Flow distortion on boom mounted cup anemometers

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Abstract (max. 2000 char.):
In this report we investigate on wind direction dependent errors in the measurement of the horizontal wind speed by boom mounted cup anemometers. The boom mounting on the studied lattice tower is performed according to IEC standard design rules, yet, larger deviations than predicted by flow models are observed. The errors on the measurements are likely caused by an underestimation of the flow distortions around the tower. In this paper an experimental method for deriving a correction formula and an in-field calibration is suggested. The method is based on measurements with two cup anemometers mounted with booms at the same height but pointing in $60^\circ$ different directions. In the examined case of a 1.9 m wide equilateral triangular lattice tower with booms protruding 4.1 m at 80 m height the measurement errors are observed to reach up to $\pm 2\%$. Errors of this magnitude are severely problematic in the measurement of wind turbine power performance, wind resource assessment and for providing purposeful in-field comparisons between different sensors, e.g. lidar anemometers. With the proposed method, direction dependent errors can be extracted and the mast flow distortion effect on the wind measurements corrected to an uncertainty estimated to better than 0.5%. This level of uncertainty is probably acceptable for the above mentioned applications.
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1 Introduction

In wind energy it is best practice to use top mounted cup anemometers for hub height wind measurements since they suffer the least from flow distortions caused by the tower. However, on many occasions the wind conditions at lower heights are measured, e.g. for detailed wind shear information. This is typically done by mounting cup anemometers on booms protruding from the tower. In the nineties several groups reported on measurement deviations from boom mounted anemometers both in free flow, in wind tunnels [1] and from actuator discs models [2]. Based on these results design rules for appropriate mounting, and specifically for appropriate boom lengths were derived. However, the experimental verification where influenced by two errors, a dominant flow distortion due to the boom and a flow distortion due to the mast. According to the IEC standard for wind turbine power performance measurements [3], it should be ensured that the influence on the wind speed measurement from the flow distortion induced by the tower is kept below 1% and the influence from the boom below 0.5%. The flow distortions due to the boom are expected to be reduced to much less than 0.5% if the cup anemometer is mounted on a support pole with a length exceeding 15 times the boom diameter.

In this paper we study a boom mounted cup anemometer setup according to the IEC standard for wind turbine power performance measurements. The measurements are taken at 80 m height and the tower is standing in extremely flat terrain. The wind conditions at the site are characterized by high speed, low turbulence and insignificant flow angles off the horizontal. The wind speed measurement of the two cups deviate depending on wind direction with up to ± 3 % in a sinusoidal pattern. From these deviations the flow distortion on an individual cup anemometer is estimated to influence the wind speed measurement with up to ± 2 %, twice the conservative design goal. Errors of this magnitude are also observed, but with lower certainty, by comparisons with lidar anemometry, a completely different measurement technique [5]. Larger than expected measurements deviations of cup anemometers mounted on tubular towers has also been reported [4].

It is unclear whether the underestimation in the IEC guidelines is due to a failure in the flow modeling or in the calculation of the tower thrust coefficient. Nevertheless, it will be difficult to increase the current boom length-tower diameter ratio due to stability issues and logistics. The need for corrections of the larger than expected direction dependent deviations is evident. It is not unlikely that lower uncertainties can be reached with a good correction method than from trying to find and experimentally verify global design rules.

In this paper we propose an experimental method for deriving corrections for the direction dependent errors caused by flow distortion. The method also provides the possibility for an in-field calibration which can reduce the uncertainty in the wind measurement. The standard deviation in the ten minute horizontal wind speed measurement difference of the two cups is decreased from 0.12 m/s to 0.03 m/s by applying this correction.

The flow distortions discussed in this paper should not be confused with the tower wake which induces 20-40% loss in wind speed measurement for the narrow range of wind directions for which the wind passes through the tower before passing the cup anemometer.
2 Investigation of flow distortion due to a lattice tower

The measurement set up and the observed deviations in the ten minute wind speed measurements are described in this section. A method for extracting a correction factor from the observed wind speed measurement ratios obtained from two boom mounted cup anemometers is presented. The results obtained from three different cup pairs are presented.

2.1 Measurement setup

The substantial lattice tower is positioned at Risø DTU’s Test Site for Large Wind Turbines in Høvsøre, Denmark [5]. The test site is located in very flat terrain, 1.8 km from the North sea in the west and to the east is mostly open farmland. The vertical wind speed is expected to be insignificant in the ten minute average for all wind directions [6]. The turbulence intensity at 80 m has a Weibull like distribution with a yearly mean of 0.07 and a standard deviation of 0.025 but varies slightly with wind sector and season.

The tower has an equilateral triangular cross-section. The western side of the tower is pointing at nominally 3°, with due north defined as 0. At 80 m height the sides are 1.9 m long and the three round legs have a diameter of 140 mm. The round cross-bracing have a diameter of 48 mm. The cross-bracing is made with a 23° angle from horizontal. The boom diameter is 50 mm and is fixed to the tower with two clamps with a 15 cm diameter.

Two P2546A cup anemometers, IEC class 1.31A, are mounted on booms at 80 m height according to the IEC standard. The booms protrude 4.1 m from the tower. The booms are nominally pointing close to the south and east-south-east (ESE) direction, referred to as 183° and 123° with due north being 0°, see Figure 1. The anemometer rotor is mounted 75 cm above the boom using a support pole, according to the IEC recommendation.

Figure 1: Boom mounted cup anemometers at 80 m height. The original P2546A cup anemometer is mounted on a boom pointing straight south. A supplementary P2546A is mounted on an identical boom pointing approximately east-south-east, at 123°.

The solidity and the thrust coefficient, $C_T$, at 80 m height are calculated according to the appendix in the IEC standard to 0.22 and 0.36 respectively. The largest flow
distortion is then predicted to induce -1.1% on the wind speed measurement for the case when the wind approaches the cup in line with the boom.

On the 9\textsuperscript{th} of September 2008 the southern cup was replaced according to Risø routines while the east-south-east cup was left on its boom. This gave an excellent opportunity to evaluate the robustness of the method, study possible aging and cup calibration accuracy. On the 9\textsuperscript{th} of September 2009 both cups where replaced giving a third period and a new pair for evaluation.

2.2 Sensors
All cups where calibrated according to Measnet certification procedures [7] in the wind tunnel at Svend Ole Hansen ApS. The uncertainty in wind tunnel calibrations is typically referred to as 0.1 m/s [3] but the repeatability in the P2546A calibration expression in a specific wind tunnel, which is the uncertainty that should be consider in the investigation of flow distortion, is typically much better, with a standard deviation of less than 0.15% over the calibration range, 4-16 m/s. The repeatability in the calibration expression of P2546A cups tested in the Svend Ole Hansen AsP wind tunnel has been evaluated from a set of 18 anemometers tested during winter and spring 2009-2010. The gain had a mean of 0.625 m/s /Hz with a standard deviation of 0.0022 m/s /Hz while the offset has a mean of 0.2156 m/s with a standard deviation of 0.013 m/s.

Results from the cup calibrations used in this study can be seen in Table 1. Note that, the calibration expressions for two of the studied cups deviate from the typical values. The offset, 0.19428 m/s, from the calibration expressions for the S pointing cup used during the first period deviates slightly from what is typical for P2546A cups calibrated in the specified tunnel, 0.216 ± 0.013 m/s, and is atypically large, 0.26212 m/s, in the post calibration. The gain, 0.62903 m/s /Hz, is larger than expected, 0.625 ± 0.002 m/s /Hz, in the ESE pointing cup used in period 1 and 2. It is uncertain whether this deviation is due to uncertainty in the cup calibration or in the manufacturing and if the calibration expression of a deviating cup can be truly trusted. The uncertainty in the calibration for those cups is estimated to be less than 0.7%.

<table>
<thead>
<tr>
<th>Test Period</th>
<th>Cup S pre-calibration</th>
<th>Cup S post calibration</th>
<th>Cup ESE pre-calibration</th>
<th>Cup ESE post calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.62225\cdot f +</td>
<td>0.61924\cdot f +</td>
<td>0.62903\cdot f +</td>
<td>0.63015\cdot f +</td>
</tr>
<tr>
<td></td>
<td>0.19428</td>
<td>0.26212</td>
<td>0.20813</td>
<td>0.2258</td>
</tr>
<tr>
<td>2</td>
<td>0.62230\cdot f +</td>
<td>0.6208\cdot f +</td>
<td>As above</td>
<td>As above</td>
</tr>
<tr>
<td></td>
<td>0.22725</td>
<td>0.22230</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.62561\cdot f +</td>
<td>Not available</td>
<td>0.62507\cdot f +</td>
<td>Not available</td>
</tr>
<tr>
<td></td>
<td>0.21421</td>
<td></td>
<td>0.22971</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Results from pre and post calibrations of the P2546A cup anemometer at the Svend Ole Hansen wind tunnel.

