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Space Charge Fields in DC Cables

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Abstract: **The space charge that accumulates in DC cables can, mathematically, be resolved into two components. One is related to the temperature and the other to the magnitude of the electric field strength. Analytical expressions for the electric fields arising from each of these space charge components are derived. Thereafter, the significance of these field components under both normal operating conditions and immediately following polarity reversal is discussed.**

INTRODUCTION

Previously, the authors have shown that the accumulation of space charge in DC cable insulation is an inherent phenomenon [1]. The presence of such space charge in the bulk of the dielectric constrains the field to be Poissonian. The corresponding field strength $\vec{E}$ can thus be expressed as

$$\vec{E} = \vec{E}_L + \delta\vec{E} \tag{1}$$

where $\vec{E}_L$ represents the basic Laplace field [2] associated with the applied voltage U, and $\delta\vec{E}$ represents the basic Poisson field [2]. This latter component is established by the space charges in the dielectric together with the associated Poissonian induced charges [3] on the electrodes.

Traditionally, electric fields associated with DC cables are evaluated directly using the functional dependence of the insulation conductivity on temperature and field strength. This circuit theory approach is used in many papers and textbooks on cable technology. In the present study, analytical expressions for the electric field are derived from a knowledge of the inherent space charge distribution: i.e. a field theoretical approach is adopted. This approach allows the significance of $\delta\vec{E}$ under steady-state operation and for conditions immediately following polarity reversal to be discussed in depth.

† deceased

SPACE CHARGE IN THE INSULATION

Consider a coaxial cable insulated with a macroscopically homogeneous dielectric of constant permittivity. The conductivity γ of this dielectric is a function of both the temperature T and the magnitude of the electric field strength $|E|$. In this study, the γ dependence is represented by an empirical relationship which is valid over the practical range of T and $|E|$ [4]; viz.

$$\gamma = \gamma_a(|E|/|E_a|)^{\nu}\exp[\alpha(T - T_a)] \tag{2}$$

where γ_a is the conductivity for a reference field strength E_a and temperature T_a. The parameter ν is a material constant which for oil-impregnated paper is approximately zero, but which for polyethylene can be ascribed values in the range 2.1 to 2.4 [4]: α is a constant and for the dielectrics used in cables the value 0.1 K^{-1} is appropriate.

For a loaded DC cable, which is in

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thermal equilibrium, it has been shown in [1] that the space charge density ρ may mathematically be considered as the sum of two components:

$$\rho = \rho_T + \rho_{|E|} \tag{3}$$

where ρ_T is the component associated with temperature while $\rho_{|E|}$ is that related to the magnitude of the electric field strength. In [1] expressions were derived for these components: viz.

$$\rho_T = \frac{\varepsilon k\beta U}{a^2} \frac{(r/a)^{k-2}}{(b/a)^k - 1} \tag{4}$$

$$\rho_{|E|} = \frac{\varepsilon k(k - \beta)U}{a^2} \frac{(r/a)^{k-2}}{(b/a)^k - 1} \tag{5}$$

where a and b are the inner and outer radii of the dielectric, and r is a cylindrical coordinate. U represents the applied DC voltage while k and β are dimensionless parameters given by

$$k = \frac{\nu + \beta}{\nu + 1} \tag{6}$$

$$\beta = \alpha(T_a - T_b)/\ln(b/a) \tag{7}$$

T_a and T_b are the temperatures at distances a and b from the cable axis. From (3), (4) and (5) it is evident that

$$\rho = \frac{\varepsilon k^2 U}{a^2} \frac{(r/a)^{k-2}}{(b/a)^k - 1} \tag{8}$$

and thus ρ_T and $\rho_{|E|}$ may be simply expressed in terms of ρ: viz.

$$\rho_T = (\beta/k)\rho \tag{9}$$

$$\rho_{|E|} = (1 - \beta/k)\rho \tag{10}$$

FIELD SOURCES

Insulation Space Charge

In evaluating the field distribution as a function of the radial distance r, it is necessary to refer to the total charge Q per unit length enclosed within a cylindrical shell of thickness $(r - a)$:

$$Q = \int_a^r \rho 2\pi r' \mathrm{d}r' \tag{11}$$

where r' is a dummy variable. Upon substitution for ρ, (8), and integrating we obtain

$$Q = 2\pi\varepsilon kU \frac{(r/a)^k - 1}{(b/a)^k - 1} \tag{12}$$

Hence on the basis of (4) and (5) we arrive at the simple relationships

$$Q_T = (\beta/k)Q \tag{13}$$

$$Q_{|E|} = (1 - \beta/k)Q \tag{14}$$

Charge Induced by the Space Charge

For the volume space charge distribution, the Poissonian induced charge q may be expressed as [2]

