Delta self-consistent field method to obtain potential energy surfaces of excited molecules on surfaces

Gavnholt, Jeppe; Olsen, Thomas; Engelund, Mads; Schiøtz, Jakob

Published in:
Physical Review B Condensed Matter

Link to article, DOI:
10.1103/PhysRevB.78.075441

Publication date:
2008

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Δ self-consistent field method to obtain potential energy surfaces of excited molecules on surfaces

Jeppe Gavnholt, Thomas Olsen, Mads Engelund, and Jakob Schiøtz

Danish National Research Foundation’s Center for Individual Nanoparticle Functionality (CINF), Department of Physics, Technical University of Denmark, DK-2800 Kongens Lyngby, Denmark

(Received 9 June 2008; revised manuscript received 15 July 2008; published 27 August 2008)

We present a modification of the Δ self-consistent field (ΔSCF) method of calculating energies of excited states in order to make it applicable to resonance calculations of molecules adsorbed on metal surfaces, where the molecular orbitals are highly hybridized. The ΔSCF approximation is a density-functional method closely resembling standard density-functional theory (DFT), the only difference being that in ΔSCF one or more electrons are placed in higher lying Kohn-Sham orbitals instead of placing all electrons in the lowest possible orbitals as one does when calculating the ground-state energy within standard DFT. We extend the ΔSCF method by allowing excited electrons to occupy orbitals which are linear combinations of Kohn-Sham orbitals. With this extra freedom it is possible to place charge locally on adsorbed molecules in the calculations, such that resonance energies can be estimated, which is not possible in traditional ΔSCF because of very delocalized Kohn-Sham orbitals. The method is applied to N₂, CO, and NO adsorbed on different metallic surfaces and compared to ordinary ΔSCF without our modification, spatially constrained DFT, and inverse-photoemission spectroscopy measurements. This comparison shows that the modified ΔSCF method gives results in close agreement with experiment, significantly closer than the comparable methods. For N₂ adsorbed on ruthenium (0001) we map out a two-dimensional part of the potential energy surfaces in the ground state and the 2π resonance. From this we conclude that an electron hitting the resonance can induce molecular motion, optically with 1.5 eV transferred to atomic movement. Finally we present some performance test of the ΔSCF approach on gas-phase N₂ and CO in order to compare the results to higher accuracy methods. Here we find that excitation energies are approximated with accuracy close to that of time-dependent density-functional theory. Especially we see very good agreement in the minimum shift of the potential energy surfaces in the excited state compared to the ground state.

DOI: 10.1103/PhysRevB.78.075441 PACS number(s): 31.15.xr, 31.50.Df, 82.20.Gk

I. INTRODUCTION

Density-functional theory (DFT) has proved to be a vital tool in gaining information on many gas-surface processes. This may be surprising, since DFT is only valid for relaxed systems in their ground state and therefore not directly applicable to dynamical situations. However, often the electrons relax much faster than the time scale of the atomic movement, such that the electron gas can be considered relaxed in its ground state at all times. Then potential energy surfaces (PES) of the ground state obtained by DFT, or any other method, can be used to describe the motion of atomic cores. This is the Born-Oppenheimer approximation.

In some situations, however, the Born-Oppenheimer approximation is not valid. This is for example the case when the electronic system is excited by a femtosecond laser or hot electrons are produced with a metal-insulator-metal junction. The Born-Oppenheimer approximation also breaks down if the time scales for the electronic and nuclear motions are comparable or if the separations between the electronic states are very small, such that transitions between the electronic states will occur. In these situations it is necessary to go beyond the Born-Oppenheimer approximation either by considering the coupling between electronic states where it becomes necessary to obtain PESs of excited states, or by an electronic friction model.

The problem of calculating excitation energies is being approached in many different ways, even within DFT. Time dependent density-functional theory (TDDFT) gives, compared to the computational cost, good agreement with experiments for excitations in atoms and molecules. However, TDDFT suffers some problems in excitations involving charge transfer. The GW approximation can be used to gain accurate excitation energies for molecules and clusters. The embedding method, which combines high-accuracy quantum chemistry methods with DFT, makes it possible to handle larger periodic systems with great accuracy. The embedding theory has been applied to estimate PESs of excited molecules on surfaces. However, the computational cost and involved complexity are still very high. Our aim has been to find a method, which at a computational cost close to the level of ground-state DFT, can estimate excited-state energies of molecules on surfaces with reasonable accuracy. Such a method would make it more feasible to consider a large range of systems in search of systems with interesting or desired properties.