Furthermore, the meteorology tower is equipped with wind direction sensors at 100 and 60 m height. The wind direction sensors are mounted on the booms which also hold the cup anemometers but closer to the tower. Possible flow distortion effects on the wind direction sensors are estimated to be smaller than 3°. The wind direction at the test height, 80 m, is linearly interpolated from these two measurements. The tower is also equipped with temperature sensors in order to screen for possible freezing effects on the anemometers.
2.3 Observed wind speed measurement ratio of two boom mounted cup anemometers

Three measurement series are used, the first from 20080417 to 20080909 taken with one set of P2546As. Another data set is taken from 20081201 to 20090701 with the same ESE pointing cup but with a replaced south pointing cup. Finally a third data set, with both cups replaced, was taken between 20090918 and 20100313.

The data sets were screened to only include ten minute averages with wind speeds in the range 4 to 16 m/s, i.e. the calibration range of the cup anemometers. Furthermore, only ten minute averages taken in temperatures higher than 2 °C were included to avoid measurements affected by freezing of the cup anemometers. Furthermore, although directional veer is limited, but not insignificant in Høvsøre, and due to the lack of a direct measurement of the wind direction at 80 m, a screen is introduced on directional veer. If the ten minute averaged wind direction differs with more than ± 5° from 60 to 100 m it is not included in the analysis. Although inter comparisons of two cup anemometers, like the one in this paper, give an indication of cup anemometers being unaffected by rain the data set has been screened to exclude ten minute periods including rain. Finally a screen was made of the time periods where the standard deviation in the ten minute wind direction at 60 and 100 m was larger than 5°.

The ratio of the 10-minute averaged horizontal wind speed measured by the S cup and the ESE cup can be seen in Figure 2.

![Figure 2: Measurement Ratio of the 10 minute average horizontal wind velocity taken by the S and ESE pointing cup as a function of wind direction. The cups are mounted at 80 m height on a lattice tower according to the IEC standard for power curve measurements. This plot shows the second data set taken in 20081201 to 20090701.](image)

Note the large deviations between the cups caused by tower wakes, centered at 290° for the ESE cup and at 10° for the south pointing cup. Also note the small bumps, of approximately 2-4%, at 240° and 60° which indicate the wind velocity loss as the wind flow passes through one cup anemometer before it hits the other. The detail of these spikes is an indication of the excellent accuracy in the wind speed measurement and the stable wind conditions in the ten minute average at 80 m height in Høvsøre, making it a veritable outdoors wind tunnel. Note that the booms seem to...
be long enough to ensure that the cup anemometers are in the far-field of the flow distortion.

The undisturbed wind direction sectors which where chosen for further analysis cover 75 – 225° and 320 – 340°. After screening 3322, 4169 and 2459 ten minute averages remained in the three tests.

By studying the observed cup ratios over different periods of the test it could be concluded that possible degradation of the cup anemometers where insignificant in all three campaigns. The larger data set for the second period was divided in two to look for cup degradation during the experiment. Degradation of either cup would strongly affect the measurement of flow distortion. However, as can be seen in Figure 3 the cup ratios seem to be identical in the two periods. The unlikely case that the two cup anemometers degrade identically, will not affect the measurement of the flow distortion around the mast.

Figure 3 Cup ratio in December 2008 - April 2009, purple, and April - July 2009, blue. No obvious sign of sensor degradation is visible.

Slightly different cup ratios were observed during the different test periods, e.g between the first and the third, shown in Figure 4. During the second and third period the cup ratio is not centered round one. As will be seen later this phenomena can be explained by the uncertainty in the cup calibration.
Figure 4 Cup ratio vs wind direction in the first, purple, and third period, blue. Also included is the fitted cup ratio obtained for the nominal boom directions taken from the geometric center, i.e. 189 and 117. The green line shows the fitted cup ratio of the later data set without using a gain difference. It is obviously an unsuitable fit to the blue data.

2.4 Model for extraction of flow distortion from measured cup ratios
The influence of the flow distortions on the individual cup anemometers has to be extracted from the observed ratio in the cup anemometer measurements. A good approach, considering the observed ratio and under the assumption of having sufficiently long booms to place the cups in the far-field of the flow distortion, is to model the flow distortion influence on the individual boom mounted cup as a sinusoidal and fit to the observed cup ratio.

The error, \( \Delta u \), in the free wind velocity, \( u \), measured with a cup anemometer is thus modeled as

\[
\Delta u = u - u_{\text{meas}} = -uA\cos(\theta)
\]

where \( A \) is the amplitude of the error due to tower wakes and \( \theta \) is the wind direction with the same coordinate system as the boom direction.

A parameter, \( \alpha \), is introduced with the intention to compensate for the uncertainty in the effective boom pointing direction, i.e. taken from the centre of the effective tower, see Figure 5. At 80 m the angle between the north corner and the geometric center of the tower, which could deviate from the equivalent tower center from a flow point of view, as seen from the south pointing cup is 6°. Note that in this setup, the line between the geometric tower center and the cup is mirror imaged for the S and ESE pointing cups. Thus giving the ideal case of \( \alpha_S = -\alpha_{\text{ESE}} \).
However, there are also considerable uncertainties in the boom mounting and in the absolute wind direction measurement. The boom directions have an uncertainty roughly estimated to $\pm 3^\circ$ and the wind direction measurement to $\pm 2^\circ$. The angle between the western side of the tower towards true north also has an uncertainty. The influence of the uncertainties coupled to direction errors can be experimentally estimated and mitigated by considering an unknown $\alpha$, for each cup.

The introduction of the $\alpha$-parameter in the model is justified by a significant reduction of the fit residuals, as seen in Figure 6, and in the spread of the estimated, supposedly constant, $A$ parameter throughout three test campaigns.

Furthermore, a parameter is introduced to compensate for potential errors in the calibrated cup anemometer gain. The introduction of the $G_{\text{diff}}$ parameter is essential to explain the observed cup ratios shifted from 1, which is observed in two out of three test campaigns. For these campaigns the inclusion of a $G_{\text{diff}}$-parameter significantly decreases the uncertainty in the extracted $A$, as seen in Figure 4. Note that the calibration expression of cup anemometers is given as $u_{\text{meas}} = k \cdot f_{\text{meas}} + m$, where $f_{\text{meas}}$ is the measured cup rotation frequency. The model presented in this paper can strictly only correct for relative deviations which uniquely depend on wind speed, i.e. $m = 0$, nevertheless it will also compensate for errors in the offset. The uncertainty of this simplification is treated in chapter 3.2.

By using the expanded model, the method proposed in this paper can also be considered for in-field calibrations and decrease the uncertainties in the absolute wind speed measurement.

The model of the ratio of the wind speed measured by two cups is thus expressed as

$$\frac{u_1}{u_2} = \frac{u + \Delta u_1}{u + \Delta u_2} = G_{\text{diff}} \frac{1 - A \cos(\theta - \alpha_1 - \beta_1)}{1 - A \cos(\theta - \alpha_2 - \beta_2)}$$

where $u$ is the free horizontal wind speed, $u_1$ is the wind speed measured by one cup and $u_2$ is the wind speed measured by the other, $G_{\text{diff}}$ is the relative gain difference between the two cup anemometers, $A$ is the amplitude of the error due to tower flow distortion, $\theta$ is the wind direction, $\alpha$ is the offset relatively to the nominal direction $\beta$ defined from the northern corner of the tower, with a positive sign defined clockwise from true north, as for $\alpha_1$ in Figure 5.
2.5 Results of extracted flow distortions from three cup anemometer pairs

The wind speed measurement of the two cups deviate, depending on wind direction, with up to 3% in a sinusoidal pattern, as seen in Figure 4. The flow distortion on an individual boom mounted cup anemometer can be extracted from the ratio of the wind velocity measured with two cups mounted on booms in different directions. A least square fitting of expression 1.2 to the observed ratio can give an estimate of $A$, $G_{diff}$, $\alpha_1$, and $\alpha_2$.

In the set up in Høvsøre the south boom is nominally pointing at 183° while the ESE points at 123°. However, fitting of equation 1.2 to the measured data does not easily converge with the default least square fit settings in Mathematica 6.0. Instead a matrix of fits are made with set $(\alpha_1, \alpha_2)$-couples and only $A$ and $G_{diff}$ as free parameters.

