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RESEARCH ARTICLE

# Trading wind energy based on probabilistic forecasts of wind generation and market quantities

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## ABSTRACT

Wind power is not easily predictable and non-dispatchable. Nevertheless, wind power producers are increasingly urged to participate in electricity market auctions in the same manner as conventional power producers. The aim of this paper is to propose an operational strategy for trading wind energy in liberalised electricity markets and to assess its performance. At first the so-called optimal quantile strategy is revisited. It is proved that without market power, i.e. under the price-taker assumption, this strategy maximises expected market revenues. Forecasts of wind power production, of spot and regulating market prices and of the system imbalance are inputs to this strategy. Subsequently, constraining of the bid that maximises the expected revenues is proposed as a way to overcome the strategy's disregard of risk and of practical limitations. Two constraining techniques are introduced: constraining in the decision space and in the probability space. Finally, the trade of a wind power producer is simulated in a test-case for the Eastern Danish (DK-2) price area of the Nordic Power Exchange (Nord Pool) during a 10 month period in 2008. The results of the test-case show the financial benefits of the aforementioned strategy as well as the consequent interaction with the electricity market. This study will support a demonstration in the framework of the EU project ANEMOS.plus. Copyright © 0000 John Wiley & Sons, Ltd.

## KEYWORDS

wind power; electricity markets; probabilistic forecasting; stochastic optimization; decision theory

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## 1. INTRODUCTION

In liberalised electricity markets, competition stands as the fundamental mechanism ensuring the efficient operation of the system. Competition is implemented through the establishment of a market (or multiple markets operating under different rules and gate-closures) where energy is traded. Bids for sale and purchase of energy are collected by the market

operators, which are responsible for optimally scheduling the dispatch of energy and allocating sufficient power reserve. The backbone of most liberalised electricity markets are the day-ahead markets, often referred to as *spot* markets\*, on which most of the energy is traded. Typically these markets offer a platform for trading energy to be delivered/withdrawn within a certain period during the upcoming day. The minimum period length is called *Program Time Unit (PTU)* and every contract covers one or more PTUs.

Although most renewables are not easily predictable and non-dispatchable, renewable power producers are increasingly urged to participate in electricity markets in the same manner as producers of conventional energy. Here we specifically concentrate on wind energy, which has been the most rapidly growing renewable energy source over the last decade. Our developments and conclusions could however be similarly applied for other types of non-dispatchable renewables e.g. solar energy.

Wind power generation is the typical example of a stochastic and non-dispatchable renewable energy source. Although the possibility of curtailing power exists, it is not economically sound as long as the electricity price remains positive. As a result, trading wind energy in a day-ahead electricity market requires forecasts of wind power production, which can be performed only with limited accuracy, as discussed in [1]. Reviews of the state of the art of wind power forecasting methods and operational tools can be found in [2] and [3], while [4] discusses their application in electricity markets.

Differences between contracted and actual energy production (e.g. due to forecasting errors) have to be settled on the intra-day and/or real-time regulating markets. Due to shorter lead-time from gate closure to delivery, these markets might reduce the revenues of producers that cause imbalance, as more flexible market players are called to equilibrate the system – generally at higher costs. Joint operation of wind and hydro power has recently emerged as a way to reduce imbalance costs among other benefits, see for instance [5] or [6]. However, this solution is only conceivable for market participants having both energy sources in their portfolio. For other producers, the most practical option for imbalance settlement is to rely on the market. Although it is sometimes possible to adjust contracts through existing intra-day markets, the volumes exchanged there are generally low, as illustrated by [7] for the main European electricity markets. Producers are therefore most often forced to settle their imbalances on the real-time market. Hence, the only way for them to reduce imbalance costs is to bid optimally into the spot market, so that the risk of facing losses on the regulating market is minimised. This bid is optimised conditioned upon the information available at the time of contracting, both in terms of future wind power production and market prices.

The penalties faced by electricity producers in the regulating market are generally asymmetric, in some cases even single sided, i.e. they are only to be paid by the producers that increase the overall imbalance with their own. This incites market participants whose portfolio includes a stochastic component to be more strategic in their approach to bidding, see [8]. Indeed it can be analytically shown that under these conditions the optimal spot market bid for a wind energy producer is a certain quantile of the distribution of wind power generation, see for instance [9], [10] and [11]. This optimal quantile is a dynamic function of the spot and the imbalance prices, which are not known a priori. Market experience shows that such optimal bids might significantly differ from the point forecasts of wind power production (consisting of the conditional expectation for each lead time). In practice, however, point forecasts are still most commonly used when contracting wind power in the day-ahead market. A more theoretical discussion about quantile forecasts being optimal bids in electricity markets can be found in [12].

