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REAL-TIME DETERMINATION OF THE SIGNAL-TO-NOISE RATIO OF PARTLY COHERENT SEISMIC TIME SERIES

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ABSTRACT

A suitable measure of the quality of signals used in exploration seismics is the signal-to- noise ratio (S/N) of the recorded signals (traces). However, the S/N of the single unstacked traces may vary considerably due to changing weather conditions during the exploration session. Since it thus is of great practical interest to be able to monitor the S/N while the traces are recorded an approach for fast real-time determination of the S/N of seismic time series is proposed. The described method is based on an iterative procedure utilizing the trace-to-trace coherence, but unlike procedures known so far it uses calculated initial guesses and stop criterions. This significantly reduces the computational burden of the procedure so that real-time capabilities are obtained.

1. INTRODUCTION

In marine seismic exploration it is a commonly experienced situation that the measurement conditions change during the recording session, for instance due to changing wether conditions. Since the signal-to noise ratio (S/N) of a single time series (trace) may be rather poor even in the case where the final averaged (stacked) traces are of perfectly satisfactory quality, and since this fact normally is established at a later time it is not at exploration time an easy task to judge wether the collected data are usable or not. This fact implies the risk that data collection in certain areas must be repeated during a new exploration session at a later time. If, however, the mentioned problem becomes known immediately new data in most cases could be gathered later during the on-going session. It is thus an important demand that a system for determination of the S/N for the traces works i real-time.

For this purpose a model may be used [1,2] which separates each seismic trace in a signal part, which is partly coherent with the signal part of a number of other traces, and a noise part which is not.

The model shown in fig.1 is used due to the fact that normally the input signals are not available in a

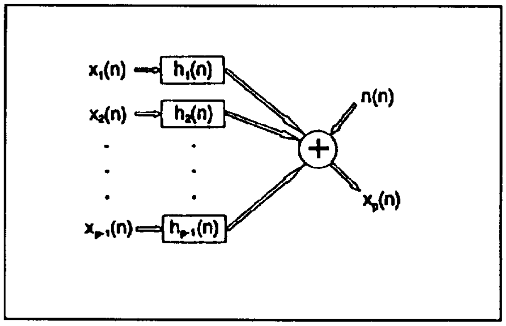

Figure 1. Noise Model for Seismic Trace

seismic experiment. From fig.1 the power density spectrum (PDS) of the output may be calculated as:

$$S_{x_p x_p}(f) = \sum_{i=1}^{p-1} H_i(f) S_{x_p x_i}(f) + S_{nn}(f)$$

where H_i are the transfer functions corresponding to the impulse responses h_i of fig.1.

Now the spectral matrix is defined as follows with the frequency parameter implied for convenience:

$$\underline{\underline{S}}_{xx} = \begin{bmatrix} S_{x_1x_1} & S_{x_1x_2} & \cdots & S_{x_1x_p} \\ S_{x_2x_1} & S_{x_2x_2} & \cdots & S_{x_2x_p} \\ \vdots & \vdots & & \vdots \\ S_{x_px_1} & S_{x_px_2} & \cdots & S_{x_px_p} \end{bmatrix}$$

The multiple squared magnitude coherence may then be

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notated [3]:

$$\gamma_p^2 = 1 - \frac{1}{S_{x_p x_p} S^{-1}_{x_p x_p}}$$

where $S^{-1}_{x_p x_p}$ is the p[th] diagonal element of the inverted spectral matrix.

Both parametric and non-parametric algorithms can be used for estimation of the spectral matrix. Normally parametric algorithms provide the best spectral resolution, but in the case of unprocessed seismic time series the two types of algorithms performe equally well in this respect because of the poor S/N. It is, however, of decisive importance that the non-parametric algorithms produce additive spectra, due to the fact that the statistical properties of the estimated multiple coherences and S/N's indicate that the calculation of the S/N should be based on the average of the multiple coherences over a number of analyzed traces. Thereby the required statistical stability is obtained and problems with singularities in the algorithm for determination of the S/N are omitted.