The $(\alpha_1, \alpha_2)$-couple matrix covers ±10° and sets out from (6°, -6°) obtained for the nominal geometric center. The average squared residual of the cup ratio fit, which describes the strength of the fit, differs slightly in the three periods. The minimum average squared residuals in the fit to the cup ratio are 8.7, 8.4 and 9.2·10^{-6} for the three periods respectively, which roughly corresponds to a standard deviation of 0.03 m/s in the difference of the corrected wind speed measurement taken by the two cup anemometers. The different magnitude of residuals obtained for the three periods can be explained by differently suiting expressions obtained during cup calibration. The shape as a function of $(\alpha_1, \alpha_2)$ is similar for the three periods and for clarity only the value for the second period is plotted in the diagram, see Figure 6.

![Figure 6: Average Residual² as a function of $\alpha_1$ and $\alpha_2$ for period 20081201 – 20090701. Note the residual valley, which fulfills $\alpha_1 = -\alpha_2 + 13$.](image)

The change in average squared residual, 8.4 to 27.7·10^{-6}, for plausible $\alpha$’s, say $\alpha_1, -\alpha_2 \in [-4°..16°]$ is considerable. However, the explanation to the difficulty for the fit algorithm lies in the residual valley. The $(\alpha_1, \alpha_2)$-couples which form the residual valley differs slightly over the different periods. Roughly estimated from the graph they lie at $\alpha_1 = -\alpha_2 + 9$, $\alpha_1 = -\alpha_2 + 13$, $\alpha_1 = \alpha_2 + 5.5$ for the three periods respectively. The difference between the periods can be explained by the uncertainty in the involved direction measurements e.g. the uncertainty in the north reference in the wind direction measurement, which changes with a replacement of
the vane. However, the deviation in offset from the nominal $\alpha_1 = -\alpha_2$ is similar for the three periods and is not likely covered even by a combination of uncertainties in boom, tower and vane directions. The deviation leaves room for the possibility of different equivalent tower centers when seen from the S cup or ESE cup position.

The $(\alpha_1, \alpha_2)$-couples which form the residual valley have non distinguishable residual sums. It is therefore not advisable to rely only on fit results to find trustworthy effective boom directions. It is expected that the ideal $(\alpha_1, \alpha_2)$-couple will change between the test periods since wind vane offsets etc will vary with mounting. The $(\alpha_1, \alpha_2)$-couple which lies in the residual valley closest to the nominal from geometric considerations, $(6, -6)$, is a reasonable guesstimate for each period. In our three examples this yields, $(10, -1)$, $(12, 1)$ and $(9, -3)$.

The uncertainty in the $(\alpha_1, \alpha_2)$-estimate is transferred to an uncertainty in $A$ and $G_{\text{diff}}$ and eventually to an uncertainty in the corrected wind speed. The sensitivity in $A$ and $G_{\text{diff}}$ towards $\alpha_1$ and $\alpha_2$ can be experimentally estimated from Figure 7.

![Figure 7: Fitted $A$ and $G_{\text{diff}}$ as functions of $\alpha_1$ and $\alpha_2$.](image)

Again the shape of the graphs is very similar. The amplitude of the flow distortion differs with less than 1 percent point with a standard deviation of 0.2 percent points over the full matrix $\alpha_1, -\alpha_2 \in [-6^\circ .. 16^\circ]$ within each period. While $G_{\text{diff}}$ varies with 0.002 for the second period with a standard deviation of 0.0006, to 0.004 with a standard deviation of 0.0009 for the third. The extracted fit parameters, with $(\alpha_1, \alpha_2)$-couples corresponding to the nominal north corner of the tower $(0^\circ, 0^\circ)$, nominal geometric center $(6^\circ, -6^\circ)$ and the guesstimates, e.g. $(9^\circ, -3^\circ)$, can be seen in Table 2.

<table>
<thead>
<tr>
<th>Period</th>
<th>$(\alpha_1, \alpha_2)$</th>
<th>$A$ [%] ± $\sigma$</th>
<th>$G_{\text{diff}}$ [ ] ± $\sigma$</th>
<th>$\sum \frac{\text{res}^2}{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>$(0^\circ, 0^\circ)$</td>
<td>$1.78 \pm 0.02$</td>
<td>$1.0014 \pm 0.0001$</td>
<td>$10.5 \cdot 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$(6^\circ, -6^\circ)$</td>
<td>$1.52 \pm 0.01$</td>
<td>$1.0014 \pm 0.0005$</td>
<td>$10.5 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>
Table 2: Extracted parameters for the experimentally estimated correction formula together with their average squared residuals. The ± signifies the 90 % confidence interval of the least square fit.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10°,-1°)</td>
<td>1.53 ± 0.01</td>
<td>0.0001</td>
<td>8.7·10⁻⁶</td>
</tr>
<tr>
<td>Second</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0°,0°)</td>
<td>2.09 ± 0.02</td>
<td>1.0057 ± 0.0001</td>
<td>15.7·10⁻⁶</td>
</tr>
<tr>
<td>(6°,-6°)</td>
<td>1.78 ± 0.02</td>
<td>1.0057 ± 0.0001</td>
<td>15.7·10⁻⁶</td>
</tr>
<tr>
<td>(12°,1°)</td>
<td>1.82 ± 0.01</td>
<td>1.0048 ± 0.0001</td>
<td>8.4·10⁻⁶</td>
</tr>
<tr>
<td>Third</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0°,0°)</td>
<td>1.84 ± 0.02</td>
<td>1.0078 ± 0.0001</td>
<td>9.4·10⁻⁶</td>
</tr>
<tr>
<td>(6°,-6°)</td>
<td>1.57 ± 0.02</td>
<td>1.0078 ± 0.0001</td>
<td>9.4·10⁻⁶</td>
</tr>
<tr>
<td>(9°,-3°)</td>
<td>1.56 ± 0.02</td>
<td>1.0072 ± 0.0001</td>
<td>9.2·10⁻⁶</td>
</tr>
</tbody>
</table>

The three periods should ideally yield identical A parameters since the tower and boom dimensions remain constant. However, the variation of 0.3 percent points between the different test periods is large and so is the variation with different (α₁,α₂)–couples. Note that the 90% confidence intervals of the least square fit is much smaller than the variation with different (α₁,α₂)–couples and consequently is a bad estimate of the total uncertainty in the model and the measurements.

The extracted flow distortion factor, i.e. the influence on the measurement, $1 - A \cos(\theta - \alpha_1 - \beta_1)$, using the α-guesstimates, on the S and ESE pointing cup as a function of wind direction are plotted in Figure 8. Note that the gain difference is omitted in these plots since it does not describe the flow distortion but is information related to the two cups.

![Figure 8: Extracted flow distortion influence on the S, blue, and SES, red, pointing cup anemometer as a function of wind direction for the three periods. The α-guesstimates from the three periods have been used in the extraction of fit parameters. In the tower wake, here blanked, a loss of 20-30% should be expected.](image-url)
The variation in gain difference with different $\alpha$-couples is comparable to the variations throughout a test period, of 0.001, as seen in Figure 11. The in-field calibration is thus not strongly dependent on actual boom direction or equivalent center of tower.

The gain difference parameter, $G_{\text{diff}}$, in contrast to $A$, should be different for each pair of anemometers and depends on the accuracy of the cup calibration. However, the value obtained in the third period is slightly larger than expected regarding the typical repeatability for Risø cups calibrated in the Svend Ole Hansen As wind tunnel.

The stability of the measurements and the model has been tested by extracting fit parameters from subsets of the full measurement period. The amplitude, gain difference and average square residual extracted from sliding sets of 1000 ten minute averaged wind speed ratio and wind direction measurements are plotted in Figure 9-Figure 11. Run id 1 indicates that the parameters where extracted from the first 1000 measurements while run id 2 used the second to the 1001st measurement etc. Run id 1 and run id 1001 are thus two completely separate studies. The variation in the fitted parameters over these different test sets gives an indication of the stability of the model and the minimum uncertainty in the extracted parameters.

![Figure 9: Average squared residual extracted from 1000 sliding measurements of cup ratio and wind direction. The blue, red and yellow plots show the results from the first, second and third period. The extractions are made with the best guesstimates on $(\alpha_1, \alpha_2)$.](image)

The average squared residuals differ maximally with a factor two over the test periods. This indicates a fairly well spread distribution of measurement variations throughout the experiment duration and that the model is representative throughout the periods.