The existing literature has already described and analysed a number of strategies for trading wind power in the spot market, with different approaches with regards to the uncertainty in production and in market prices. As a basic approach, some authors consider that traditional point forecasts of wind power generation may be used for analysing the value of wind energy in electricity markets, e.g. [13, 14, 15]. Furthermore, [16] models wind generation uncertainty through Markov probability tables and chooses, in a discrete decision space, the bid that minimises the expected costs. Alternatively, [17] suggests the construction of a utility cost function to model the financial risk of wind power producers participating in the market, using persistence forecasting of wind power and average values as price forecasts. The stochastic optimisation

\* This holds in Europe only, while in the USA the term *forward* markets is used, while *spot* markets indicate real-time markets. The European convention is adopted in this article.

algorithm described in [18] uses scenarios of wind power production as input along with historical imbalance prices. Besides, [11] makes use of probabilistic forecasts of wind power and yearly or quarterly average values of imbalance prices in order to determine the optimal quantile bid, in a fashion resembling that of [9]. The same strategy is implemented in [19], using probabilistic forecasts and measured data for wind speed and yearly averages as estimates of the spot and regulation prices. Finally, [20] proposes a linear programming technique for optimising the trade of wind energy in spot, intra-day and real-time markets. The uncertainty in both wind power production and market prices is modelled through simple ARIMA/ARMA models. All these works and strategies either only account for uncertainty in wind power generation but disregard uncertainty in the market quantities, or include both but make use of simple forecasting methods.

In this work, we revisit and generalise the quantile strategy described in [9] and [11] by considering state-of-the-art probabilistic forecasts of both wind power generation and market quantities as input. These market quantities include the regulation sign, which can be down-regulation, up-regulation or no regulation, as well as the unit regulation costs. This strategy is formulated in Section 2 as a stochastic optimisation problem, which aims at the maximisation of the expected revenues (or utility) of the market participant. This approach is hereafter referred to as Expected Utility Maximisation (EUM). Being its objective the expected value of the revenues, such an approach directly relates to a long-term optimisation of the market performance of the wind power producer. It is also shown through an example that, due to the uncertainties involved and potentially large forecast errors, such a strategy may occasionally lead to severe losses from a single contract. For instance this might occur when the regulation sign forecast wrongly assigns a high probability to an imbalance direction that is not realised. It is therefore proposed in Section 3 to constrain the EUM bid in terms of deviations from the point forecasts, either in the quantity space or in the probability space. The two constraining methods are proposed with two different ranges of the allowed interval in the decision space. There are both theoretical and practical reasons for this constraining. From a theoretical point of view, and without regard to the market, the conditional expectation should be the most risk-averse bid. This is because the point forecast is estimated under a least-squares criterion. Therefore by setting a constraint on the allowed deviation from the point forecast, the trader decides on its level of risk-aversion. The practical reason for constraining the bid is that system operators are reluctant to allow large deviations from the point forecasts. This is because efficient system planning requires market bids to closely reflect the actual delivery of energy. Moreover, since point forecasts have been used as operational bids since wind energy started to be traded on electricity markets, such point forecasts act as anchors in the mind of the operators. Next, in Section 4, the participation of a wind power portfolio in the Nord Pool market (Eastern Denmark price area) over a period of 10 months in 2008 is considered in order to evaluate the actual performance of the aforementioned trading strategies. To our knowledge a test-case of such length, combining state-of-the-art forecasts of wind power production, spot and imbalance prices, as well as observed wind production and market data, has never been performed. The results of the exercise show the possibility for wind power producers to significantly reduce their imbalance costs and control the risk of dramatic losses. The work performed here will support and serve as the basis for a real-world demonstration of stochastic approaches to wind power participation in electricity markets in the framework of the EU project ANEMOS.plus.

## 2. THE EXPECTED UTILITY MAXIMISATION (EUM) BIDDING STRATEGY

In electricity spot markets, power producers have to indicate the amount of energy they are willing to deliver at any program time unit (PTU) through a bid submitted to the market operator. Bids are collected with a certain lead-time to the physical delivery of energy. For example, at the Nord Pool spot market the deadline for submission is at noon on the day prior to delivery. Let  $\widetilde{W}_k$  denote the amount of energy contracted in the spot market and let  $W_k$  be the stochastic production of wind energy, both for the  $k$ -th PTU. The power producer will then have to correct the stochastic imbalance  $W_k - \widetilde{W}_k$  on the real-time market. This is because the possibility of trading on the intra-day market is disregarded, due to its general illiquidity. Hence, the total revenues of the generator,  $\rho_k$ , can be expressed as the sum of the revenues,  $\rho_k^{(S)}$  and  $\rho_k^{(T/L)}$ ,