$$q = -\int \lambda(r)\rho \mathrm{d}\Omega \tag{15}$$

where $\mathrm{d}\Omega$ is a volume element. The parameter λ is the proportionality factor between the space charge accumulated in the insulation and the charge induced on the conductor in question. The λ-function is a solution of the general Laplace equation [2]

$$\vec{\nabla}\cdot(\varepsilon\vec{\nabla}\lambda) = 0 \tag{16}$$

in which ε denotes permittivity. The relevant boundary conditions are $\lambda = 1$ for $r = a$ and $\lambda = 0$ for $r = b$. For the present simple case of the coaxial geometry with a homogeneous dielectric, the appropriate solution of Laplace's equation for the λ-function is

$$\lambda = 1 - \frac{\ln(r/a)}{\ln(b/a)} \tag{17}$$

Owing to the axial symmetry of the space charge distribution, it is however more appropriate to consider q as the charge induced per unit length. Hence for a cylindrical shell of radius r and

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thickness dr we have

$$q = -\int_a^b \lambda\rho 2\pi r \mathrm{d}r \tag{18}$$

Upon combining (17) with (8) and integrating, we can express q as

$$q = 2\pi\varepsilon kU\left[1 - \frac{(b/a)^k - 1}{k\ \ln(b/a)}\right] \tag{19}$$

The components of q follow directly: viz.

$$q_T = (\beta/k)q \tag{20}$$

$$q_{|E|} = (1 - \beta/k)q \tag{21}$$

As the induced charge is of *opposite* polarity to the source charge, see (18), this implies that the field components associated with Q and q are in opposition.

ELECTRIC FIELDS

In view of the symmetry of the space charge distribution, the basic Poisson field strength δE at a distance r is given by

$$\delta E = \frac{Q + q}{2\pi\varepsilon r} \tag{22}$$

Owing to symmetry, all vector quantities will be directed either radially away from or towards the axis. We can therefore replace all vector equations with scalar equations. The direction of the field is away from the axis for E positive. Upon introducing (12) and (19) into (22), we obtain

$$\delta E = \frac{U}{r\ln(b/a)}\left[\frac{k\ \ln(b/a)}{(b/a)^k - 1}(r/a)^k - 1\right] \tag{23}$$

and the associated components of δE are

$$\delta E_T = (\beta/k)\delta E \tag{24}$$

$$\delta E_{|E|} = (1 - \beta/k)\delta E \tag{25}$$

For the coaxial geometry, the basic Laplace field is simply given by

$$E_{\mathrm{L}} = \frac{U}{r\ln(b/a)} \tag{26}$$

Thus on the basis of (1), the net steady state field E can be expressed as

$$E = \frac{U}{a}\,\frac{k(r/a)^{k-1}}{(b/a)^k - 1} \tag{27}$$

It should be noted that this field expression is identical with that derived in [4] using the traditional approach.

The reversal of the applied voltage polarity can occur on a time-scale which is very short in comparison to the relaxation time constant associated with the accumulation of space charge in the cable insulation. Thus, immediately following such a reversal of polarity, the magnitude and distribution of the space charges are virtually unaffected. Consequently, although the basic Laplace field has changed polarity, the basic Poisson field is effectively unchanged. Hence following polarity reversal, the *initial* field E_{R} may be expressed as

$$E_{\mathrm{R}} = -\frac{U}{a}\left[\frac{2}{(r/a)\ln(b/a)} - \frac{k(r/a)^{k-1}}{(b/a)^k - 1}\right] \tag{28}$$

APPLICATION OF THEORY

To illustrate the order of magnitude of the effects of the accumulated space charge in a coaxial DC cable, we consider the 250 kV, 500 MW XLPE cable proposed by Fukagawa *et al.* [5]. As ν can attain values between 2.1 and 2.4, we assume for expediency a value of 2.25. The parameter β can be evaluated if the power dissipated in the conductor per unit length P_{c} is known [1]. The relationship is

$$\beta = \frac{\alpha P_{\mathrm{c}}}{2\pi\kappa} \tag{29}$$

We assume that the temperature of the conductor has the specified maximum value of 80 $^{\mathrm{o}}$C, and that the thermal

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conductivity κ of XLPE is 0.3 W/Km. Thereafter, on the basis of the data given in [5], we obtain $\beta = 4.53$.

Using (6), (8), (9) and (10), the radial variations of ρ, ρ_T and $\rho_{|E|}$ were derived. Results are shown in Figure 1, in which x is the radial distance from the surface of the inner conductor; i.e. $x = r - a$. On letting d denote the insulation thickness, we have $d = b - a$. From Figure 1, it is evident that the space charge distributions are only slightly non-uniform in that these distributions deviate by <5% from their average values. Moreover $|\rho_T|$ attains values of about twice $|\rho_{|E|}|$. However, as these components are of opposite polarity, the net space charge ρ is about $(\rho_T/2)$ such that ρ lies in the range of $12 < \rho/(\mathrm{pC/mm^3}) \lesssim 13$.