The amplitude, which represents the magnitude of the flow distortion around the tower, should ideally be identical in all tests. The variation in $A$ throughout each test is small, with standard deviations of 0.04-0.08 percent points, as seen in Figure 10.
Figure 10: Fitted amplitudes extracted from sliding data sets of 1000 measurements of cup ratio and wind direction. The blue, red and yellow plots show the results from the first, second and third period. The extractions are made with the best guesstimates on $(a_1, a_2)$.

The difference in $A$ for test period 2 versus test period 1 and 3 does not seem to be explained by the stochastic variance in the measurements since the red and the blue/yellow plots never cross. Nor is it likely that the explanation lies in the lack of the offset term in the in-field calibration expression, which is estimated to give a less than 0.05% bias to the amplitude estimate, see chapter 3.2. Although likely to explain a reasonable amount of the deviation it is unlikely that the $a$-couple uncertainty can fully account for the difference in the extracted A-parameter.

The gain difference on the other hand is expected to differ for the different cup anemometer pairs. The $G_{\text{diff}}$ extracted from subsets of 1000 ten minute average measurements are plotted in Figure 11.

Figure 11: Gain difference extracted from 1000 sliding measurements of cup ratio and wind direction. The blue, red and yellow plots show the results from the first, second and third period. The extractions are made with the best guesstimates on $(a_1, a_2)$.

Wind tunnel calibration of class 1.3 cup anemometers is estimated to have a Gaussian based standard uncertainty of approximately 0.1 m/s [3]. However, as previously mentioned the relative deviation of the same brand of cup anemometer calibrated in the same wind tunnel is typically smaller, 0.15%, although it is likely to
be slightly larger for these specific cup anemometers which have calibrations expressions which deviate slightly from typical. In addition to calibration uncertainty comes for example individual over speeding response to turbulence.

The Gaussian distribution of the gain difference between two cups has a standard deviation which is $\sqrt{2}$ larger. For the three tested couples the estimated gain differences of 0.1 %, 0.5 % and 0.7 % appear plausible. It should also be noted that the small variation throughout the individual experiments, within standard deviations of 0.0001-0.0006, indicates very consistent results.

The robustness of the results obtained in this paper are supported by comparisons with the relatively noisy wind speed measurements taken by lidar anemometry, the standard deviation of lidar measurement – cup measurement in the tests was 0.085 m/s [5]. More wind directions have to be excluded from the lidar cup comparison due to tower wakes entering the lidar probe volume. Also note that the lidar and cup gain differ, i.e. a displacement around 1. This is expected since they do not have the same reference source. A sinusoidal, with a maximum deviation at about 180° and 120° in comparison with the S and ESE cup, is clearly visible in the lidar-cup ratio versus wind direction and the amplitude is estimated to lie in the region of 1.5-2%, see Figure 12.

![Figure 12: Cup/lidar ratio at 80 m as a function of wind direction.](image)

Similar results are obtained towards the cup anemometers mounted at 60 and 100 m, which according to the IEC guidelines should have centre line deviations of less than 0.8 and 0.7%. The cup-lidar ratio deviates from the sinusoidal at 40 m, where the centre line deviation is estimated to 1.3% from IEC-guidline calculations of $C_T$. This could be due to near-field flow effects but could also be explained by a number of other factors related to the lidar.

### 2.6 Impact of corrections

To show on the impact of flow distortion corrections, the difference between the wind speed measured by the S and ESE pointing cups are compared before and after correction formulas are applied, see Figure 13, Figure 14 and Table 3. The best guesstimates for the $\alpha$'s have been used for this correction.
Figure 13: Observed cup ratio, blue, and after correction, red, for the first period.

Figure 14: Histogram over the difference of the 10 minute horizontal wind speed measurements taken by the south and the east-southeast pointing cup anemometers during the first period. As observed, top, and after corrections, bottom.

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean before correction [m/s]</th>
<th>Mean before correction [%]</th>
<th>St.dev before correction [m/s]</th>
<th>St.dev after correction [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>-0.02</td>
<td>-0.1%</td>
<td>0.12 or 1.2%</td>
<td>0.025 or 0.3%</td>
</tr>
<tr>
<td>Period 2</td>
<td>0.01</td>
<td>0.2%</td>
<td>0.12 or 1.2%</td>
<td>0.026 or 0.3%</td>
</tr>
<tr>
<td>Period 3</td>
<td>0.09</td>
<td>1.1%</td>
<td>0.11 or 1.1%</td>
<td>0.028 or 0.3%</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; corrected by formula extracted from 2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>0.01 After corr.</td>
<td>0.1% After corr.</td>
<td>0.12 or 1.2%</td>
<td>0.033 or 0.4%</td>
</tr>
</tbody>
</table>

Table 3: Mean and standard deviations of the difference of the 10 minute horizontal wind speed measurements taken by the south and the east-southeast pointing cup anemometers, before and after corrections. Since the data sets are corrected with the
least square fits generated on the full data set the mean difference after correction is always 0 m/s. The last row shows the results if period 1 is corrected by the \( A \) and the \( \alpha \)s obtained from period 2, note that the mean difference here is calculated after corrections.

Note that the mean differences are small in period 1 and 2 since the data set for the selected wind sectors are well balanced. However, during period 3 the mean difference is significant before corrections.

As seen in Figure 13 and Figure 14 and in Table 3 the spread in wind speed measurements is significantly reduced by the correction. The standard deviation for the experimental data sets are reduced with more than 4 times, even when the correction formula obtained from period 2 is applied to the data set from period 1. The standard deviation of the uncorrected experimental data set is in good agreement with that of a sinusoidal distribution with an amplitude of 1.7\%.
3 Sensitivity analysis

Several unknowns introduce uncertainties in the extrapolated $A$ and $G_{\text{diff}}$. Initially, it is not evident to model the flow deviation proportionally to the wind speed and the model therefore needs to be tested in different wind speed regimes. Secondly, the in-field calibration procedure can mitigate errors in the cup calibration expression, but it only covers wind speed proportional deviations, while the possibility for errors in the cup calibration offset terms are excluded. Furthermore, more common uncertainties exist, like actual boom directions which differ from the intended, errors in the wind direction measurement and uncertainty in defining a center of the mast as seen from a far field flow distortion point of view this was treated in section 2.5. That section ends with a study of the variation within three long test periods. These plots give a minimum estimate of the uncertainty in the model.

3.1 Validity at different wind speed ranges

It is not obvious to model the flow deviation proportionally to the wind speed, as $u_{\text{meas}} = u_{\text{true}} + A_{\text{true}} \cos(\text{dir})$, and not as $u_{\text{meas}} = A_{\text{true}} \cos(\text{dir})$, i.e. modeling the flow proportionally to wind speed directly. The latter is more in terms with standard flow modeling but cannot explain the observations. Fitted parameters from different wind speed ranges are presented in Table 4. An $(\alpha_1, \alpha_2)$-couple of (3,3) was used in this analysis.

<table>
<thead>
<tr>
<th>First period, Wind speed range</th>
<th>$A$</th>
<th>$G_{\text{diff}}$</th>
<th>$N$</th>
<th>$\sum \frac{\text{res}^2}{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-8</td>
<td>1.76</td>
<td>1.0004</td>
<td>1215</td>
<td>$14.0 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>8-12</td>
<td>1.85</td>
<td>1.0027</td>
<td>1692</td>
<td>$7.8 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>12-16</td>
<td>1.75</td>
<td>1.0029</td>
<td>415</td>
<td>$3.7 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>4-16</td>
<td>1.78</td>
<td>1.0019</td>
<td>3322</td>
<td>$11.0 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second period, Wind speed range</th>
<th>$A$</th>
<th>$G_{\text{diff}}$</th>
<th>$N$</th>
<th>$\sum \frac{\text{res}^2}{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-8</td>
<td>1.98</td>
<td>1.0066</td>
<td>1442</td>
<td>$20.6 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>8-12</td>
<td>2.08</td>
<td>1.0062</td>
<td>1877</td>
<td>$12.6 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>12-16</td>
<td>1.92</td>
<td>1.0042</td>
<td>850</td>
<td>$10.2 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>4-16</td>
<td><strong>2.06</strong></td>
<td><strong>1.0060</strong></td>
<td><strong>4169</strong></td>
<td>$15.5 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 4: Fitted flow distortion parameters in different wind speed ranges. $N$ is the number of ten minute averages in the wind speed range and res the fit residuals.