obtained at the spot and the regulating market respectively

$$\rho_k = \rho_k^{(S)} + \rho_k^{(\uparrow/\downarrow)} \quad (1)$$

The revenues at the spot market can be determined as the multiplication of the contracted energy  $\widetilde{W}_k$  with the spot market price  $\pi_k^{(S)}$

$$\rho_k^{(S)} = \pi_k^{(S)} \widetilde{W}_k \quad (2)$$

The regulating market revenues are positive if  $W_k > \widetilde{W}_k$  (energy surplus to be sold) and negative if  $W_k < \widetilde{W}_k$  (energy deficit to be purchased)

$$\rho_k^{(\uparrow/\downarrow)} = \begin{cases} \pi_k^{(\downarrow)} (W_k - \widetilde{W}_k), & W_k \geq \widetilde{W}_k \\ \pi_k^{(\uparrow)} (W_k - \widetilde{W}_k), & W_k < \widetilde{W}_k \end{cases} \quad (3)$$

In this expression,  $\pi_k^{(\downarrow)}$  ( $\pi_k^{(\uparrow)}$ ) represents the unit down(up)-regulation price which is paid to (by) an overproducing (underproducing) generator. At Nord Pool regulation prices are restricted such that

$$\begin{aligned} \pi_k^{(\downarrow)} &\leq \pi_k^{(S)} \\ \pi_k^{(\uparrow)} &\geq \pi_k^{(S)} \end{aligned} \quad (4)$$

at all times. Then depending on the total imbalance of the system, the inequality sign is substituted by an equality sign in at least one of the two inequalities in Equation (4). More specifically, let the system net imbalance be denoted as

$$(\widetilde{G}_k - G_k) - (\widetilde{L}_k - L_k) \quad (5)$$

where  $\widetilde{G}_k$  and  $G_k$  denote the total energy production, contracted and realised respectively, for the  $k$ -th PTU. Similarly,  $\widetilde{L}_k$  and  $L_k$  represent the total contracted and realised consumption respectively. During hours of power surplus, i.e. when the system net imbalance in Equation (5) is  $< 0$ , the following holds for the prices

$$\begin{aligned} \pi_k^{(\downarrow)} &\leq \pi_k^{(S)} \\ \pi_k^{(\uparrow)} &= \pi_k^{(S)} \end{aligned} \quad (6)$$

This situation is commonly referred to as down-regulation. Conversely during hours of power deficit (when the system net imbalance in Equation (5) is  $> 0$ ), commonly termed up-regulation, it holds that

$$\begin{aligned} \pi_k^{(\downarrow)} &= \pi_k^{(S)} \\ \pi_k^{(\uparrow)} &\geq \pi_k^{(S)} \end{aligned} \quad (7)$$

Finally during hours of perfect balance between load and production then

$$\pi_k^{(S)} = \pi_k^{(\downarrow)} = \pi_k^{(\uparrow)} \quad (8)$$

In this way, only the producers contributing to the overall system imbalance risk being penalised, while the ones acting to reduce it receive the spot price for their realised production, when transactions on both the spot and the real-time markets are combined. The rationale behind this choice of market design is that producers should not be allowed to profit from their imbalances. However, it should be pointed out that there are exceptions to this. For instance, the Dutch APX electricity market is just one example of a market where energy imbalance can actually be rewarded.

Now Equation (1) can be reformulated as:

$$\rho_k = \pi_k^{(S)} W_k + C_k^{(\uparrow/\downarrow)} \quad (9)$$

Assuming that the wind power producer is a price-taker individually, which is reasonable if it does not hold a significant share of the total production, the term  $\pi_k^{(S)} W_k$  in Equation (9) is independent of its decision. That is, neither the spot price  $\pi_k^{(S)}$  nor the wind power production  $W_k$  are influenced by the bidding policy adopted in the spot market<sup>†</sup>. This term represents the revenues that the producer could achieve if it had perfect information on its future wind power production (i.e. if contracted power and wind power production are equal:  $\widetilde{W}_k = W_k$ ). The second term in Equation (9) can be made explicit as

$$C_k^{(\uparrow/\downarrow)} = \begin{cases} \psi_k^{(\downarrow)} (W_k - \widetilde{W}_k), & W_k \geq \widetilde{W}_k \\ \psi_k^{(\uparrow)} (W_k - \widetilde{W}_k), & W_k < \widetilde{W}_k \end{cases} \quad (10)$$

where the variables  $\psi_k^{(\downarrow)}$  and  $\psi_k^{(\uparrow)}$  represent the unit regulation costs for positive and negative imbalances and are given by