Constrained DFT (Refs. 7, 18, and 19) and Δ self-consistent field (ΔSCF) (Refs. 20 and 21) are two different approaches, which both can be considered as small extensions of ground-state DFT, such that the computational cost lies close to that of ground-state DFT. In constrained DFT an additional potential is introduced and varied until a certain constraint on the electrons is fulfilled. The simplest approach is to lower (or increase) the potential in a certain part of space until you have the desired number electrons in this area. A different approach is to introduce potentials on the orbitals in a localized basis set, which depends on the orbitals’ positions in space. In Sec. III we will argue that when...
considering molecular resonance states on surfaces it may be problematic with such a strict constraint on the electrons, since a part of the charge may return to the surface on a much shorter time scale than the lifetime of the resonance.

In the $\Delta$SCF scheme the positions of the electrons are controlled by controlling the occupation of the Kohn-Sham (KS) states as the system reaches self-consistency. The $\Delta$SCF scheme has for a long time been justified in cases, where the excited state corresponds to the lowest state of a given symmetry. The scheme has, however, often been applied to more general cases. More recently, Görling extended the KS formalism to include excited states, such that $\Delta$SCF gets a formal justification in the general case, although a special unknown orbital-dependent exchange-correlation potential should be used for the excited states. In practical implementations standard exchange-correlation potentials from ground-state DFT are typically used.

This traditional way of just controlling the occupation of the KS orbitals has some limitations. For example when a molecule is placed on a metallic surface the molecular orbitals will hybridize with the orbitals in the surface, such that the molecular orbitals will be spread over several KS states. For such systems there is no good way of representing a resonance on the molecule as a change in the occupations of the KS orbitals. The optimal thing one can do within this scheme is to occupy the KS orbital with the largest overlap with the molecular orbital in question, but this overlap can be quite small and highly system size dependent. This problem was also pointed out by Hellman et al. and Behler et al.

In this paper we modify the $\Delta$SCF approach, such that electrons are allowed to occupy arbitrary linear combinations of KS orbitals. In this way one achieves much better control on the position of the excited electron. As is the case for traditional $\Delta$SCF some knowledge of the resonance is needed in order to apply the method. The method is especially relevant in Newns-Anderson-type systems, where a resonance can be attributed to a known single level, which has been hybridized through interactions with other states. This includes systems with molecules adsorbed on metal surfaces and molecules trapped between to metal contacts.

The modification we propose only has minor implications on the way practical calculations are performed, which is very similar to performing an ordinary ground-state DFT calculation. In the following we will go through the details of the method and apply it to a few diatomic molecules on metallic surfaces. The obtained results will be compared to the ordinary $\Delta$SCF method, spatially constrained DFT, and inverse-photoemission spectroscopy (IPES) measurements. Finally we present some tests on the performance of the $\Delta$SCF approach on $N_2$ and CO in the gas phase.

II. METHOD

In the following we go through the differences between the linear-expansion $\Delta$SCF method we propose, ordinary $\Delta$SCF, and standard DFT. We start by stating the modification of the KS equations when considering an electron excited from the Fermi level to a higher lying state. Then we show how this affects the energy calculation. Finally we generalize the approach to other types of excitations.

A. Kohn-Sham equations

The fundamental KS equations represent a practical way of finding the ground-state electron density for a given external potential and a given number of electrons through an iterative process

$$
\left[ -\frac{\nabla^2}{2} + v_{KS}[n](\mathbf{r}) \right] \psi_i(\mathbf{r}) = E_i \psi_i(\mathbf{r}), \tag{1}
$$

where $v_{KS}$ is the KS potential, $E_i$ is the energy of the $i$th electron, and $n(\mathbf{r})$ is the density.

In the linear-expansion $\Delta$SCF method we propose, the excited electron is not forced to occupy a KS orbital, but can occupy any orbital that is a linear combination of empty KS orbitals

$$
\psi^{exc}(\mathbf{r}) = \sum_{i=N}^M a_i \psi_i(\mathbf{r}), \tag{5}
$$

where $\psi_i(\mathbf{r})$ is the $i$th KS orbital, $\psi^{exc}(\mathbf{r})$ is the excited electron, and $a_i$ are the expansion coefficients.