As seen there is no trend with the extracted amplitude changing with wind speed. However, there is a trend with a changing gain difference, although with opposite directions for the two test cases. This could be a reflection of the uncertainty in the offset term in the cup anemometers’ calibration expressions. This is explained in the section of uncertainty in in-field calibration. These tendencies can also be seen in the scatter plot showing the fit residuals as a function of wind speed, Figure 15.
Figure 15 The residual, predicted-measured, of the cup ratio fit versus wind speed shows a slight tendency to increase with wind speed in the first data set, above. During the second period the fit residuals decrease with wind speed. The dependence between residuals and wind speed can be explained by the uncertainty in the offset term in the cup calibration expression which isn’t fully resolved by the in-field calibration possibility in the model.

The deviations in A lie within the uncertainties of the test, see for example Figure 10. The flow distortion is therefore probably well described by the proposed model while the in-field calibration has some room for further improvement.

3.2 Sensitivity to errors in the cup calibration offset term.

Cup anemometers are generally calibrated in wind tunnels and each unit is given a calibration expression $u_{true} = u_{meas} \cdot k + m$. However, these calibrations are made with an absolute uncertainty for example estimated to lie within ±0.1 m/s [3]. The proposed model finds an estimate of the wind speed proportional difference in the cup measurement, represented as $G_{diff}$. This value can be used to improve on the cup
calibration k. As will be seen $G_{\text{diff}}$ can even to some extent balance an error in m. However, the imperfections in the in-field calibration procedure is linked with uncertainties which are transferred to uncertainty in the fitted A, and $G_{\text{diff}}$ parameters of the model.

In an attempt to estimate the uncertainty introduced on A and $G_{\text{diff}}$, wind climates have been produced as 1000 data points from a normal distribution with a mean of 10 m/s and a standard deviation of 1.8 m/s. The wind directions have been taken from a uniform distribution between 0 and 360. To simulate boom mounting, the flow distortion was modeled as 2 %, without any offset on boom directions and a separation between the booms as 60 degrees. A second data set is generated by introducing an offset in the wind speed of the first data set. This new data set simulates measurements taken with a cup anemometer with a large error on m. A and $G_{\text{diff}}$ are then fitted to the measurement ratio of the erroneous cup anemometer and an ideal anemometer. An example of the obtained cup ratio using an error offset of 0.1 m/s and an $(\alpha_1, \alpha_2)$-couple (0,0) was used in this analysis is given in Figure 16.

![Figure 16: Cup ratio taken on a simulated wind climate, where one of the cup anemometers has an error in the cup calibration offset term of 0.1 m/s.](image)

As can be seen in Figure 17 the fitted amplitude differs only slightly from the intended 2 % due to the introduced offset error. The exact deviation depends on the stochastic nature of the simulated wind climate but in no run did the deviation exceed 0.05 percent points when the error offset lied within ± 0.1 m/s. The uncertainty introduced on amplitude is thus estimated to be negligible and covered by the uncertainties in the experimental study.

![Figure 17: Deviation in fitted A-parameter from the expected 2% as a function of the error in the offset term introduced on the simulated wind climate.](image)
On the other hand does the omission of an uncertainty in the offset term in the cup calibration expression give a large influence on the fitted $G_{\text{diff}}$ parameter, as seen in Figure 18. Independently of the stochastic nature of the simulated wind climate, the outcome is essentially identical and the fitted $G_{\text{diff}}$ is a tenth of the offset error. This is due to the effect that the simulated wind climate has a mean of 10 m/s. In a different wind climate where the mean wind is, say 6 m/s, $G_{\text{diff}} \approx \text{Offset error}/6$.

![Figure 18: Deviation of the fitted $G_{\text{diff}}$ parameter from the expected 1 as a function of the error in offset term introduced on the simulated wind climate with mean 10 m/s. The results of the three runs plotted in the graph more or less overlap.](image)

The errors on $G_{\text{diff}}$ are substantial. However, under these circumstances a large deviation from the expected $G_{\text{diff}}$ does not necessarily equal a large error on the wind speed measurement. Some of the error in $m$ is corrected by the estimate of $G_{\text{diff}}$. So despite the error introduced on $G_{\text{diff}}$ the wind speed measurements taken by the two cups become more equal, than if the error on $m$ is left uncorrected. That is $G_{\text{diff}}$ does no longer only correspond directly to errors in $k$ but becomes a best fit to the mixture of error in $k$ and $m$. Especially in wind measurement this works fairly well since a wind climate typically is reasonably symmetric around its mean and the dynamic range is small in comparison to the expected errors in offsets of less than 0.1 m/s, and even more so if we only consider the 4-16 m/s sensitive range of a wind turbine.

For example, if a cup calibration lacks 0.05 m/s in the offset term then correcting this lack with an estimated $G_{\text{diff}}$ will not fully compensate the measurement of weak wind. Say that the mean wind is 10 m/s, then the extracted $G_{\text{diff}}$ becomes in the order of 0.995. The 0.05 m/s undermeasurement of a 5 m/s wind, an error of 1 %, is then corrected with 0.025 m/s, leaving an error of 0.5 %. Correspondingly, strong winds will be slightly overestimated while typical winds in the studied wind climate will be fairly well represented. The option is to have all winds underestimated. Note that in a real case it will not be possible to determine if the error lies in one, both or the other cup and the corrections will therefore be applied in order to get the average of the two measurements.

### 3.3 Wind direction variability during the ten minute average

Wind velocity measurements are generally reported as a ten minute averages in wind energy applications. The non-linear cup ratio will therefore have a certain spread due to wind direction variability. The wind flow distortions influence on boom mounted cups is most accurately caught from a data set which is heavily screened on wind direction variability. An alternative is to use shorter time averages but in this case other issues like cup inertia becomes important. However, for wind energy...
applications it is important to provide corrections which give a valid picture in typical wind conditions including wind direction variability. The choice between the two approaches is not evident, but the impact of the choice will typically be small. Corrections which are applied to the tower in Høvsøre have been extracted from a dataset where ten minute averages with a standard deviation of wind direction > 5° have been excluded.

Figure 19: Histogram of wind direction variability in Høvsøre September 2008 to May 2009.
4 Estimates of the uncertainty in the wind velocity measurement taken by boom mounted cup anemometers

It is important to estimate the uncertainty in the corrected measurement. In the previous sections several parameters which can influence the results of the proposed method have been discussed. Sensitivity analyses have been performed as well as an attempt to estimate the variation in the results over a relatively large set of good data. However, the uncertainty analysis is made complicated by the correlation between the model parameters. Each test period show on a large stability and low residuals throughout that period, still the fit results can deviate slightly, but significantly more than the statistical uncertainties from one test to another. The residual valley and the slightly larger deviations in best guess estimate $\alpha$ than expected from the nominal should also be taken into account. An uncertainty analysis in the corrected and infield calibrated wind speed measurement is performed in this chapter.

The results described in this paper is only strictly true for the metrology tower in Høvsøre and the boom and tower dimensions at 80 m. Similar studies at other locations would give a better idea of the generality.

The uncertainties in $A$, $\alpha$ and $G_{\text{diff}}$ are discussed in this section. From these considerations the uncertainty in the flow distortion and the correction factor is estimated. Finally the uncertainty in the uncertainty in the corrected and infield calibrated wind speed measurement is estimated. The presented graphs of uncertainties as a function of wind direction apply to a south pointing boom with a flat of the tower facing west.

4.1 The uncertainty in the $A$ parameter and the flow distortion

The uncertainty in the $A$ parameter originates mainly from possible biases in the wind direction measurement and the boom direction and from the uncertainty in the actual position of the equivalent tower center. The introduction of an $\alpha$-couple reduces the uncertainty in $A$ by statistically attempting to estimate these deviations. The uncertainty in the estimate of the $A$ parameter due to the uncertainty in the offset term in the cup anemometer’s calibration expression is negligible in comparison.

In Figure 10, with a fixed $\alpha$-couple, the extracted $A$ differs within $\pm 0.1$ percent points during testing of one cup pair and with max $\pm 0.15$ percent points between different pairs, giving an estimate of the uncertainty in the fit to measurement data of $\pm 0.25$ percent points. However, a fairly large range of $\alpha$-parameters will provide statistically sound solutions.