$$\psi_k^{(\downarrow)} = \pi_k^{(\downarrow)} - \pi_k^{(S)} \quad (11)$$

$$\psi_k^{(\uparrow)} = \pi_k^{(\uparrow)} - \pi_k^{(S)} \quad (12)$$

The quantity in Equation (10) therefore accounts for negative revenues, which represent the losses for the producer contracting  $\widetilde{W}_k$  at the spot market in comparison to the case of perfect information. At Nord Pool it holds that  $C_k^{(\uparrow/\downarrow)} \leq 0$  at all times. Elsewhere (e.g. APX in the Netherlands),  $C_k^{(\uparrow/\downarrow)} > 0$  might occur. Regarding the latter case, economists argue that although situations where producers can gain from their imbalance are possible, this cannot be exploited in the sense of strategic bidding. The argument is that the expectation  $\mathbb{E} \{ C_k^{(\uparrow/\downarrow)} | \chi \}$  of the losses given the information available at the moment of bidding is negative. As a consequence, the producers are expected to suffer losses from their imbalance in the long run, although in some PTUs they might be able to gain from it. Interested readers are referred to [21] for a detailed discussion.

As one can see, at Nord Pool  $\psi_k^{(\downarrow)} \leq 0$  and  $\psi_k^{(\uparrow)} \geq 0$ , and they are equal to zero in the cases of up- and down-regulation respectively. It should also be noted that both the unit regulation costs in Equations (11) and (12) are stochastic variables as the spot price and the imbalance prices are not known in advance by the power producer.

It is assumed from now on that the wind power producer is *rational* (see e.g. [22] for a conceptual introduction) and that its objective is the maximisation of the expected value of its total revenues. The set of bids  $\widetilde{\mathbf{W}}$  maximising the total revenues is

$$\widetilde{\mathbf{W}} = \arg \max_{\widetilde{\mathbf{W}}} \mathbb{E} \left\{ \sum_{k=i_{PTU}}^{f_{PTU}} \rho_k \right\} \quad (13)$$

where  $i_{PTU}$  and  $f_{PTU}$  are the shortest and the longest lead-times considered in the optimisation, respectively. Here the commonly accepted assumption of independence of decisions for different PTUs is followed. However it may be argued that market dynamics should be accounted for, see for instance [23, 24, 25]. Under the assumption of time-independent decisions over time, the maximisation of the sum of the revenues over time is equal to the maximisation of the revenues obtained at each single  $k$ . The optimal bid at the spot market is then

$$\widetilde{W}_k = \arg \max_{\widetilde{W}_k} \mathbb{E} \{ \rho_k \} \quad (14)$$

<sup>†</sup> The production  $W_k$  could actually be curtailed by the producer. Nevertheless, it has already been pointed out that this behaviour would be uneconomic as long as the price  $\pi_k^{(\downarrow)}$  for energy in excess is positive.

Since the first term in Equation (9) is not dependent on the decision on the spot market, the maximisation of the expected revenues in Equation (14) is equivalent to the maximisation of the expectation of the regulation costs, which are non-positive

$$\widetilde{W}_k = \arg \max_{\widetilde{W}_k} \mathbb{E} \left\{ C_k^{(\uparrow/\downarrow)} \right\} \quad (15)$$

The problem in Equation (15) is a variant of the well known linear terminal loss problem (also called the newsvendor problem), see for instance [26], in which the imbalance costs to be borne by the decision maker are stochastic, asymmetric and piecewise linear. Under the assumption that the unit up- and down-regulation costs are independent of the power producer's imbalance, these stochastic costs can be replaced by *certainty equivalents* in the optimisation problem. Assuming that the considered wind power producer is relatively small, such a simplification seems quite reasonable as the producer is a price-taker. Nevertheless, it is clear that some variables could influence wind power production and regulation costs at the same time. This could be the case of e.g. weather related variables in a relatively small power system. This issue goes beyond the scope of this article, but it certainly calls for future research in modelling variables influencing both prices and wind power production.

According to the theory of certainty equivalents, see [26], the rational decision maker can determine the optimal decision without taking into account the whole distribution function of the unit costs. Instead an equivalent problem is solved, in which the stochastic unit costs are substituted by certain deterministic functions of the unit costs themselves. It is proved below that maximising  $C_k^{(\uparrow/\downarrow)}$  in Equation (10) is equivalent to maximising the expectation of the following function with deterministic unit costs

$$\overline{C}_k^{(\uparrow/\downarrow)} = \begin{cases} \widehat{\psi}_k^{(\downarrow)}(W_k - \widetilde{W}_k) & W_k \geq \widetilde{W}_k \\ \widehat{\psi}_k^{(\uparrow)}(W_k - \widetilde{W}_k) & W_k < \widetilde{W}_k \end{cases} \quad (16)$$

where  $\widehat{\psi}_k^{(\downarrow)}$  and  $\widehat{\psi}_k^{(\uparrow)}$  denote the expected values of the unit regulation costs  $\psi_k^{(\downarrow)}$  and  $\psi_k^{(\uparrow)}$ . The expectation of the imbalance costs in Equation (10) can be expanded as