In the linear-expansion $\Delta$SCF method we propose, the excited electron is not forced to occupy a KS orbital, but can occupy any orbital that is a linear combination of empty KS orbitals

$$
n(\mathbf{r}) = \sum_{i=1}^{N-1} n_i \psi_i(\mathbf{r}) \psi_i^*(\mathbf{r}) + \sum_{i,j=N}^M a_i^* a_j \psi_i^*(\mathbf{r}) \psi_j(\mathbf{r}), \tag{6}
$$

where $n_i$ is the density of the $i$th KS orbital, $a_i$ are the expansion coefficients, and $M$ is the number of KS orbitals in the calculation.
\[ \psi^{\text{res}}(\mathbf{r}) \text{ resembles the relevant molecular orbital as much as possible, i.e.,} \]
\[ a_i = \frac{\langle \psi | \phi \rangle}{\left( \sum_j |\langle \psi | \phi \rangle|^2 \right)^{1/2}}, \]  
\[ \text{where } \phi \text{ is the molecular orbital. This is consistent with a Newns and Anderson picture, where the resonance corresponds to an electron getting in the molecular orbital, but the resonance broadening and energy shift are due to hybridization with the metallic bands and an image charge effect.} \]

In calculations with \( k \)-point sampling the linear expansion is performed independently in all \( k \) points. In the linear-expansion \( \Delta \text{SCF} \) one then avoids the difficulties one can encounter in choosing which KS state to occupy in each \( k \) point in the traditional way of performing \( \Delta \text{SCF} \) calculations. For example, one may risk occupying different bands in each \( k \) point, when just choosing the KS orbital with the largest overlap with the molecular orbital in each \( k \) point.

**B. Energy**

The energy calculation, which is performed after the KS equations have reached self-consistency, is not significantly different in the linear-expansion \( \Delta \text{SCF} \) scheme compared to ordinary DFT. The Hartree energy is evaluated directly from the density, which is also the case for the exchange-correlation energy if an orbital independent functional is used. So in linear-expansion \( \Delta \text{SCF} \) these terms are evaluated exactly as in ordinary DFT. In ordinary DFT the kinetic energy is evaluated as

\[ T[n(\mathbf{r})] = \sum_{i=1}^{N} \frac{\nabla^2}{2} |\phi_i\rangle = \sum_{i=1}^{N} \epsilon_i - \int v_{\text{KS}}[n](\mathbf{r}) n(\mathbf{r}) d\mathbf{r}, \]

where the last equality is seen directly from Eq. (1). Similarly the expression for the kinetic energy in the linear-expansion \( \Delta \text{SCF} \) is found to be

\[ T[n(\mathbf{r})] = \sum_{i=1}^{N-1} \epsilon_i + \sum_{i=N}^{M} |a_i|^2 \epsilon_i - \int v_{\text{KS}}[n](\mathbf{r}) n(\mathbf{r}) d\mathbf{r}. \]

For orbital-dependent exchange-correlation functionals some effort must be put into ensuring that the exchange-correlation energy is evaluated correctly. This should however be quite straightforward since all the occupied orbitals are known.

**C. Gradients**

Gradients of PESs are easily evaluated in ordinary DFT due to the Hellman-Feynman theorem. The Hellman-Feynman theorem, however, only applies to eigenstates and not linear expansions of eigenstates. Due to this there is no easy way of gaining the gradients in a linear-expansion \( \Delta \text{SCF} \) calculation. In Sec. IV C we will show that the Hellman-Feynman gradients do in fact not match the true gradients.
The resonance energy is the total-energy difference between a resonant calculation and a ground-state calculation, both performed with atomic positions corresponding to the minimum of the ground-state PES (vertical resonance energies). We minimize the energy in the ground-state calculations by keeping all surface atoms frozen and found that the nitrogen molecule is placed on top with the two nitrogen atoms placed 2.084 and 3.201 Å above the surface. In the resonance calculation the $2\pi_v$ orbital of the N$_2$ molecule has been expanded on all KS states above the Fermi energy. This expansion has been used as $\psi^{\text{res}}$ in Eq. (5). Although an extra electron is placed on the molecule we keep the total number of electrons unchanged, such that the unit cell is neutral. This is reasonable because a charged molecule will form an image charge in the surface, keeping the entire system neutral.

The resonance energy is converged to within 0.1 eV at a surface unit cell of (2,2). The rather large variation in energy for smaller unit cells is probably due to dipole interactions between periodic images. This is confirmed by a simple estimation of the dipol-dipol interaction energies. The resonance energy is not influenced significantly by the number of layers in the ruthenium, indicating that the charge redistribution only occurs very near to the surface. That the charge redistribution is local is confirmed by Fig. 2, which shows the change in charge between the resonance calculation and the ground-state calculation for four different surface unit cells.

