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A PARAMETRIC LTR-SOLUTION FOR DISCRETE-TIME SYSTEMS.

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ABSTRACT.

A parametric LTR-solution for discrete-time compensators incorporating filtering observers which achieve exact recovery will be presented for both minimum and non-minimum-phase systems.

First the recovery error, which define the difference between the target loop transfer and the full loop transfer function, is manipulated into a general form consisting of the target loop transfer matrix and the fundamental recovery matrix. Based on the recovery matrix a parametric LTR-solution will be developed. At last it will be shown that the LQG/LTR solution is included in this new parametric solution as a special case.

1 INTRODUCTION.

The LQG/LTR feedback design methodology for robust model-based compensation for both continuous-time and discrete-time systems has received much attention in the recent years, see f.ex. [1-10].

For continuous-time minimum-phase systems the asymptotic recovery techniques works very effective. Unfortunately a similar asymptotic recovery technique is not feasible in discrete-time. When the LQG/LTR-solution is applied to the discrete-time case, the recovery error will in general remains finite when a prediction observer is used, [6]. However, it is still possible to make LTR, but other methods must be applied. Based on the equation for the recovery error, a parametric exact LTR-solution has been developed in [9] for minimum-phase as well as non-minimum phase systems.

If the processing time of computing the control signal is negligible in comparison to the sampling interval, a filtering observer can be used in the compensator instead of a prediction observer. It is then possible to achieve exact recovery in this case by using the LQG/LTR-solution when the system is minimum phase and has maximal number of zeros [7]. However, when these conditions isn't satisfied the LQG/LTR-solution will result in a finite recovery error, but it will still be possible to make exact recovery by applying the principle from the parametric LTR result in [9] to this case. It will then be possible to make exact recovery for both non-minimum-phase systems as well as for systems with less than the maximal number of zeros.

2 RECOVERY EQUATION.

In the following square discrete-time systems $S(A,B,C)$ are considered. The plant transfer matrix $G(z)$ is given by:

$$\begin{aligned} G(z) &= C\Phi(z)B, \dim G(z) = m \times n \\ \Phi(z) &= (Iz - A)^{-1}, \dim \Phi(z) = n \times n \end{aligned} \quad (1)$$

It will be assumed that the model is minimal and let the number of transmission zeros be p . The plant is controlled by using a feedback compensator consisting of a filtering observer in serie with a state-feedback controller.

The compensator transfer matrix $H(z)$ can be rewritten as [7]:

$$H(z) = zK(Iz - (I - F^fC)(A - BK))^{-1}F^f \quad (2)$$

where F is the full-order observer gain, F^f is the filtering observer gain (the relation between F and F^f is: $F = AF^f$, [7]) and K is the full-state feedback gain.

Now let the two step LTR-design method [1,2,3,6,7] be used for the design of $H(z)$ which consist of a target design of one

of the compensator gains (K or F) followed by a recovery design of the other gain.

In order to formulate the loop-shape robustness constraints the uncertainties (disturbance, noise and modelling errors) are reflected to the plant input [6]. The target loop transfer function is then the full-state loop transfer $K\Phi B$ and the full loop transfer is HG [2,3]. Let $E_I(z)$ define the recovery error as [6,9]:

$$E_I(z) = K\Phi(z)B - H(z)G(z) \quad (3)$$

In order to have exact recovery it is required that $E_I(z) = 0$ for all z . Now let eq. (3) be rewritten in an equivalent form:

$$E_I(z) = M_I(z)[I + M_I(z)]^{-1}[I + K\Phi(z)B] \quad (4)$$

where $M_I(z)$ is the recovery matrix defined by:

$$M_I(z) = K^f(A - FC)(Iz - A + FC)^{-1}B \quad (5)$$

where $K = K^fA$.

Proof of eq. (4) is given in the appendix.

Eq. (4) for the recovery error is equivalent to the equation defined in [6,9] when a prediction observer is used instead. The only difference is the equation for the recovery matrix.