$$\begin{aligned} \mathbb{E} \left\{ C_k^{(\downarrow/\uparrow)} \right\} &= \int_0^{+\infty} \int_0^{\widetilde{W}_k} \psi_k^{(\uparrow)}(W_k - \widetilde{W}_k) dP_{W_k} dP_{\psi_k^{(\uparrow)}} \\ &\quad + \int_{-\infty}^0 \int_{\widetilde{W}_k}^{W^{(max)}} \psi_k^{(\downarrow)}(W_k - \widetilde{W}_k) dP_{W_k} dP_{\psi_k^{(\downarrow)}} \end{aligned} \quad (17)$$

where  $W^{(max)}$  is the installed capacity of the wind power producer. Still assuming independence between the unit regulation costs and wind power production the integrations can be separated so that one gets to

$$\begin{aligned} \mathbb{E} \left\{ C_k^{(\downarrow/\uparrow)} \right\} &= \int_0^{+\infty} \psi_k^{(\uparrow)} dP_{\psi_k^{(\uparrow)}} \int_0^{\widetilde{W}_k} (W_k - \widetilde{W}_k) dP_{W_k} \\ &\quad + \int_{-\infty}^0 \psi_k^{(\downarrow)} dP_{\psi_k^{(\downarrow)}} \int_{\widetilde{W}_k}^{W^{(max)}} (W_k - \widetilde{W}_k) dP_{W_k} \end{aligned} \quad (18)$$

This is by definition equal to

$$\begin{aligned} \mathbb{E} \left\{ C_k^{(\downarrow/\uparrow)} \right\} &= \widehat{\psi}_k^{(\uparrow)} \int_0^{\widetilde{W}_k} (W_k - \widetilde{W}_k) dP_{W_k} \\ &\quad + \widehat{\psi}_k^{(\downarrow)} \int_{\widetilde{W}_k}^{W^{(max)}} (W_k - \widetilde{W}_k) dP_{W_k} \end{aligned} \quad (19)$$

which is equal to the expected value of the equivalent loss in Equation (16).

The problem of maximising the expectation of the utility in Equation (16) is a standard linear terminal loss problem, which can then be treated as the general case in [26]. The proof is omitted here and only the expression for the Expected

Utility Maximisation (EUM) bid is given

$$\widetilde{W}_k = F_{W_k}^{-1} \left( \frac{|\widehat{\psi}_k^{(\downarrow)}|}{\widehat{\psi}_k^{(\uparrow)} + |\widehat{\psi}_k^{(\downarrow)}|} \right) \quad (20)$$

where  $F_{W_k}$  is the cumulative distribution function of the wind power production  $W_k$ . Therefore, the EUM bid  $\widetilde{W}_k$  is a quantile of the distribution of the stochastic variable  $W_k$  corresponding to the probability given by the fraction

$$\widetilde{r}_k = \frac{|\widehat{\psi}_k^{(\downarrow)}|}{\widehat{\psi}_k^{(\uparrow)} + |\widehat{\psi}_k^{(\downarrow)}|} \quad (21)$$

From Equation (20) it follows that the determination of the optimal bid requires forecasts of both wind power production and imbalance costs.

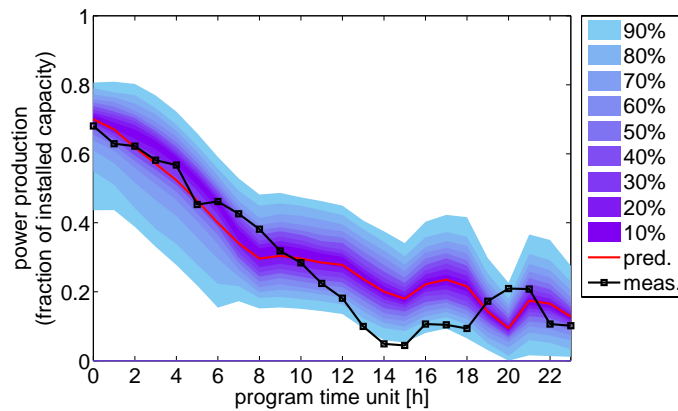
As far as wind power production is concerned, a probabilistic forecast is needed, as the distribution  $F_{W_k}$  of the generation  $W_k$  appears in Equation (20). Here the non-parametric probabilistic tool described in [27] and [28] is considered. This tool provides the user with a set of forecast quantiles of the wind power distribution for each PTU. Let us denote the  $\alpha$ -quantile of wind power production at time  $k$  with  $q_{W_k}(\alpha)$ , such that