For the larger unit cells, where the resonance energy has converged, a clearly localized image charge is seen below the nitrogen molecule and above the first layer of ruthenium atoms. The area with extra charge clearly resembles the $2\pi$ orbital of nitrogen, indicating that the $2\pi$ orbital is well represented by the linear expansion of KS orbitals. Figure 2 also reveals that some charges are redistributed within the molecule.

In order to get an estimate of the size of the charge redistribution we also performed Bader decomposition$^{37,38}$ on the density found in the ground-state calculation and the resonance calculation. The two bottom panels in Fig. 1 show the extra charge assigned to the nitrogen molecule in the resonance calculation compared to the ground-state calculation as a function of system size. The converged value is close to 0.5 electron charge, i.e., only half of the electron is placed on the nitrogen molecule according to the Bader decomposition. This discrepancy could either be due to the ambiguity in the way one chooses to assign charge to the atoms or a more physical effect of charge going back into the surface when extra charge is placed on the molecule. The former reason is very likely, since the image charge is located very close to the molecule.

In order to investigate the effect of charge going back into the surface we start by considering the $2\pi$ orbital itself. Figure 3 shows the density of KS states and the projected density of states (PDOS) for the $2\pi$ orbital for the ground-state calculation and the resonance calculation. In the ground-state calculation a part of the long tail of the PDOS goes below the Fermi energy, i.e., a small part of the $2\pi$ orbital is occupied here. In the resonance calculation the PDOS has moved upward in energy such that the tail no longer goes below the Fermi energy, i.e., some charges go back into the surface as charge is placed on the molecule. Similar effects are seen for the other molecular orbitals as visualized in Fig. 4, which shows the PDOS for the $3\sigma$, $4\sigma$, $1\pi$, and $5\sigma$ orbitals. Again it is seen that all the PDOSs are shifted up in energy as more
charge is placed on the molecule. Almost the entire PDOSs are still under the Fermi level, but small ripples can be seen above the Fermi level, also contributing to the amount of charge going back into the surface.

This backtransfer of charge is not an unwanted effect, since we try to model the long-lived resonance state, i.e., the reasonably localized peak in the PDOS in Fig. 3. The backtransfer of charge is due to some on the energy scale very delocalized bands, indicating a much shorter lifetime, i.e., the backtransfer is expected to happen on a much shorter time scale than the decay of the resonance. It is however clear from Figs. 3 and 4 that the charge backtransfer in this case is far from the 0.5 electron indicated by the Bader decomposition. We then conclude that the main part of the discrepancy in this situation can be assigned to the ambiguity in the way charge is assigned to the different atoms. We also find that one gets significant different results by assigning charge in a different manner, for example, by dividing the charge by a flat plane midway between the surface and the molecule.

B. Comparison with inverse-photoemission spectroscopy experiments

In Table I we have tested the linear-expansion $\Delta$SCF method against inverse-photoemission spectroscopy measurements and compared the results to spatially constrained DFT and ordinary $\Delta$SCF calculations. The modified $\Delta$SCF values are all calculated in exactly the same manner as for $\mathrm{N}_2$ on ruthenium in Sec. III A. In all cases the molecules sit on top, and all surface atoms were kept fixed during the minimization of the molecular degrees of freedom. For the

<table>
<thead>
<tr>
<th>System</th>
<th>Constrained DFT</th>
<th>$\Delta$SCF (orig.)</th>
<th>$\Delta$SCF (this work)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathrm{N}_2$ on Ni(001)</td>
<td>2.2</td>
<td>3.5</td>
<td>4.0</td>
<td>4.4$^a$</td>
</tr>
<tr>
<td>CO on Ni(001)</td>
<td>2.2</td>
<td>3.2</td>
<td>4.2</td>
<td>4.0/4.5$^b$</td>
</tr>
<tr>
<td>NO on Ni(001)</td>
<td>2.2</td>
<td>0.6</td>
<td>1.4</td>
<td>1.6/1.5$^c$</td>
</tr>
<tr>
<td>CO on Ni(111)</td>
<td>2.8</td>
<td>4.3</td>
<td>4.4</td>
<td>4.4$^c$</td>
</tr>
<tr>
<td>NO on Ni(111)</td>
<td>2.7</td>
<td>0.5</td>
<td>1.4</td>
<td>1.5$^b$</td>
</tr>
<tr>
<td>CO on Pd(111)</td>
<td>4.6</td>
<td>4.1</td>
<td>4.9</td>
<td>4.7$^d$</td>
</tr>
<tr>
<td>CO on Pd step</td>
<td>2.8</td>
<td>3.2</td>
<td>4.5</td>
<td>4.0$^d$</td>
</tr>
</tbody>
</table>