The electric fields associated with the space charge were evaluated using (6) (23) to (27) and (29) and the results are illustrated in Figure 2. The behaviour of δE, δE_T and $\delta E_{|E|}$ can be understood with reference to the corresponding space charge distributions, where we recall that the induced charge is of opposite polarity to the space charge. As a consequence of this latter feature, the values of δE, δE_T and $\delta E_{|E|}$ are zero for

$$(r/a) = \left[\frac{(b/a)^k - 1}{k\ \ln(b/a)}\right]^{1/k} \tag{30}$$

see (23). With respect to Figure 2, we have the relationship

$$(x/d) = \frac{(r/a) - 1}{(b/a) - 1} \tag{31}$$

Upon using the present values of k and (b/a), we find that $(x/d)_0 = 0.47$, see Figure 2. As a result of the space charge field δE changing polarity at $(x/d)_0$, this field opposes E_L for $(x/d) < (x/d)_0$,

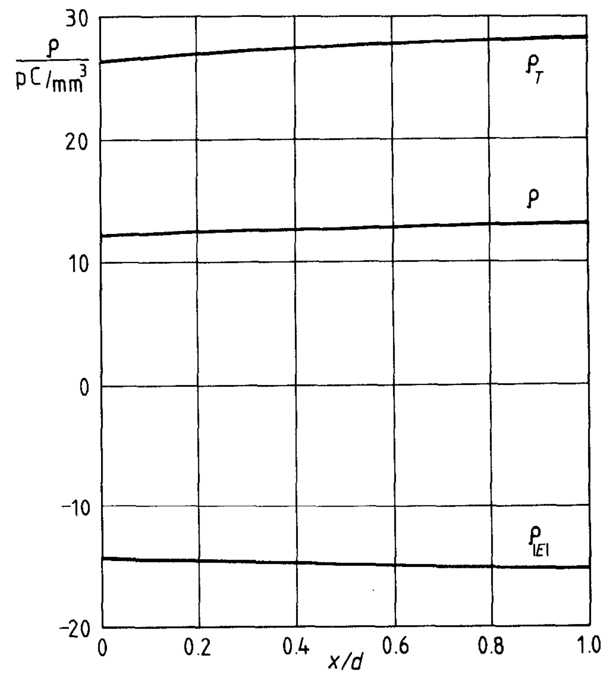

Figure 1. Radial variation of space charge ρ and its components in a loaded DC cable.

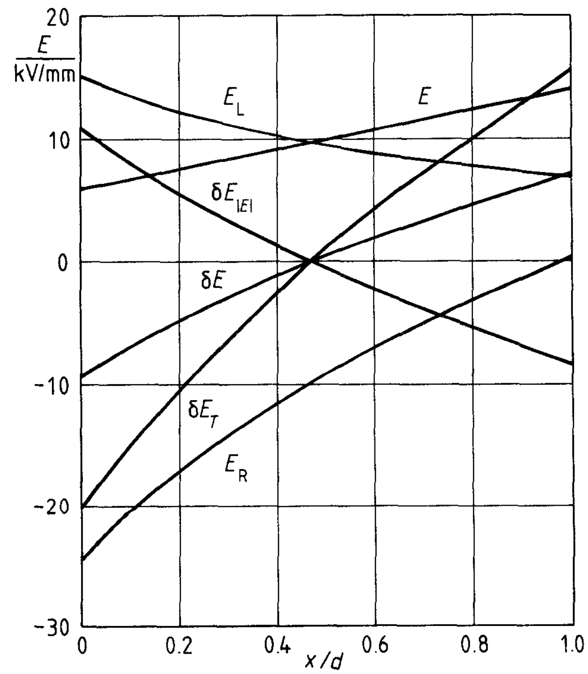

Figure 2. Radial variation of electric field distributions E in a loaded DC cable.

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whereas δE augments E_L for $(x/d) > (x/d)_0$ such that we obtain stress inversion: i.e. the greatest stress now occurs at the cable sheath.

For the present ν and β values, the exponent of the radial distance, $(k-1)$, is 1.08. As a result, the steady-state field E varies essentially linearly with the radial distance. Following polarity reversal, the magnitude of the field at the inner conductor is seen to be increased by 61% above that of E_L, see Figure 2.

CONCLUSION

From a knowledge of the inherent space charge distribution in a DC cable, it is possible to identify the influence upon the electric field distribution of the parameters which control the insulation conductivity. For a loaded cable, it is shown that the major influence on the field distribution is associated with temperature, although this is seen to be counteracted partially by that of the electric field. This behaviour suggests that the conductivity parameters could be selected such as to minimise the resultant space-charge-generated electric field. Such an approach would reduce overstressing of the insulation which otherwise would certainly be encountered on polarity reversal.

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