A valid range of plausible $\alpha_1$ goes from about $-5^\circ$ to $+15^\circ$. In Figure 7 the $A$ parameter covers $\pm 0.5$ percent points over that range. However, the guesstimate of the $A$ and the $\alpha$-couple are correlated. As the $\alpha_1$ parameter increases, from roughly $-5^\circ$ to $+15^\circ$, the estimate of the $A$ parameter decreases from roughly $2.5\%$ to $1.5\%$ in our examples. The difference in the flow distortion factor, $1 - A \cos(\theta - \alpha - \beta)$, is thus much smaller for many wind directions than what is implied by the $\pm 0.75$
percent points uncertainty in $A$ over different $\alpha$ including the measurement uncertainties.

Instead, Uncertainty boundaries for the flow distortion factor can be estimated by studying the differences obtained when using $A = 2.5\% \pm 0.25\%$ with an $\alpha_1 = -5^\circ$ and $A = 1.5\% \pm 0.25\%$ with an $\alpha_1 = 15^\circ$. The highest uncertainty in the flow distortion of 0.8 percent points appears at 160° and 340°, while the smallest uncertainty of 0.16 percent points appears at 60° and 240°, as seen in Figure 20.

![Figure 20](image)

Figure 20: Uncertainty boundaries for the flow distortion factor, top graph, are estimated by studying the difference in the correction factors obtained with $A = 2.5 \pm 0.25\%$ and $\alpha_1 = -5^\circ$, bottom graph, blue plots, and with $A = 1.5 \pm 0.25\%$ and $\alpha_1 = +15^\circ$, bottom graph, red plots.

### 4.2 The uncertainty in the G\text{diff} parameter

The uncertainty in the extracted $G_{\text{diff}}$ parameter is seen to vary within $\pm 0.1$ percent points over a test period in Figure 11. Since $G_{\text{diff}}$ is expected to be different for each cup pair the variation in different tests does not give an estimate of the variance. The estimate of $G_{\text{diff}}$ depends to a small degree on the set $\alpha$-couple, in Figure 7 the $G_{\text{diff}}$ varies with less than $\pm 0.1$ percent points. The estimate of $G_{\text{diff}}$ typically is influenced with less than $\pm 0.5$ percent points with errors in the offset term in the cup’s calibration expression with $\pm 0.05$ m/s.

### 4.3 The uncertainty in the correction factor
The central uncertainty in this study is the uncertainty in the correction factor, \[ \frac{2}{1+G_{\text{diff}}(1+A \cos(\theta - \alpha_1))} \]. However, it is not trivial to translate the uncertainty in the \( G_{\text{diff}} \) factor, caused by an error in the offset term in the calibration, into an uncertainty in the corrected wind speed measurement. The fact that the extracted, strictly speaking erroneous, \( G_{\text{diff}} \) still will decrease the magnitude of the errors in the wind speed measurement should somehow be accounted for, see section 3.2.

Due to these difficulties a rough guess on effective uncertainty boundaries of the \( G_{\text{diff}} \)'s implication on the correction factor would be \( \pm \sqrt{0.1^2 + 0.3^2} = \pm 0.32 \) percent points.

The uncertainty boundaries in the correction factor as a function of wind direction can then be estimated by studying the largest difference in the correction factors obtained with the most extreme feed parameters, \( A, G_{\text{diff}} \) and \( \alpha_1 \). The extreme correction factors and the correction factor uncertainty are plotted as functions of wind direction in Figure 21.

![Figure 21: Uncertainty boundaries for the correction factor, left graph, are estimated by studying the difference in the correction factors obtained with \( A = 2.5 \pm 0.25\% \), \( \alpha_1 = -5^\circ \) and \( G_{\text{diff}} = 1 \pm 0.0032 \) right hand graph blue plots, and with \( A = 1.5 \pm 0.25\% \), \( \alpha_1 = +15^\circ \) and \( G_{\text{diff}} = 1 \pm 0.0032 \) right hand graph red plots.](image)

The maximal uncertainty of 1.01\%, appears at about 160° and at 340° where the wind falls approximately along the boom direction while the minimal uncertainty of 0.32\%, as expected occurs when the wind direction is close to perpendicular to the effective boom direction at 60° and 240°.
4.4 The uncertainty in the corrected and in-field calibrated absolute wind speed measurement

The uncertainty in the corrected and in-field calibrated absolute wind speed measurement can basically be found by adding on the uncertainty in the cup calibration, improved with a factor $1/\sqrt{2}$, and the uncertainty in the reference, here the wind tunnel, towards the absolute wind speed. The uncertainty due to tilted mounting etc. is partly included in the extracted boom correction parameters. Conservatively they are fully accounted for, with 1%, in the final uncertainty calculation.

The repeatability in the infield wind speed measurement after calibration in the wind tunnel at Svend Ole Hansen A/S is roughly described by a standard uncertainty of 0.5%, although probably slightly larger for the cups used in these tests. While the uncertainty in the reference is typically estimated to have a standard uncertainty of 0.1 m/s or 1.4% for a 7 m/s speed. Conservatively a uniform distribution of the correction factor has been assumed and the standard uncertainty is thus calculated as the uncertainty boundary $/\sqrt{3}$.

Since the non-linear correction factor uses ten minute averaged wind direction measurements as input an uncertainty has to be added which takes into account the wind direction variation at the site. Optionally the correction factor could be based on a data set which is not screened for large wind direction variations. The variations in the wind direction are thus indirectly taken into consideration for that specific data set, but to a cost of larger uncertainties in the extracted model parameters. The data set was screened for large wind direction variations in this study. The uncertainty in the corrected wind speed measurement induced by the wind direction variation at the site is roughly estimated to 0.2 %.

Uncertainties in the wind speed measurement due to the influence of off horizontal flow etc are treated in [9] but are not accounted for here since they are specific for the site and the cup anemometer.

The total uncertainty in the corrected wind speed measurement is thus calculated as

$$\sqrt{0.014^2 + \left(\frac{0.005}{\sqrt{2}}\right)^2 + 0.01^2 + 0.002^2 + \sigma^2_{\text{Correction factor(\theta)}}}$$

and is plotted in Figure 22. These uncertainties can be compared to those of a top mounted P2546A cup anemometer of 1.8% at 7 m/s.
Figure 22: Estimated uncertainty in the corrected wind speed measurement, blue plot, are compared to the uncertainties of a top mounted cup anemometer, red. Here given for 7 m/s.

The uncertainty in the absolute wind speed measurement is dominated by the uncertainty in the reference wind tunnel. Perhaps counter-intuitively, the uncertainties in the corrected and infield calibrated wind speed measurement estimated in this study are lower for certain directions, than those obtained with a top mounted cup anemometer. This is due to the fact that a measurement with a cup anemometer only seems to be slightly perturbed when taken perpendicularly to the wind direction, while the significant uncertainties in the cup calibration seems to be considerably reduced by the in-field calibration.

By averaging the measurements from the two cups the uncertainty due to tilted mounting etc can be reduced. By calibrating both cup anemometers in several tunnels the uncertainties in the reference source can be reduced.

5 Examples of the implications of uncorrected errors

Examples, in a few typical measurement situations in the wind energy industry, of the implications of leaving the influence of the flow distortion uncorrected are given in this section. For all these examples an \((\alpha_1, \alpha_2)\)-couple of \((0,0)\) is used.

5.1 Impact on power curve measurement

Although top mounted cup anemometers are preferred, the IEC acknowledges power curve measurements taken with boom mounted cup anemometers as long as the design guidelines are followed. The wind sector used in a power curve measurement is often very narrow due to restrictions caused by turbulence from turbines both on the sensor and on the turbine. This means that the error observed on boom mounted cups are not averaged to a large degree. Furthermore, is it typically the case that more than one wind turbine in a park is acceptance tested using the same mast. Since the accepted wind sector often is quite different for each tested turbine the errors will differ and thus giving a large spread in the observed performance even if the turbines are identical.

In Figure 23 three power curves of a 2 MW Vestas turbine are plotted. The blue curve shows the nominal power curve while the red shows a power curve obtained with the wind speed error on a boom mounted cup anemometer pointing towards the narrow accepted wind sector. The green curve shows the result obtained if the boom points about 150 degrees away from the previously accepted wind sector.
Figure 23: Top Graph: Power curves obtained from measurements with a cup anemometer which experiences errors of 0% (blue), -2% (green) and +1.5% (red). Bottom Graph: Error in the power curve if the wind sensor makes a -2% (green) or +1.5% (red) error.

Note that the deviations in the power curves are in the order of 5% in the wind speed range of 7-13 m/s where the power increases rapidly with wind speed. Such results could easily fail an acceptable and pass an unacceptable turbine during acceptance testing which typically puts demands on an AEP on a nominal wind climate which deviates with less than 5% from the expected.