$^a$Johnson and Hulbert (Ref. 39).
$^b$Reimer et al. (Ref. 40).
$^c$Reimer et al. (Ref. 41).
$^d$Rogozik and Dose (Ref. 42).
TABLE II. The positions of the molecules in the systems from Table I. All positions are relative to the closest surface atom. The z direction is normal to the surface. At the Pd step the CO molecule is tilted over the step, which is the reason for the composit in the y direction. All numbers are in Angstroms.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Molecule</th>
<th>Pos. of 1. atom</th>
<th>Pos. of 2. atom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni(001)</td>
<td>N$_2$</td>
<td>N: (0,0,1.638)</td>
<td>N: (0,0,2.798)</td>
</tr>
<tr>
<td></td>
<td>CO</td>
<td>C: (0,0,1.456)</td>
<td>O: (0,0,2.621)</td>
</tr>
<tr>
<td></td>
<td>NO</td>
<td>N: (0,0,1.404)</td>
<td>O: (0,0,2.580)</td>
</tr>
<tr>
<td>Ni(111)</td>
<td>CO</td>
<td>C: (0,0,1.774)</td>
<td>O: (0,0,2.941)</td>
</tr>
<tr>
<td></td>
<td>NO</td>
<td>N: (0,0,1.758)</td>
<td>O: (0,0,2.935)</td>
</tr>
<tr>
<td>Pd(111)</td>
<td>CO</td>
<td>C: (0,0,1.904)</td>
<td>O: (0,0,3.064)</td>
</tr>
<tr>
<td>Pd step</td>
<td>CO</td>
<td>C: (0,0,586,1.801)</td>
<td>O: (0,0,844,2.934)</td>
</tr>
</tbody>
</table>

Ni (001) surface we used three atomic layers, for the Ni (111) and Pd surfaces we used two atomic layers. The positions of the molecules in their minimized position are given in Table II. All resonance energies are vertical from the minimum of the ground-state PES. The relevant resonance for all the considered systems is the 2$\pi$ resonance.

The spatially constrained DFT method was suggested by Wu and Van Voorhis. The calculations we perform here we divide the space into two areas divided by the flat plane mid between the surface and the lowest atom in the molecule. We apply a potential $V(z)\propto \exp(-z^2/\Delta z^2)$, with $\Delta z=0.2$ Å and $z_0$ being the z value of the dividing plane. $V_0$ is varied until an extra electron is placed on the molecules side of the dividing plane compared to the unconstrained calculation. The energy is then calculated as described by Wu and Van Voorhis. The results using the original $\Delta$SCF method have all been obtained by forcing an electron in the KS orbital with the largest overlap with the 2$\pi$ orbital.

The results obtained with our proposed modification of the $\Delta$SCF method are seen to agree quite well with the experimental results, better than the spatially constrained DFT and the original $\Delta$SCF methods. All the results obtained by the original $\Delta$SCF approach lie too low, which is due to the fact that the large hybridization of the molecular orbitals makes it impossible to place sufficient charge on the molecule. However, a significant problem with this method is that PESs often become discontinuous if one chooses to occupy the KS orbital with the largest overlap with the molecular orbital, since this can be different orbitals at different configurations.

The major problem with the spatially constrained DFT method seems to be that it in some cases is a too strict criterion to force an extra electron on the molecule, which reflects itself in similar resonance energies for CO and NO. We find that the backtransfer of charge discussed in Sec. III A is significant for adsorbed NO and essential to obtain the resonance energies we find with the modified $\Delta$SCF method. This indicates that the spatially constrained DFT approach is more suited for systems with a smaller coupling than one has on the metallic surfaces considered here. The good agreement between our modified $\Delta$SCF method and experiments indicates that this method is preferable for these kinds of systems and that the backtransfer effect is indeed physically reasonable.

C. Potential energy surfaces for N$_2$ on ruthenium

In Fig. 5 we have mapped out a part of the potential energy surfaces for a nitrogen molecule on a ruthenium (0001) surface in the ground state and the 2$\pi$, resonance as a function of the distance between the two nitrogen atoms and the distance from the surface to the center of mass of the nitrogen molecule. The energies are in eV. The small dots represent the points where the energy has been calculated in order to generate the surfaces. The black arrow represents a possible trajectory of the system in the resonance state (see text).