$$F_{W_k}(q_{W_k}(\alpha)) = \alpha \quad (22)$$

The provided forecasts are then

$$\widehat{q}_{W_k}(\alpha) = \mathbb{E}\{q_{W_k}(\alpha) | M, \theta, \chi_t\} \quad (23)$$

for different values  $\alpha \in [0, 1]$ . The expectation on the right side of Equation (23) is conditioned on the choice of the model  $M$ , on its estimated parameters  $\theta$  and on the information  $\chi_t$  available at the time  $t$  when the forecast is issued. It holds trivially that  $t < k$ . In the example of Nord Pool  $t$  might be 11am (one hour before the deadline for bidding), while  $k$  could be any of the hours in the following day. From now on the condition on the expectation is discarded in order to lighten the notation. However, the reader should keep this in mind whenever a forecast is defined. An example of quantile forecast can be seen in Figure 1. The complete forecast of the function  $F_{W_k}$  can then be obtained from the set of forecast quantiles



**Figure 1.** Example of probabilistic forecast of production for a wind power portfolio in Eastern Denmark. The forecast was issued on the previous day at 11am.

$\widehat{q}_{W_k}(\alpha)$  by linear interpolation.

The expected values of the regulation costs  $\widehat{\psi}_k^{(1)}$  and  $\widehat{\psi}_k^{(\uparrow)}$  need to be forecast as well. Methods for forecasting the spot market price  $\pi_k^{(S)}$ , as well as the unit imbalance costs  $\psi_k^{(1)}$  and  $\psi_k^{(\uparrow)}$ , conditioned upon the regulation sign<sup>‡</sup>, are described in [29]. The following forecasts are therefore available

$$\widehat{\pi}_k^{(S)} = \mathbb{E} \left\{ \pi_k^{(S)} \right\} \quad (24)$$

$$\widehat{\psi}_{k|\psi_k^{(1)} < 0}^{(1)} = \mathbb{E} \left\{ \psi_k^{(1)} | \psi_k^{(1)} < 0 \right\} \quad (25)$$

$$\widehat{\psi}_{k|\psi_k^{(\uparrow)} > 0}^{(\uparrow)} = \mathbb{E} \left\{ \psi_k^{(\uparrow)} | \psi_k^{(\uparrow)} > 0 \right\} \quad (26)$$

[29] also presents a method for estimating conditional prior probabilities of imbalance in each direction being penalised at any given time  $k$ , defined as

$$P_k^{(1)} = P \left\{ \psi_k^{(1)} < 0 \right\} \quad (27)$$

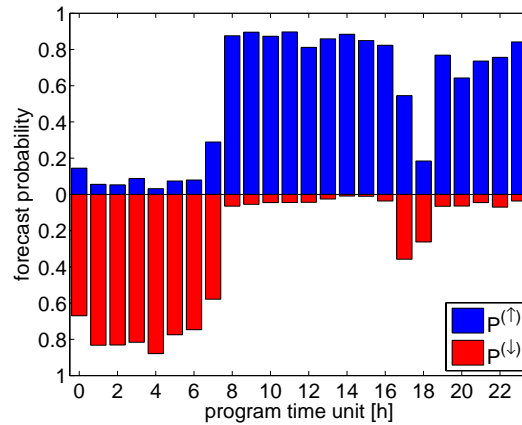
$$P_k^{(\uparrow)} = P \left\{ \psi_k^{(\uparrow)} > 0 \right\} \quad (28)$$

From a pure trading perspective this is equivalent to predicting the sign of the actual imbalance as the trader is indifferent to imbalances he/she is not penalised for.

An example of forecasts of the regulation signs is shown in Figure 2. It should be noticed that the two probabilities in the figure do not sum to 1. Indeed, the probability of no regulation  $P_k^{(0)}$  might also be positive, and at any time  $k$  it holds

$$P_k^{(\uparrow)} + P_k^{(1)} + P_k^{(0)} = 1 \quad (29)$$

The expected values  $\widehat{\psi}_k^{(1)}$  and  $\widehat{\psi}_k^{(\uparrow)}$  can then be determined according to the law of total expectation



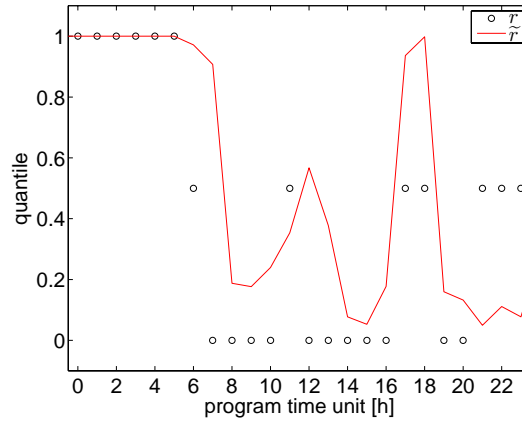
**Figure 2.** Example of forecast probabilities of up ( $P^{(\uparrow)}$ ) and down ( $P^{(1)}$ ) regulation in DK-2.