5.2 Impact on the estimation of energy potential

The impact of the wind direction dependent errors in the wind speed measurement by a boom mounted cup anemometer on the production potential of a site depends on the wind climate at the specific site and on the choice of boom direction. However, since the wind climate in a site generally often is roughly known, the boom and tower are in the best case directed so that periods influenced by tower wakes are restricted in as high degree as possible. A further lenient factor is that in many cases the wind direction is dominantly in two opposite directions, which often is perpendicular to the least occurring wind direction.
For example, in Høvsøre, westerly winds are dominant while northerly winds are scares. The choice in Høvsøre to minimize the occurrence of tower wakes by having the main cup anemometer pointing south, thus automatically also reduced the occurrence of wind speed measurements for which the largest underestimation due to flow distortions from the tower occur.

However, the impact of the corrections will vary for each site and the production potential will vary with the choice of boom direction. With corrections these differences are mitigated. As an example the influence of flow distortions in our site are gathered in Table 5, where $\sigma$ is the standard deviation of the two cup measurements, mean denotes the average relative difference, i.e. $\text{mean}(\text{abs}(\text{cup } S - \text{cup } SES)/\text{true wind speed})$ and $\text{EP}$ the estimated production of a standard 2 MW Vestas turbine. The turbine used in this example cuts in at 4 m/s.

<table>
<thead>
<tr>
<th>Data set</th>
<th>$\sigma$ [m/s]</th>
<th>Mean [%]</th>
<th>EP [MW]</th>
<th>$\Delta$EP</th>
</tr>
</thead>
<tbody>
<tr>
<td>20080417 - 20080909 uncorrected</td>
<td>0.12</td>
<td>1.2</td>
<td>2294 with S</td>
<td>-2.0 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2307 with SES</td>
<td>-1.4 %</td>
</tr>
<tr>
<td>20080417 - 20080909 corrected and in-field calibrated</td>
<td>0.03</td>
<td>0.2</td>
<td>2338 with S</td>
<td>-1.2 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2334 with SES</td>
<td>-1.8 %</td>
</tr>
<tr>
<td>20081201 – 20090701 uncorrected</td>
<td>0.12</td>
<td>1.2</td>
<td>3017 with S cup</td>
<td>-1.2 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3000 with SES</td>
<td>-1.8 %</td>
</tr>
<tr>
<td>20081201 – 20090701 corrected and in-field calibrated</td>
<td>0.04</td>
<td>0.3</td>
<td>3057 with S</td>
<td>-1.2 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3052 with SES</td>
<td>-1.8 %</td>
</tr>
</tbody>
</table>

Table 5: Difference in wind speed and predicted power output from a 2 MW turbine.

The improvement in accuracy of each ten minute period measurement is in the order of 3-6 times. However, due to the averaging effect of up and down stream winds in this particular test case, the estimated power production is not drastically different, although errors of 2% are significant in an economic evaluation.

### 5.3 Impact on shear measurements and extrapolated wind speeds

The flow distortions around metrological towers also have an impact on the measurement of wind shear and thus for extrapolated wind speeds at higher altitudes. The errors are site specific, in addition to wind direction and speed distribution, it also depends on the shear distribution which typically depends on wind direction. When measurements are to be used to extrapolate the wind velocity at a higher height it is not evident that the general guideline of trying to point the boom perpendicular to the prevailing wind direction will ensure the lowest uncertainty.

As an example the wind speed at 140 m in Høvsøre has been extrapolated from the south pointing boom mounted cup measurement at 80 m and a top mounted cup measurement at 116.5 m. The ten minute averaged wind speed as a function of height, $u(z) = b \cdot z^a$, is found by fitting $a$ and $b$ to the ten minute measurements at 80 and 116.5 m. In this specific test case the wind shear is in general small when the measurement errors are large, i.e. southerly winds, and shears larger when the errors are small, easterly and westerly winds, thus giving a limited effect, see Figure 24.
Figure 24: The fitted shear parameter, $a$, as a function of wind direction at 80 m. The blue line shows the correction factor to be applied, in percent.

Corrected 80 m values are used to give a theoretically true value at 140 m and compared to the uncorrected extrapolation in Figure 25 and Table 6.

Figure 25: “True” wind speed at 140 m minus the wind speed at 140 m extrapolated from uncorrected measurements at 80 m.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20080417 - 20080909</td>
<td>3824</td>
<td>3849</td>
<td>+ 0.65 %</td>
</tr>
<tr>
<td>20081201 – 20090701</td>
<td>5140</td>
<td>5159</td>
<td>+ 0.37 %</td>
</tr>
</tbody>
</table>

Table 6: Estimated production of a 2 MW turbine with a hub height at 140 m using corrected and uncorrected measurements at 80 m.
Due to the averaging effect of positive and negative errors and the limited wind shears in the test case the deviations in estimated production are small. Note that the deviation from the true wind speed at 140 m extrapolated from a power law formula is probably larger than the errors introduced by the faulty measurement, but we will not go further into this matter in this report.

5.4 Impact on comparisons with remote sensors

For proper use of remote wind sensors it is important that their accuracy in retrieving wind velocity can be traced to calibrated cup anemometers. In order to verify that remote sensors measure identically at several heights they are typically tested both towards a cup mounted anemometer and at several levels with boom mounted cups.

Remote sensors have traditionally been compared with cup anemometers in linear regression analysis. A calibration expression for the remote sensor has typically been given as

\[ u_{\text{lidar}} = G_r \cdot u_{\text{cup}} + m_r \]

where \( G_r \) is the remote sensor gain and \( m_r \) an offset term. It is typically stipulated that the uncertainty in \( u_{\text{cup}} \) is considerably smaller than in the remote sensor and for traceability purposes can be considered as \( u_{\text{true}} \).

This analysis method disregards the important uncertainty in sensing height combined with the use of remote sensors but it will be used to illustrate the impact of errors in boom mounted cup anemometry.

In the analysis of cup mounted errors we have seen that the deviations in the wind speed measured by cup anemometers can reach 2%. However, this does not mean that in a study of an ideal lidar the error in the observed lidar gain, \( G_r \), is limited to 2%.

For example, consider the situation with a tower placed such that one acceptable measurement sector covers inland winds with a medium wind speed of 6 m/s. In the opposite direction another acceptable test sector covers strong coastal winds with a medium of 10 m/s. In this example the two wind distributions are generated from a normal distribution with a standard deviation of 1 m/s. Now consider that the boom mounted cup underestimates the wind speed with 1.5% for the slower section and thus overestimates the wind speed in the other direction with 1.5%. The linear regression of the data in Figure 26 yields a \( G_r \) of 0.95 and a \( m_r \) of 0.3 m/s. If the boom instead was mounted in the opposite direction, reversing the sign of the boom mounted error, the linear regression yields a \( G_r \) of 1.05 and a \( m_r \) of -0.3 m/s. Note that the \( R^2 \) number, a parameter traditionally given as a measure of accuracy but connected with several problems, of 0.999 is comparable to several reported tests [8].
Figure 26: Linear regression analysis of the wind speed measured by an ideal lidar and by an uncorrected boom mounted cup anemometer, according to scenario 1. The plotted line indicates the best fit \( \text{u}_{\text{lidar}} = 0.95 \cdot \text{u}_{\text{cup}} + 0.3 \).

Errors of this magnitude are much too important to use in calibration and testing of remote sensors. They practically make any conclusions and debugging of system errors impossible since they will hide other errors. Since it has been assumed that the uncertainty in the wind speed measurement was significantly lower, as long as design guidelines for booms where respected, care must be taken not to over interpret results indicating large uncertainties in remote sensing.

6 Practical application of correction formula and infield calibration

Several approaches are available when it comes to the practical application of the proposed correction and in-field calibration method for generation of a corrected wind data set. In this section those choices are discussed and a recommendation for best practice is suggested.

6.1 Recommendations for application of method

To obtain the smallest uncertainties in the extracted correction formula it is recommended that two boom mounted cup anemometers are used for the entire measurement period. \( G_{\text{diff}} \), \( A \) and \( \alpha \) are obtained from fits to the full dataset after screening of selected wind sectors. Post correction of the data set is done by applying equation 1.3 to the data set. For wind directions where cup one is exposed to a tower or cup wake, \( u_1 \) is set to 0, while \( u_2 \) is doubled, and vice versa.

\[
\text{u}_{\text{corr}} = \frac{2}{1 + G_{\text{diff}} \left( 1 + A \cos(\theta - \alpha_1 - \beta_1) \right) + \frac{1}{1 + \frac{1}{G_{\text{diff}}} \left( 1 + A \cos(\theta - \alpha_2 - \beta_2) \right)}}
\]

(1.3)

If a cup fails in post correction, or otherwise is set under doubt, the cup which is trusted can be used as a reference and the extracted \( G_{\text{diff}} \) is only used to calibrate the
problematic cup, which probably still has significantly better measurements for the
sector where the good cup is affected by the tower wake. However, it must be made
clear that the data set has not suffered from degradation.