It is simple to see by using the form of $E_I(z)$ in eq. (4) that:

$$E_I(z) = 0 \text{ iff } M_I(z) = 0 \quad (6)$$

The recovery matrix M_I will only be used for making exact recovery in this paper, but M_I have also other attractive properties. By using eq. (3) and (4) it is easy to derive that [10]:

$$S_I(z) = S_{TFL}(z)(I + M_I(z)) \quad (7)$$

where S_I and S_{TFL} are the input sensitivity functions for the full loop and the target full loop transfer function. For more details, please see [10].

3 THE PARAMETRIC LTR-SOLUTION.

Now let the equation for the recovery matrix, eq. (5), be rewritten in the residual form, [6]:

$$\begin{aligned} M_I(z) &= K^fVW^T(Iz - VW^T)^{-1}B \\ &= K^fE \sum_{i=1}^n \frac{v_i v_i^T B}{z - \tau_i} \end{aligned} \quad (8)$$

where $V = [v_1, \dots, v_n]$, $W = [w_1, \dots, w_n]$, $\Lambda = \text{diag}(\tau_1, \dots, \tau_n)$, v_i and w_i^T are right and left eigenvectors associated with the eigenvalue τ_i of $A - FC$. $VW^T = WT V = I$

It is then easy to show that:

$$\begin{aligned} M_I(z) &= 0 \text{ iff} \\ K^f v_i &= 0 \text{ or } \tau_i = 0 \text{ or } w_i^T B = 0, i = 1, \dots, n \end{aligned} \quad (9)$$

if $A - FC$ is non-defective.

From eigenstructure assignment it is known that the left eigenvector w_i^T with the eigenvalue τ_i of $A - FC$ are given by [11]:

$$[w_i^T \ z_i^T] \begin{bmatrix} \tau_i I - A \\ -C \end{bmatrix} = 0, i = 1, \dots, n \quad (10)$$

$$w_i^T F = -z_i^T$$

The last condition in eq. (9) implies that:

$$[w_{10}^T \ z_{10}^T] \begin{bmatrix} \tau_{10}I - A & B \\ -C & 0 \end{bmatrix} = 0 \quad (11)$$

Maximally p eigenvectors w_{10}^T can satisfy this condition, if τ_{10} is selected as a transmission zero of $S(A,B,C)$, [14]. Let these p eigenvalues/vectors be selected from eq. (11) so the p conditions in eq. (9) are satisfied.

The second condition in eq. (9), $\tau_i = 0$, can be satisfied by placing maximally m eigenvalues τ_i at the origin.

Again from eigenstructure assignment [11] it is easily found that with $\tau_i = 0$:

$$w_i^T = z_i^T C A^{-1}, \quad i = p+1, \dots, p+m \quad (12)$$

The choice of z_i^T depends on the type of the system which is treated: A square uniform rank system or a square non-uniform rank system.

The non-uniform rank case.

The square non-uniform rank case is defined as:

$$C A^i B = 0, \quad i = 1, \dots, a-2, \quad C A^{a-1} B \neq 0, \quad \det[C A^{a-1} B] = 0 \quad (13)$$

The m eigenvectors w_{10}^T corresponding to the m zero eigenvalues are given by, [13]:

$$w_{10}^T = e_{i-p}^T E C A^{-1}, \quad i = p+1, \dots, p+m \quad (14)$$

where E is a $m \times m$ non-singular matrix of the appropriate coefficients of the linear combination of the rows of $C A^{-1}$, see Shaked [13] for calculating E .

Again F can be parametrized by:

$$F = \begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}^{-1} \begin{bmatrix} z_1^T \\ \vdots \\ z_n^T \end{bmatrix} \quad (15)$$

$$z_i^T = z_{10}^T, \quad w_i^T = z_{10}^T C (\tau_{10}I - A)^{-1}, \quad i = 1, \dots, p$$

$$z_i^T = e_{i-p}^T E, \quad w_i^T = z_i^T C A^{-1}, \quad i = p+1, \dots, p+m$$

and $(\tau_i, z_i^T, i=p+m+1, \dots, n)$ are free design parameters.

The uniform rank system.

The square uniform rank case is defined as, see [13]:

$$C A^i B = 0, \quad i = 1, \dots, a-2, \quad \det[C A^{a-1} B] \neq 0 \quad (16)$$

The solution for the uniform rank case is given by eq. (15) with $E = I$.