$$\widehat{\psi}_k^{(1)} = \widehat{\psi}_{k|\psi_k^{(1)} < 0}^{(1)} P_k^{(1)} + \widehat{\psi}_{k|\psi_k^{(1)} = 0}^{(1)} (1 - P_k^{(1)}) = \widehat{\psi}_{k|\psi_k^{(1)} < 0}^{(1)} P_k^{(1)} \quad (30)$$

$$\widehat{\psi}_k^{(\uparrow)} = \widehat{\psi}_{k|\psi_k^{(\uparrow)} > 0}^{(\uparrow)} P_k^{(\uparrow)} + \widehat{\psi}_{k|\psi_k^{(\uparrow)} = 0}^{(\uparrow)} (1 - P_k^{(\uparrow)}) = \widehat{\psi}_{k|\psi_k^{(\uparrow)} > 0}^{(\uparrow)} P_k^{(\uparrow)} \quad (31)$$

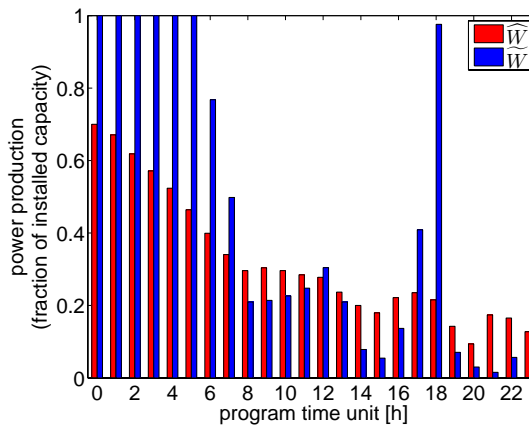
<sup>‡</sup> In [29] a given hour is defined as up-regulation hour if  $\psi_k^{(\uparrow)} > 0$  and a down-regulation hour if  $\psi_k^{(1)} < 0$ .

In the cases when both  $\hat{\psi}_k^{(l)}$  and  $\hat{\psi}_k^{(r)}$  are zero the ratio in Equation (21) is not defined. In these cases the producer might bid the median, corresponding to the 0.5 quantile, which maximises the expected market revenues in the general case where the forecast penalties in the two regulation directions are equal. Figure 3 plots an example of forecast,  $\tilde{r}_k$ , and measured,  $r_k$ , ratios in Equation (21) for a power producer in Eastern Denmark participating in Nord Pool. The resulting



**Figure 3.** Optimal forecast ( $\tilde{r}$ ) and measured ( $r$ ) ratios for a wind power portfolio in DK-2 on a selected day.

bid maximising the expected revenues is shown in Figure 4. As one can see from the scale employed on the  $y$ -axis of the figure, the bid is shown as a fraction of the total installed capacity. The point forecast, which is currently the reference for wind power producers participating in spot markets, is also shown for comparison.



**Figure 4.** Example of point forecast ( $\hat{W}$ ) and EUM bid ( $\tilde{W}$ ) for a wind power portfolio in DK-2.

## 2.1. Testing the EUM bid

This section presents the setup and the results obtained in a test-case simulating energy trading in Nord Pool. Its aim is to assess the performance of the EUM bidding strategy compared to the traditional point forecast bidding. Afterwards, the main drawbacks of the EUM strategy are discussed, along with the reasons motivating the introduction of more risk-averse strategies, which are presented in Section 3.

In this test-case, the DK-2 (Denmark East) market area has been considered as the geographic location of the wind power plants of a virtual power producer. Data and forecast availability motivate the choice of a 10-month period of simulation, spanning from the 1st March 2008 to the 31st December 2008. The size of the producer is not defined, and all the results

are scaled to its installed capacity. It is assumed, though, that the producer is a price-taker, i.e. that changes in its bidding policy do not influence the market. This implies that its size is small relatively to the total installed capacity in the region.

The data set used consists of measured wind power production, point and probabilistic forecasts of wind power production, observed regulation costs and the market forecasts previously described. All data refer to the DK-2 market area and have a temporal resolution of 1 hour. Based on point forecasts issued by WPPT, see [30, 31], probabilistic wind power forecasts are obtained by the method described in [28] and [27] while market forecasts have been obtained as outlined in [29]. All observations used are publicly available on [www.energinet.dk](http://www.energinet.dk).