Another possibility, which is likely to be connected with larger uncertainties, is to
find global correction formulas which are used on the specific tower-boom
dimension or even scale the flow distortion to other dimensions by for example using
the IEC results as a base. However, it is likely that the uncertainties connected with
applying a scaled correction formula to another tower-boom dimension, are
significantly smaller than those in using uncorrected measurements. This is
supported by comparisons with remote sensors [5].

6.2 Recommended set up

It is clear that a wind direction measurement directly at the boom mounted level will
decrease the uncertainties in the extracted parameters. Further work has to be done to
estimate to which degree the uncertainty in the correction factor decreases with the
introduction of yet further booms and cup anemometers at the same height level. It is
also clear that the booms should be long enough to place the cup anemometer
outside the near flow-field to obtain the sinusoidal cup ratio versus wind direction.

Equation 1.2 experiences the highest dynamic range, and the uncorrected
measurements the largest extreme deviations, when the two booms are mounted in
opposite directions. Theoretically this will give the highest accuracy in the estimates
of \( A, \alpha \) and \( G_{\text{diff}} \) since stochastic measurement deviations will have a smaller
influence on the extracted parameters. However, the extremes of the sinusoidal are
not available due to the tower wakes. From a practical point of view a 120°
separation is interesting since it gives a relatively high dynamic range while the
booms can be fastened and aligned to a tower side.

If boom corrections are not applied, the measurement uncertainty can still be
mitigated by mounting two cup anemometers with a 90° separation on the booms.
This set up has the advantage that one cup experiences minimal distortion, while the
other experiences maximal. The final data set can thus be constructed by accepting
the ten minute measurements from the cup anemometer which is mounted on the
boom which is nearly perpendicular to the ten minute wind direction.

Another approach is to simply increase the guidelines for boom lengths and verify,
for example with the proposed model, that the uncertainty due to the flow deviations
are smaller than the uncertainties in the correction formula. It is likely that such
booms will be very long and other uncertainties like significant vibrations, and
influence from support wires has to be estimated.

The uncertainty in the corrected wind speed due to flow distortions can roughly be
estimated to have a standard uncertainty of 0.3%. To achieve this uncertainty in the
wind speed measurement, assuming a uniform distribution of wind directions, the
booms would, at 80 m, have to protrude 7 m from the 1.9 m wide tower, according to
IEC guidelines. However, the IEC guidelines are underestimating the maximal flow
deviation with about 75% for the examined set up. As a simple approximation the
IEC guidelines can be scaled with this result. The predicted need of the cup-tower
distance, which can achieve lower uncertainties than from using corrections,
becomes about 10 m.
Measuring with two cups has several other advantages. Supplementary cups are for example often applied in a fork setup even at top level. Two cups ensures that the full 360 wind sector can be measured since tower wakes can be avoided. Data from the supplementary cup can be used in case of cup failure. Furthermore, the corrected measurement result can be used for quality control, e.g. to indicate and flag for periods where the cup anemometers are frozen or otherwise influenced and for detection of degradation.
7 Discussion and recommendations

From the investigations in this report, it is clear that the errors on boom mounted cup anemometers can be significant even though the set up follows IEC design rules. The flow distortion around a lattice tower is measured to be about 75% larger than expected; the maximum error is an underestimation of about 2% which appears when the wind is directed towards the tower in a line from the cup anemometer. However, when the wind approaches perpendicularly to the effective boom direction the flow distortions are insignificant.

7.1 Model choices

Modeling the flow distortion as a sinusoidal is sensible taking into account the observed cup ratios and the assumption that the cup is placed in the far-field so that the tower can be considered to be symmetrical. The model has been expanded to include an \( \alpha \)-parameter which takes into account that the equivalent boom direction, i.e. including uncertainty in wind direction bias, centre of equivalent tower etc, differs from the intended boom direction measured from the tower corner. A few unexplained issues with the \( \alpha \)-parameter, the deviation from geometrical considerations and the residual valley which forces a guesstimate of a most suitable \( \alpha \)-couple, increases the uncertainty of the results. Yet the experiences from the Høvsøre tower, a significant reduction in the residuals and a smaller variation in the extracted parameters, encourage the introduction of an \( \alpha \)-couple.

Important to note is that a measured cup ratio does not contain information on drag, here defined as an offset to a symmetrical sinusoidal flow distortion. I.e. if each individual cup anemometer is affected by an equal drag it will not show up in the measured cup ratio. However, the important difference between top mounted set up, which might over speed due to increased flow over the tower in comparison with the free wind velocity, and a boom mounted set up which might experience drag, has to be estimated by other means, i.e. by remote sensors or top mounted cup anemometers on a nearby tower. Early indications give estimates of these differences to be considerably smaller than 1% [5].

A further notion to take into account is that some cup anemometer brands, including the P2546A cups used at Høvsøre, have had a non-symmetric flow angle response measured in wind tunnels [9]. This means that they are suspected to over speed in certain wind directions and under speed in others when inclined. Severe tilting of the cup anemometer could thus single handedly explain the observations. However, it is unlikely that the pattern would iterate during three test periods and seem to be similar also at other heights when compared to lidar anemometry but tilted mounting could explain some of the differences observed between the three periods. Nevertheless, the effects of tilted mounting will, to some degree, be mitigated by a correction formula extracted according to the recommendations in this paper for the data set in question.

Finally a parameter was introduced to explain observations of significant offsets from 1 in the mean cup ratio, a phenomena which is contributed to an uncertainty in the cup anemometer calibration. It is strongly recommended to include a \( C_{\text{airf}} \) parameter in the fit formulas in order to obtain a better accuracy in the estimate of the flow distortion amplitude. However, it might not be straightforward to apply the result. Although no other physical phenomenon has been found which can explain
the observed \( G_{\text{diff}} \), there might be some uncertainty in the claim that \( G_{\text{diff}} \) is representing the difference in measurement gain between the cup pair. Tilted mounting of either cup, for example, isn’t likely to give a shift of the cup ratio which is independent of wind direction, nor would an obstacle. Simultaneous post calibration in a wind tunnel of a few test pairs could shed more light on this matter and increase the confidence in using \( G_{\text{diff}} \) for infield calibration. In either way, it is probably a good idea to continuously extract a \( G_{\text{diff}} \) parameter throughout a measurement campaign and flag for a very large \( G_{\text{diff}} \), e.g. above 1 %, and then perform a post calibration and/or simply replace cups as soon as possible.

8 Conclusion

The wind speed measurements taken by two cup anemometers, mounted according to current IEC guidelines, on a lattice tower with booms at the same height but with different pointing directions are observed to deviate, depending on wind direction, in a sinusoidal pattern with up to 3% The deviation is considered to mainly be caused by flow distortion around the tower.

The flow distortion effect on the individual cup anemometers is extracted by fitting a modeled error to the measured cup ratio and wind direction. The measurement error is estimated to reach up to \( \pm 2\% \). The errors are on average about 75% larger than those predicted from the IEC standard for power performance measurements.

Errors of that magnitude are severely problematic in the measurement of wind turbine power performance and for providing purposeful traceable measurement accuracy of lidar anemometers.

The uncertainty in the wind speed measurement taken by a boom mounted anemometer can be significantly reduced by the method proposed in this article. The proposed correction method is estimated to reduce the standard uncertainty in the measurement due to the flow distortion from the tower with about 3 times. The method can also be used for in-field calibration of the sensors to mitigate the uncertainty in the calibration expressions found in wind tunnels.

In three periods using different cup pairs the repeatability in the wind speed measurement was improved about 4 times. The deviation between the corrected cup anemometer measurements was Gaussian like with a standard deviation lower than 0.03 m/s.

The proposed method can be used to significantly improve the measurement of wind shear when a mixture of top and boom mounted anemometers are used. It can also improve the prediction of a wind resource assessments and the measurement of a wind turbines power performance. Furthermore, it can significantly improve the accuracy in the traceable measurements of remote sensors or in comparison between different methods, like sonic or propeller anemometry, or different brands of cup anemometers.

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