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Problem Setting

GBD Principle

GBD Results

Local Failure Criteria

Summary and Conclusions

Global Optimal Design of Composite Laminates Including Failure Criteria Using Decomposition Techniques

Eduardo Muñoz, Mathias Stolpe

DTU Mathematics, Denmark

May 17, 2010

Problem Setting

Discrete material optimization (DMO) with 0-1 design variables.



- Material selection among a discrete set of candidates materials.
 - t:= Density of material block (design variables):
 - t_{ij} = Density of the material *j* used in the element *i*.
 - u_l := Displacement due to the load condition f_l , l = 1, ..., m.
 - M:= Available amount of material.

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• Material selection among a discrete set of candidate materials.



Composite Laminates Including Failure Criteria Using Decomposition Techniques

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- Materials are defined by the specific stress-strain relationship.
- Material candidates could for example, be the same material oriented in specific angles (orthotropic materials).
- In this case, the problem becomes an angle selection problem.

Problem Formulation

Minimum compliance, multi-material, local Failure problem:

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minimize	$\max_{1 \le l \le m} \{f_l^T u_l\}$		(Compliance)
s.t.	$K(t)u_l = f_l,$	$l = 1, \ldots, m$	(Equilibrium)
(P)	$\rho^T t \leq M,$		(Mass)
	$t_{ij}\in\{0,1\},$	$\forall i, j.$	(0-1 cond.)
	$\sum_{i} t_{ij} = 1$,	$i = 1,, n^c,$	(Mat. Select.)
	$F(x, u_I) \leq 0,$	$l=1,\ldots,m.$	(Local Failure.)

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 $\mathcal{K}(t) = \sum_{i \ i} t_{ij} \mathcal{K}_{ij}, \quad \mathcal{K}_{ij} \succeq 0,$ linear elasticity.

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• Solve (P) to global optimality is a quite difficult task, even if no failure is considered.

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- Solve (P) to global optimality is a quite difficult task, even if no failure is considered.
- The task becomes even more difficult if the considered failure criterion does not have useful mathematical properties.

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Global Optimization

- Solve (P) to global optimality is a quite difficult task, even if no failure is considered.
- The task becomes even more difficult if the considered failure criterion does not have useful mathematical properties.
- Only few existing works in this area (and none for multimaterial problems).

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Mixed Integer formulation: Features

• We propose to use the Generalized Benders' Decomposition (GBD) algorithm to attack problem (P).

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Mixed Integer formulation: Features

- We propose to use the Generalized Benders' Decomposition (GBD) algorithm to attack problem (P).
- The GBD method in general does not converge to global optima. Nevertheless, it does for the problem (P) when no failure criterion is considered.

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- We propose to use the Generalized Benders' Decomposition (GBD) algorithm to attack problem (P).
- The GBD method in general does not converge to global optima. Nevertheless, it does for the problem (P) when no failure criterion is considered.
- The main reason is the convexity of the compliance function as a function only in the design variable *t*.

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- We propose to use the Generalized Benders' Decomposition (GBD) algorithm to attack problem (P).
- The GBD method in general does not converge to global optima. Nevertheless, it does for the problem (P) when no failure criterion is considered.
- The main reason is the convexity of the compliance function as a function only in the design variable *t*.
- This issue must be taken into account when attacking the problem (P) (i.e., including local failure).

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Figure: Generalized Benders' Decomposition Method

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 $t \in \{$

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GBD Principle

GBD: Relaxed Master Problem (m=1)

$$\begin{array}{ll} \underset{\in \{0,1\}^n, y \in \mathbb{R}}{\text{minimize}} & y \\ s.t. & \nu^{k}{}^T t - y \leq -2f{}^T u^k \qquad k = 1, ..., N \\ & \sum_{i,j} \rho_j t_{ij} \leq M, \\ & \sum_j t_{ij} = 1 \qquad \forall i \end{array}$$
(RMP)

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• $\nu^{k^{T}} = -u^{k^{T}} \nabla_{t} K(t) u^{k} = -(u^{k^{T}} K_{1} u^{k} u^{k^{T}} K_{2} u^{k} \dots u^{k^{T}} K_{n} u^{k})$

- u^k is the displacement solution to $K(t^k)u^k = f$.
- The *t^k*'s is the solution of the (*k* 1)-th Relaxed Master Problem (RMP)

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GBD Performance

- Heuristics to find candidate solutions improve the performance of the method.
 - Solve Sub-MIP problem by GBD.
- Designs without failure criterion:
 - C. Hvejsel, today 18h00, Solution with less than 2 % global optimality gap, for a multilayered problem of 23.000 design variables.

Local Failure Criteria

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- Local failure criteria + Global Optimization: Possible?
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- · Conoral types of failure criteria for composite structures
- General types of failure criteria for composite structures
- Max stress, max strain, Tsai-Hill, Tsai-Wu, Puck, Cuntze, Hashin, etc
- It is not clear which is the most convenient failure criteria for composite structures.
- In general, all local failure criteria functions are not convex.
- GBD does not guarantee global solutions in this case.