If $p < n-m$ the remaining $n-p-m$ conditions in eq. (9) must be satisfied by selecting K^f suitable. The first condition in eq. (9) implies:

$$K^f [v_1, \dots, v_n] = [Q \ 0] \quad (17)$$

with $\dim Q = m \times (p+m)$ but otherwise it is arbitrary. Now

$$K = [Q \ 0] V^{-1} A = Q \begin{bmatrix} w_{10}^T \\ \vdots \\ w_{p+m}^T \end{bmatrix} A = Q \Gamma A \quad (18)$$

with $\dim \Gamma = (p+m) \times m$. Γ consist of the left eigenvectors w_{10}^T constrained in eq. (11) and in eq. (14) and is thus a matrix of fixed elements.

A simple parametrization of the controller matrices which achieves exact recovery are found. An equivalent LTR result can be found in [9] when a prediction or a minimal-order observer are used.

4 DISCUSSION.

In this section a few important consequences of exact LTR are discussed.

If the $\text{rank}[CB] = m$, i.e. $S(A,B,C)$ have maximal number of zeros, and $S(A,B,C)$ is minimum-phase, the n recovery conditions in eq. (9) can be satisfied by the selection of F , and K will be free to select. A more simple equation for F than eq. (15) is given by:

$$F = AB(CB)^{-1} \quad (19)$$

which is also the LQG/LTR solution [7], which show that the LQG/LTR solution is included as a special case.

If the LTR-results from sec. 3 are used on a non-minimum phase system $G(z)$, the resulting controller will be unstable. It is, however, still possible to achieve exact recovery for non-minimum phase systems. In order to facilitate exact recovery, note that in the selecting of F , only a subset j of the eigenvectors constrained by eq. (11) need to be chosen. The consequence is, of course, a reduction of the free parameters in K . Therefore such a selection is only advisable for non-minimum phase system if only the system's j minimum phase zeros are used in eq. (11). By doing this the following equation will be satisfied by a stable compensator:

$$K\Phi(z)B = H(z)G(z) \quad (20)$$

The non-minimum phase zeros of $G(z)$ are not cancelled out on the right hand side. Hence $H(z)$ and $K\Phi(z)$ are both non-minimum phase. This result is in agreement with the result in [15].

The selecting of F is only constrained by eq. (15) and stability can always be achieved for the observer. The parametrization of the state-feedback matrix K imply that all the closed loop eigenvalues cannot be assigned freely, and no stability guarantees are available.

If a prediction observer is used, it is simple to develop dual result for the plant output loop breaking point [6,9], which be due to the duality between the prediction observer and the state-feedback scheme. However, in spite of the missing duality when a filtering observer is used, it is still possible to develop dual result for the plant output loop breaking point. This can easily be shown by applying the observer gain F as the target design instead of K . The recovery error defined at the plant output loop breaking point is then given by:

$$E_0(z) = C\Phi(z)F - G(z)H(z) \quad (21)$$

Eq. (21) can also be rewritten into a form equivalent to eq. (4).

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APPENDIX.

Proof of eq. (4).

$$\begin{aligned} E_1(z) &= K[I - z(I - F^f C)(A - BK)]^{-1} F^f C [Iz - A]^{-1} B \\ &= K[Iz - (I - F^f C)(A - BK)]^{-1} [Iz - (I - F^f C)(A - BK) - zF^f C] [Iz - A]^{-1} B \\ &= K[Iz - (I - F^f C)(A - BK)]^{-1} (I - F^f C) (Iz - A + BK) [Iz - A]^{-1} B \\ &= K^f (Iz - (A - FC)(I - BK^f))^{-1} (A - FC) (B + BK(Iz - A)^{-1} B) \\ &= K^f (A - FC) (Iz - (I - BK^f)(A - FC))^{-1} B (I + K\Phi(z)B) \\ &= K^f (A - FC) (Iz - A + FC)^{-1} (I + BK^f(A - FC)(Iz - A + FC)^{-1})^{-1} B (I + K\Phi(z)B) \\ &= K^f (A - FC) (Iz - A + FC)^{-1} B (I + K^f(A - FC)(Iz - A + FC)^{-1})^{-1} (I + K\Phi(z)B) \\ &= M_1(z) (I + M_1(z))^{-1} (I + K\Phi(z)B) \end{aligned}$$