FIG. 5. (Color) Potential energy surfaces (PES) for a nitrogen molecule on a close-packed ruthenium surface in the ground state and the 2$\pi$, resonance as a function of the distance between the two nitrogen atoms and the distance from the surface to the center of mass of the nitrogen molecule. The energies are in eV. The small dots represent the points where the energy has been calculated in order to generate the surfaces. The black arrow represents a possible trajectory of the system in the resonance state (see text).
generate states without getting convergence problems, i.e., only advantage of our modified response time-dependent density-functional theory calculations they make it possible to compare to more accurate linear-response calculations on N₂ and CO. These small systems have the advantage that the mentioned image charge effect.

The resonance state until the resonance decays and the system returns to the ground-state PES. The potential energy after the electron event in this optimal situation is approximately 1.5 eV higher than before the event. This is seen to be more than enough to desorb the molecule. A more detailed analysis involving calculations of the possible vibrational excitations and the probabilities of exciting them will be the topic of a future publication. Such an analysis will have to take all six degrees of freedom of the molecule into account.

The PES show that the center of mass is shifted away from the surface when the resonance is occupied. This may seem counterintuitive since the charged molecule is attracted to the generated image charge in the surface. However, the resonance weakens the bond between the nitrogen atoms, such that the distance between them increases, which shifts the center of mass outwards as the lower atom is not free to move closer to the surface. This effect is more significant than the decrease in the ruthenium-nitrogen distance due to the resonance weakens the bond between the nitrogen atoms, since the lower atom is not free to move closer to the surface. This effect is more significant than the decrease in the ruthenium-nitrogen distance due to the mentioned image charge effect.

### IV. SMALL MOLECULES

In the following we present some small tests performed on N₂ and CO. These small systems have the advantage that they make it possible to compare to more accurate linear-response time-dependent density-functional theory calculations. When possible we also compare to experiments. The only advantage of our modified ΔSCF compared to ordinary ΔSCF for these molecules is the possibility of handling degenerate states without getting convergence problems, i.e., the following should be viewed as a test of the ΔSCF approach rather than a test of our modification. We are especially interested in confirming the ability to predict the shift of the minimum when going from the ground-state PES to the excited-state PES, which we in Sec. III C argued is very important when considering molecular motion induced by an electron hitting a molecular resonance.

#### A. Excitation energies

We have used the linear-expansion ΔSCF in combination with the multiplet sum method to calculate excitation energies for different excitations in the N₂ and CO molecules. The results are presented in Tables III and IV, respectively. The 4σ and 5σ states are both represented by a single KS orbital. The 1π and 2π states are both double degenerate, so they are both represented in a linear combination of two KS orbitals: |ψ⟩= |a⟩|π₁KS,a⟩+ |b⟩|π₁KS,b⟩, where |π₁KS,a⟩ and |π₁KS,b⟩ are the two degenerate KS orbitals. The imaginary unit i has been included in order to get the correct angular momentum of the excited states (Π and Δ). This would not be possible using traditional ΔSCF, where one only has the freedom to change occupation numbers of the KS states. Due to the rotational symmetry of the density found from these states the calculations do not suffer from any convergence difficulties. That is not the case if one just occupies one of the degenerate KS orbitals. Only the Δ states are included in the 1π → 2π transitions in Tables III and IV, since the Σ states cannot be estimated by the multiplet sum method. This is, however, not a problem for the kinds of systems for which

<table>
<thead>
<tr>
<th>State</th>
<th>Transition</th>
<th>ΔεKSᵃ</th>
<th>TDDFTᵇ (ALDA)</th>
<th>ΔSCF (LDA)</th>
<th>ΔSCF (RPBE)</th>
<th>Exp.ᶜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>a¹Π</td>
<td>5σ → 2π</td>
<td>9.23</td>
<td>8.75</td>
<td>8.58</td>
<td>9.31</td>
<td></td>
</tr>
<tr>
<td>B¹Π</td>
<td>1π → 2π</td>
<td>1.61</td>
<td>1.20</td>
<td>1.06</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Singlet-triplet splitting:</td>
<td>1.61</td>
<td>1.20</td>
<td>1.06</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>w¹Δ</td>
<td>1π → 2π</td>
<td>10.27</td>
<td>10.50</td>
<td>10.52</td>
<td>10.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Singlet-triplet splitting:</td>
<td>1.36</td>
<td>1.56</td>
<td>1.73</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>o¹Π</td>
<td>4σ → 2π</td>
<td>13.87</td>
<td>11.97</td>
<td>12.40</td>
<td>13.63</td>
<td></td>
</tr>
<tr>
<td>C¹Π</td>
<td>4σ → 2π</td>
<td>11.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Singlet-triplet splitting:</td>
<td>3.43</td>
<td>1.60</td>
<td>1.79</td>
<td>2.44</td>
<td></td>
</tr>
</tbody>
</table>

ᵃKS eigenvalue differences.
ᵇLinear-response calculations taken from Grabo et al. (Ref. 44).
ᶜComputed by Oddershede et al. (Ref. 45) using the spectroscopic constants of Huber and Herzberg (Ref. 46).
this method is intended, such as molecules on surfaces where high-accuracy alternatives are still lacking.