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- Max strain: $Au \leq b$
- Tsai- Wu/Tsai-Hill:

$$\frac{1}{2}u^{\mathsf{T}}W_j(t)u+w_j(t)u\leq Y_j \qquad \forall j=1,...,n$$

$$W_j(t) = \sum_i t_{ij} W_{ij}, \quad w_j(t) = \sum_i t_{ij} w_{ij}$$

- W(t) positive semidefinite matrix.
- If the failure is non convex a convex reformulation (if possible) is necessary for using GBD or any Global Optimization technique.

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• ''Big-M" reformulation of bilinear equations (Stolpe and Svanberg 2001)

 $z_{ij} = t_{ij}u$

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• ''Big-M" reformulation of bilinear equations (Stolpe and Svanberg 2001)

 $\displaystyle \Longleftrightarrow \ t_{ij}c_{ij}^{min} \leq z_{ij} \leq t_{ij}c_{ij}^{max} \ (1-t_{ij})c_{ij}^{min} \leq u-z_{ij} \leq (1-t_{ij})c_{ij}^{max}$

 $z_{ii} = t_{ii} u$

c^{min}, c^{max} are convenient bounds.

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GBD for Problem (P)

- The ''Big-M'' reformulation can be applied to create GBD cuts for the max strain, max stress, Tsai-Hill/Wu failure criteria.
- However, the constraints generated by this method are in general too weak (there is no impact in performance).
- Algorithm becomes slower instead of becoming faster.
- Only possible for a few local failure criteria (if a convex reformulation exists).
- Even a feasible design is very difficult to find.

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Toy Example: Topology Design, 24 DV

• No failure criterion, minimum compliance, connectivity imposed, optimal solution found



• Optimality gap: 0.5 %, CPU-time: 38[s], 54 iterations.

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maximum strain value: 5.397

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• Strategy: Use this value as reference for setting the limit for the strain.

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- Strategy: Use this value as reference for setting the limit for the strain.
- Strain limit imposed

 $\| \epsilon \| \le 5.397 - 0.001 = 5.396.$

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• Attack (P) for a max strain failure criterion with the proposed GBD algorithm.

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- Attack (P) for a max strain failure criterion with the proposed GBD algorithm.
- Result: Algorithm runs for 12[h], and not a single feasible solution is found.

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- Strategy: Use this value as reference for setting the limit for the strain.
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 $\parallel \epsilon \parallel \le 5.397 - 0.001 = 5.396.$

- Attack (P) for a max strain failure criterion with the proposed GBD algorithm.
- Result: Algorithm runs for 12[h], and not a single feasible solution is found.
- Conclusion: Failure feasibility constraints from the GBD method are too weak for expecting an acceptable performance.

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- Alternative: Solve the problem (P) in two stages. First, without the local failure criterion.
- When the problem is solved to optimality, reset the Upper bound to +∞. Restart the GBD algorithm considering the local failure criterion in the problem formulation.
 - If the current design at iteration k, t^k , is infeasible for the local failure, include a single linear constraint preventing t^k (and only t^k) to be feasible in the master problem.

$$c^T t^k \leq b^k, \quad c \in \mathbb{R}^{n+1}, b^k \in \mathbb{R}.$$

• The method can be used for any kind of failure function $F(x, u_l)$ (no special properties are needed).



- Angle selection problem: +45, -45, 0, 90.
- 400 FE discretization.
- 100 design element discretization × 4 candidate angles: 400 DV.

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No Failure Solution

• No failure criterion, minimum compliance, solution found.



- Optimality gap: 2.64 %, compliance: 15.4716.
- maximum strain value: 5.74397E-03. CPU-time: 12[h]

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- Strategy: Use this value as reference for setting the limit for the strain.
 - Strain limit imposed

$$\| \epsilon \| \le 5.74397 \cdot 10^{-3} - 1 \cdot 10^{-8} = 5.74396 \cdot 10^{-3}.$$

• Attack (P) with the proposed GBD algorithm.

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• Failure criterion included, solution found.



- Optimality gap: 2.87 %, Objective Value: 15.4902.
- maximum strain value: 5.74375 · 10⁻³. CPU-Time: 43[h]

Comparison

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(a) No local failure



(b) Max strain failure criterion

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Summary and Conclusions

- Generalized Benders' Decomposition can solve medium size structural design problems to optimality.
 - The inclusion of local failure criteria makes the feasible set smaller.
 - → Faster convergence is expected. However, the opposite occurs.

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• The minimum compliance problem problem (P) + local failure.

can be attacked by Generalized Benders' Decomposition.

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• The minimum compliance problem problem (P) + local failure.

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 $ullet \Longrightarrow$ The algorithm converges theoretically to a global minimum

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• The minimum compliance problem problem (P) + local failure.

can be attacked by Generalized Benders' Decomposition.

 $ullet \Longrightarrow$ The algorithm converges theoretically to a global minimum

solution in a finite number of steps, or stops with an infeasibility flag (if the problem is infeasible).

• The method can be used for almost any failure criterion, independently of convexity assumptions.

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Thanks for your attention!

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