2. THE SIGNAL-TO-NOISE RATIO (S/N)

Before the definition of the S/N one has to realize that the time series x_j is a mix of two signal types: the signal part s_j which is a deterministic time series with finite energy and a noise part n_j which is a stochastic time series with finite power. In order to be able to compare these two signal types a window is introduced which is placed around a seismic event. The S/N of a trace k is then defined as the ratio between the energies of the two signal types measured within the window:

$$\rho_k(f) = \frac{S_{s_k s_k}(f)}{S_{n_k n_k}(f)}$$

The PDS (or more strictly the energy density spectrum) of the considered trace may now be written:

$$S_{x_k x_k}(f) = S_{n_k n_k}\left(1 + \frac{S_{s_k s_k}}{S_{n_k n_k}}\right)$$

$$= S_{n_k n_k}(f)(1 + \rho_k(f))$$

where ρ_k is the S/N as defined above.

From this expression it is possible to establish a connection between the S/N and the multiple coherence [1]:

$$\gamma_k^2 = \frac{\rho_k}{1+\rho_k} * \frac{Z-\rho_k}{1+Z-\rho_k}$$

$$Z = \sum_{i=1}^{p} \rho_i$$

where p is the number of traces considered.

3. CALCULATION OF THE S/N

Determination of the S/N based on the described model requires the estimation of a spectral matrix from which multiple coherences are calculated and an iterative computation of the S/N from the calculated multiple coherences, since an analytical solution is not possible for three or more traces.

This iterative process may constitute a problem in two respects. First, if numerous iterations become necessary it may be a problem with reasonably simple portable hardware to meet the real-time requirement. Second, numerous iterations are very often leading to problems with numerical stability.

In order to limit the computational burden of the iterations necessary three things are essential:

The accuracy of the initial guess and its computational complexity

The number of iterations and their computational complexity

The used stop criterion

In [1] it is stated that the quantity

$$\frac{\sum_{i \neq k} \rho_i}{1 + \sum_{i \neq k} \rho_i}$$

normally is close to 1 corresponding to a high S/N. It is therefore suggested that 1 is chosen as the initial guess (for the k[th] trace). However, since a more accurate initial guess will assure much faster convergence of the iteration, a novel and more accurate initial guess has been derived. In the derivation Z is considered unknown independent of ρ_k. From this the legal parameter interval of Z is found. Since the equation for γ_k^2 must have real roots the minimum of Z as a function of γ_k^2, $Z_{\min,k}$, can be found from [4]:

$$Z_{\min,k} = \frac{4\,(1-\gamma_k^2)\,\gamma_k^2 \pm \sqrt{16\,(1-\gamma_k^2)^2\,\gamma_k^2}}{2\,(1-\gamma_k^2)^2}$$

$$= 2\,\frac{\gamma_k^2 + \sqrt{\gamma_k^2}}{1-\gamma_k^2}$$

In the equation above a negative solution must be rejected and since $Z_{\min,k}$ is an increasing function of γ_k^2 for $0 \le \gamma_k^2 < 1$ the minimum of Z for all of the traces, $Z_{\min}$, is found by substituting the coherence of the trace with the highest coherence into the equation above. Now ρ_k can be found from:

$$\rho_k = \frac{Z}{2} \pm \frac{\sqrt{Z^2 - 4\,(Z+1)\,\gamma^2/(1-\gamma^2)}}{2}$$

$$= \frac{Z}{2}\left(1 \pm \sqrt{1 - 4\,(Z+1)\,\gamma_k^2\,/\,(Z^2\,(1-\gamma^2)}\,\right)$$

For $k \neq k_{\max}$ where $k_{\max}$ corresponds to the trace with the largest coherence one gets:

$$\rho_k^{(1)} = \frac{Z}{2}\left(1 - \sqrt{1 - \frac{4\,(Z+1)\,\gamma_k^2}{Z^2\,(1-\gamma_k^2)}}\right)$$

where superscript (1) indicates that it is the first iteration. For $k = k_{\max}$ one gets:

$$\rho_{k_{\max}}^{(1)} = \frac{\gamma_{k_{\max}}^2\left(1 + \sum_{i \neq k_{\max}} \rho_i^{(1)}\right)}{\sum_{i \neq k_{\max}} \rho_i^{(1)} + \gamma_k^2\left(1 + \sum_{i \neq k_{\max}} \rho_i^{(1)}\right)}$$

where the summations are made over the S/N's for all traces except the trace with the largest coherence. With these initial guesses for the S/N of the traces it turns out that normally extremely few iterations are necessary. As a matter of fact in most cases the initial guess itself may be sufficiently good so that no iterations at all are necessary.