In general the excitation energies found by the linear-expansion SCF method look quite good for the low-lying excitations. The accuracy is only slightly worse than that of lrTDDFT and significantly better than just taking KS eigenvalue differences. The singlet-triplet splittings are also rather close to the experimental values. The method however seems to struggle a bit more in the higher lying transitions. This could indicate that the method should only be applied to low-lying excitations. Changing the exchange-correlation functional from LDA to RPBE does not affect the accuracy significantly although a small tendency toward better performance is seen for the higher lying excitations. We note, however, that the intended application of SCF do not include simple diatomic molecules, where more accurate quantum chemical methods are available.

**B. Excited potential energy surfaces**

The shapes of the potential energy surfaces can in some cases be more important than the exact height of them, i.e., a constant error is not so critical. This is for example the case when considering chemistry induced by hot electrons. In order to get an idea of the accuracy with which the linear-expansion SCF method reproduces correct shapes of potential energy surfaces we have calculated the potential energy surfaces for the ground state and two excited states in the \( \text{N}_2 \) molecule. These are plotted in Fig. 6 together with results from lrTDDFT calculations.

The small differences between the two ground-state curves are due to the fact that they have been calculated with two different codes. Both codes are real-space codes, but gpaw uses the PAW formalism to represent the core electrons whereas OCTOPUS uses norm-conserving pseudopotentials. The calculations have been made with the same grid spacing.

![FIG. 6.](image-url) The energy as a function of bond length for the \( \text{N}_2 \) molecule in the ground state and two excited states. The black lines correspond to \( \Delta \text{SCF} \) calculations, the gray (online: light blue) lines correspond to linear-response calculations. The linear-response calculations have been made using OCTOPUS (Refs. 35 and 36). The vertical lines indicate the positions of the minima.
FIG. 7. (Color online) The energy as a function of bond length for the N\textsubscript{2} molecule in the ground state and two excited states. The short thick lines indicate the size of the gradients.

and unit-cell size and with the same exchange-correlation potential (LDA/ALDA).

The shapes of the potential energy surfaces found from the two different methods are seen to be very similar. Especially the predicted positions of the minima are seen to agree very well. The shifting of the minima toward larger bond lengths is also the expected behavior, since an electron is moved from a bonding orbital to an antibonding orbital. When going to bond lengths beyond 2 Å we start having problems with convergence problems in the ∆SCF calculations, since the 2\pi orbital ceases to exist. This is not a problem we have encountered in the systems with a molecule on a surface.

The good agreement between ∆SCF and lrTDDFT probably reflects that ∆SCF and ignoring the history dependence of the exchange-correlation potential in TDDFT are related approximations. For example, the density obtained in ∆SCF would be stationary if evolved in time with TDDFT.

C. Gradients

As mentioned in Sec. II C the Hellman-Feynman theorem does not apply in the linear-expansion ∆SCF method. This is verified by the calculations shown in Fig. 7. Here the energies of the ground state and two excited states in the N\textsubscript{2} molecule are plotted as a function of the bond length. The short thick lines indicate the gradient given by calculated Hellman-Feynman forces. For the ground state the agreement is as expected perfect, but for the excited states there is a clear mismatch. Unfortunately this implies that it is computationally heavy to do dynamics or minimizations in the excited states.

V. SUMMARY

We have extended the ∆SCF method of calculating excitation energies by allowing excited electrons to occupy linear combinations of KS states instead of just single KS states. This solves the problems encountered for molecules near surfaces, where the molecular orbitals hybridize, such that none of the KS orbitals can be used to represent an extra electron placed on the molecule. The method has been implemented in gpaw\textsuperscript{27,28} and applied to several systems.

From calculated potential energy surfaces of N\textsubscript{2} on a ruthenium surface we concluded that an electron hitting the 2\pi resonance in this system can induce molecular dynamics due to the different positions of the minima of the ground-state PES and the resonance PES. Through a simple analysis we found that one electron can optimally place 1.5 eV in the atomic motion, more than enough to desorb the molecule.

