x} \Bigg|_{\theta = \hat{\theta}(t_i)}$$

$$\alpha(t_i + 1) = 1/(1 + i), \qquad i = 1, 2, 3, \cdots. \tag{10-p}$$

The same treatment will be used when H_i is an identity matrix and has the same dimension as the state vector x. In this case the $P(t_i)$ matrix will not be calculated any longer and is replaced by $\hat{\Lambda}(t_i)$.

IV. EXAMPLE

The ability of the nonlinear RPE method to estimate parameters and states of a nonlinear system of practical importance is demonstrated in this example. The continuous-discrete version of the nonlinear filter derived above is compared to the corresponding linear algorithm by Gavel and Azevedo [4]. The results demonstrate the advantages in terms of bias correction of the nonlinear filter.

The nonlinear system considered is an equivalent to the ship speed equation. The parameters identified will, for the real ship, mean hull resistance and efficiency in utilizing the prime mover of the vessel for forward thrust. Both values are of major technical importance and as they change over time, they have vast impact on the ship's fuel economy and efficiency. The criteria for maintenance of the ship's hull, propeller, and prime mover system can be directly derived from these parameters, and it is hence of prime importance that they are estimated without bias.

$$\frac{d}{dt} x(t) = ax^2(t) + bu(t) + v(t)$$

$$Q_1 = 0, \ Q_2 = 0.01, \ Q_{12} = 0. \tag{11}$$

The second-order nonlinearity type of system is furthermore technically important when identifying propulsion losses of ships at sea aiming at autopilot and steering gear performance evaluation, Blanke [1], Blanke and Sórensen [3], Blanke [2].

The response and parameter estimates below were obtained using a square wave perturbation to the input $u(t)$. The amplitude of the perturbation is 10 percent of its steady-state value. The practical equivalent to this experiment would be a stepwise increase/decrease in propeller thrust.

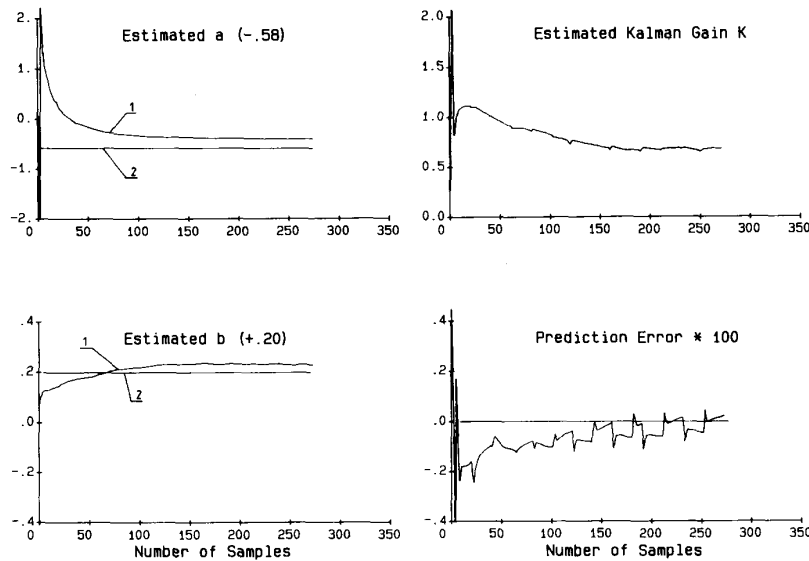The matrices B_x, $\bar{W}_x^*$, $\bar{M}_\theta$, and N in the algorithm (10-a)–(10-l)

Fig. 2. Identification of parameters a and b of (11) with similar excitation as in Fig. 1.
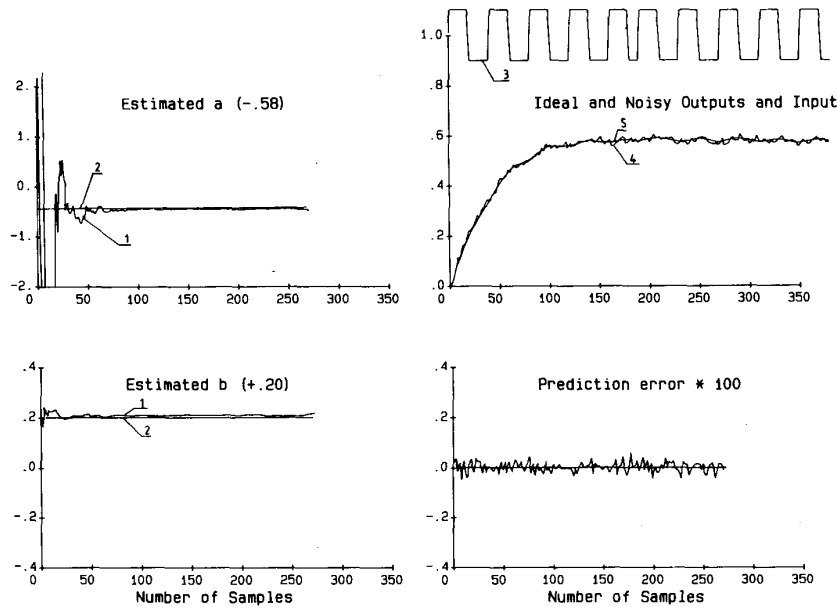The linear estimator is used. (1) is estimate, (2) is true value.