The decision wether more iterations are necessary or not depends of course on the chosen stop criterion. Since the estimated coherence which is used for the calculation of the S/N has a certain variance it seems reasonable to stop the iteration procedure when the calculated S/N corresponds to a coherence which relative to this variance does not deviate considerably from the mentioned estimated coherence. This leads to the following stop criterion:

$$\left(\gamma_k^2 - \frac{\rho_k^{(j)}}{1+\rho_k^{(j)}} * \frac{\sum_{i \neq k} \rho_i^{(j)}}{1 + \sum_{i \neq k} \rho_i^{(j)}}\right)^2 \le \frac{C}{BT}\,2\,\gamma_k^2\,(1-\gamma_k^2)^2$$

where C is a constant which e.g. can be chosen to 1/64 so that the error from the iteration becomes suitably insignificant compared to the variance on the estimated coherence. BT is the time-bandwidth product of the seismic signal.

4. ITERATION ALGORITHMS

A number of iteration algorithms has been investigated and it turns out that an algorithm which handles the trace with the highest coherence as the last trace has the best performance in the actual situation where especially the total computational burden of the calculation is essential. This is despite the fact that the mentioned algorithm converges slower than e.g. a Newton-Raphson algorithm.

For $k = 1,\ldots, k_{\max}-1, k_{\max}+1,\ldots, p$ the S/N after the j'th iteration becomes:

$$\rho_k^{(j)} = \frac{\gamma_k^2\,(1+\Sigma)}{\Sigma - \gamma_k^2\,(1+\Sigma)}$$

where

$$\Sigma = \sum_{i<k} \rho_i^{(j)} + \sum_{i>k} \rho_i^{(j-1)}$$

For $k = k_{\max}$ one gets:

$$\rho_{k_{\max}}^{(j)} = \frac{\gamma_{k_{\max}}^2(1+\Sigma)}{\Sigma - \gamma_{k_{\max}}^2(1+\Sigma)}$$

where

$$\Sigma = \sum_{i \neq k_{\max}} \rho_i^{(j)}$$

5. EXPERIMENTS

The described method has been implemented [5] using a scalable DSP multi-processor system based on PC

ISA/EISA-bus compatible interconnected DSP building blocks [6]. Since in a practical situation one would monitor only a limited number of seismic traces with respect to their S/N real-time capabilities are easily obtained in any normal seismic experiment. An example of the determined S/N as a function of frequency for a trace recorded in the North Sea is shown on fig. 2. The analysis window was placed around a seismic event and the S/N for this single unstacked trace was rather poor (from approximately 5 dB to approximately -20 dB).The spectral matrix was calculated using a modified periodogram and a Parzen window.

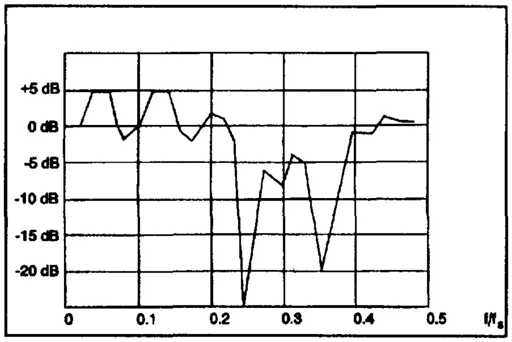

Figure 2. Signal-to-Noise Ratio for Unstacked Seismic Trace. The sampling frequency f_s = 1 kHz.

6. CONCLUSIONS

In this paper a novel approach for fast real-time determination of the signal-to-noise ratio of seismic time series is proposed. Unlike procedures known so far it uses a calculated initial guess and stop criterion. This significantly reduces the computational burden of the procedure. Furthermore, the use of coherences averaged over a number of seismic traces increases the statistical stability and removes hitherto known numerical problems originating from singularities in the used spectral matrix.

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