We find good agreement between the model and inverse photoemission experiments for several diatomic molecules on different metallic surfaces. For the considered systems we find significantly better agreement with experiments using the modified ∆SCF method compared to spatially constrained DFT and traditional ∆SCF.

Finally we applied the method to N\textsubscript{2} and CO in their gas phases we found that excitation energies are estimated with quite good accuracy for the lower lying excitations, comparable to that of TDDFT. Especially the shape of the potential energy surfaces and the positions of the minima agree well with TDDFT results.

ACKNOWLEDGMENTS

The authors wish to thank Jens Jørgen Mortensen and Anders Hellman for fruitful discussions. The Center for Individual Nanoparticle Functionality (CINF) is sponsored by the Danish National Research Foundation. This work was supported by the Danish Center for Scientific Computing.

APPENDIX: OVERLAPS USING PAW PSEUDOWAVE FUNCTION PROJECTIONS

The projector augmented wave\textsuperscript{29} method utilizes that one can transform single-particle wave functions |\psi\rangle oscillating wildly near the atom core (all-electron wave functions) into smooth well-behaved wave functions |\tilde{\psi}\rangle (pseudowave functions) which are identical to the all-electron wave functions outside some augmentation sphere. The idea is to expand the pseudowave function inside the augmentation sphere on a basis of smooth continuations |\tilde{\phi}\rangle\rangle of partial waves |\phi\rangle centered on atom \textit{a}. The transformation is

\begin{equation}
|\psi\rangle = |\tilde{\psi}\rangle + \sum_{i,a} \left( |\phi\rangle - |\tilde{\phi}\rangle\right) |\tilde{\phi}\rangle |\tilde{\psi}\rangle.
\end{equation}

where the projector functions |\tilde{p}\rangle\rangle inside the augmentation sphere \textit{a} fulfills

\begin{equation}
\sum_{j} |\tilde{p}_{i}^{\textit{a}}\rangle \langle \tilde{p}_{j}^{\textit{a}}| = 1, \quad \langle \tilde{p}_{i}^{\textit{a}}| \tilde{p}_{j}^{\textit{a}}\rangle = \delta_{ij}, \quad |\mathbf{r} - \mathbf{R}^{\textit{a}}| < r_{c}.
\end{equation}

Suppose we have an atom adsorbed on a metal surface and we wish to perform a ∆SCF calculation where a certain atomic orbital |\alpha\rangle is kept occupied during the calculation. If the orbital is hybridized with the metal states we need to find the linear combination which constitutes the orbital. This can
always be done if a sufficient number of unoccupied KS orbitals is included in the calculation

\[ |i⟩ = \sum_n c_n |ψ_n⟩, \quad c_n = ⟨ψ_n | i⟩. \tag{A3} \]

Since the partial waves are typically chosen as atomic orbitals we just need to consider the quantity

\[ ⟨ψ_n | ϕ^o_j⟩ = (ϕ_j^o | p_j^o⟩) + \sum_{j,a} (ϕ_j^o | p_j^a⟩) (⟨ϕ^a_j | ϕ^o_j⟩) \]

\[- (⟨ϕ^o_j | ϕ^a_j⟩) = ⟨ψ_n | p_j^a⟩. \tag{A4} \]

If we were just considering a single atom, the last equality would be exact inside the augmentation sphere since the partial waves would then be orthogonal and the pseudopartial waves are dual to the projectors in Eq. (A2). When more than one atom is present there are corrections due to overlap of partial waves from neighboring atoms and noncomplete-ness of projectors/pseudopartial waves between the augmentation spheres. However using \( ⟨ψ_n | p_j^a⟩ \) is a quick and efficient way of obtaining the linear combination, since these quantities are calculated in each step of the self-consistency cycle anyway. The method can then be extended to molecular orbitals by taking the relevant linear combinations of \( ⟨ψ_n | p_j^a⟩ \).

*Present address: Department of Micro- and Nanotechnology, Technical University of Denmark, DK–2800 Kongens Lyngby, Denmark.

†schiotz@fysik.dtu.dk


26Often an electronic temperature is introduced to improve convergence, such that the electrons are distributed by a Fermi-Dirac distribution. This can also be done in the proposed method.


28The GPAW code including the implementation of ΔSCF is available as a part of the CAMPOS software, www.camp.dtu.dk/Software.


35OCTOPUS is a freely available real-space time-dependent density-functional theory code (see www.tdfft.org/programs/octopus/).