Fig. 3. Same example as Fig. 1 with the nonlinear filter, but measurement corrupted
with noise. (1) is estimate, (2) is true value, (3) is input, (4) is output with noise, and
(5) is noise free output.

corresponding to the example are calculated below. Note that the parameter vector used is (a, b, k)

$$B_x(t|t_i) = ap(t_i) = a\hat{\Lambda}(t_i)$$

$$W_x^*(t|t_i, \hat{\theta}) = 2a\hat{x}(t_i) \frac{d}{d\theta} \hat{x}(t, \hat{\theta})$$

$$\bar{M}_\theta(t_i = [(\hat{x}^2(t_i) + P(t_i)), u(t_i), 0] = [(\hat{x}^2(t_i) + \hat{\Lambda}(t_i)), u(t_i), 0]$$

$$N(t_i) = [0, 0, \epsilon(t_i)]. \qquad (12)$$

Fig. 1 shows results of identifying the parameters a and b in the nonlinear

equation using the nonlinear filter. The curves plotted in Fig. 2 illustrate the performance of a linear RPE filter applied to the same nonlinear equation. Although the driving signal's perturbation is only 10 percent of its average, the bias of the linear estimator is apparent, and the superior performance of the nonlinear filter is obvious. This is also the case when measurement noise is added, as shown in Figs. 3 and 4.

V. CONCLUSIONS

This note has presented two algorithms for identifying parameters of a nonlinear discrete state-space system model and a nonlinear continuous-
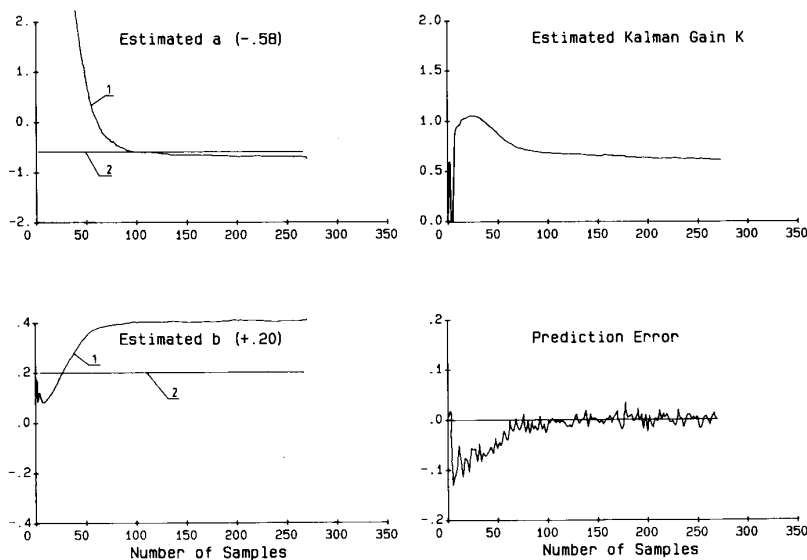
Fig. 4.   Same example as Fig. 2 with the linear filter, but measurement corrupted with noise. (1) is estimate, (2) is true value.

discrete state-space system model. Both versions are treated using a linear discrete measurement equation. These algorithms were investigated with reference to the theory of linear RPE methods and the theory of nonlinear filtering. The innovations model formulation was found to be attractive, and the algorithms were implemented and tested against computer simulations showing excellent convergence and bias properties that by far exceed those of a linear continuous/discrete filter.

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Failure Detection and Identification

MOHAMMAD-ALI MASSOUMNIA, GEORGE C. VERGHESE, AND ALAN S. WILLSKY

Abstract—Using the geometric concept of an unobservability subspace, a solution is given to the problem of detecting and identifying control system component failures in linear, time-invariant systems. Conditions are developed for the existence of a causal, linear, time-invariant processor that can detect and uniquely identify a component failure, first for the case where components can fail simultaneously, and second for the case where they fail only one at a time. Explicit design algorithms are provided when these conditions are satisfied. In addition to time domain solvability conditions, frequency domain interpretations of the results are given, and connections are drawn with results already available in the literature.

I. INTRODUCTION

Failure detection and identification (FDI) is currently the subject of extensive research, and is being used in the design of highly reliable control systems. An FDI process essentially comprises two stages: residual generation and decision making. In this note we concentrate on residual generation, and refer the reader to the extensive literature on the decision-making phase of FDI (see [21], [10], and [19] for comprehensive surveys). All our discussion will be for finite-dimensional, linear, time-invariant (LTI) systems.

The output of a residual generator is, by definition, a function of time