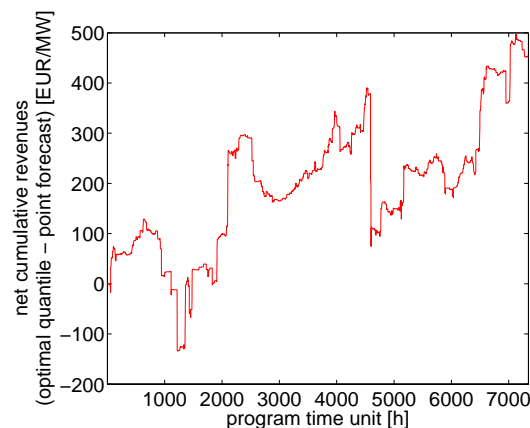
For the sake of performing a realistic test-case, the forecasts of wind power production and of spot and regulating market prices used in this study were issued before 11am of the previous day. In fact, these forecasts are the result of all (and only) the information available at the moment of bidding on the spot market. On the other hand, 1-hour-ahead forecasts were used for the regulation sign due to reasons of availability. Therefore, the wind power producer in the test-case has more information on the regulation sign than it would have in a realistic situation. As a consequence, the reader should keep in mind that the test-case may overestimate the performance of the EUM strategy. Having said that, it must be emphasised that this choice affects neither the validity of the described methodology nor the effectiveness of the test-case.

Table I shows the economic results of the wind power producer in both the cases of point forecast bidding and of EUM bid. The third column represents the reduction in the imbalance costs in Equation (10) with respect to the case of point forecast bidding. Its value on the first row is then trivially 0, while one can notice that the improvement obtained with the EUM is approximately 11%.

Strategy	Net revenue per installed MW (€/MW)	Imbalance cost per installed MW (€/MW)	Imbalance cost reduction (%)	Price per MWh (€/MWh)
Point forecast	94436.40	4076.51	0.00	54.48
EUM	94890.16	3622.75	11.13	54.74

**Table I.** Economic results for the wind power producer in the test-case performed from the 1st March 2008 to the 31st December 2008 with real market data and forecasts issued for the DK-2 market area.

Figure 5 shows the subtraction of the cumulative revenues obtained with the EUM strategy and the cumulative revenues obtained with the point forecast bid for each PTU in the test-case. As one can notice, the trend is clearly increasing. This signals the fact that the EUM bid is outperforming the point forecast bid in the long run. On the other hand, some steep decreases appear, for instance around the 1200th and the 4500th hours in the figure. This suggests that the producer adopting the EUM strategy is exposed to the risk of significant losses stemming from a single contract. It can be shown



**Figure 5.** Subtraction of the cumulative revenues per installed MW using the EUM bid and the cumulative revenues using the point forecast. Its positive value signals an improvement in the performance.

that the losses are due to incorrect forecasts of the regulation costs or sign. What the EUM aims at is, essentially, to set the spot market bid on the “safe” side of the decision space, i.e. on the imbalance direction that will not be penalised at the regulating market and paid at the spot price  $\pi_k^{(S)}$ . As Figure 3 shows, by doing this the optimal ratio  $\tilde{r}_k$ , and therefore the EUM strategy, results in being somewhat “extreme”. In fact, when the forecasts indicate that one regulation direction is far more likely than another,  $\tilde{r}_k$  tends to the extreme values 1 or 0. Figure 4 shows that this yields a bid close to the nominal capacity (as in the first hours in the figure) or to zero (as in the last hours in the figure). Hence the producer is in the situation of probably having a great imbalance in the forecast “safe” regulation direction. In the case that the forecasts leading to  $\tilde{r}_k$  are correct, the imbalance is paid at the spot price  $\pi_k^{(S)}$ , with no loss for the producer. On the other hand, if the forecast turns out to be incorrect the producer will have to pay regulation costs for a high amount of energy, resulting in one of the significant losses shown in Figure 5.

### 3. CONSTRAINING THE EUM BID

As an extension to the EUM strategy, a risk-aversion parameter is introduced in this section as a way to avoid severe losses in the market due to incorrect forecasts.

As a matter of fact, the point forecast bid is a robust decision from this point of view. Indeed, the point forecast minimises the expectation of the squared deviation from the energy production  $W_k$

$$\widehat{W}_k = \arg \min_x \mathbb{E} \{ (x - W_k)^2 \}$$

It is then likely that the producer will not have a high level of imbalance if  $\widehat{W}_k$  is bid. Therefore, a compromise between the EUM bid, which is risk-neutral, and the point forecast, which is far from being optimal but risk-averse, is sought. Moreover, energy traders are somehow bound to the point forecast, which has traditionally been bid on the spot market and has proved to be reliable over the years. For this reason it is desirable for an operational strategy not to deviate too much from it.

The main idea in this section is that the bid should somehow be bounded to some values around the point forecast. In this way extreme bid values - and hence extreme losses - are avoided. Constraints can be imposed in the decision space, so that the bid  $\widehat{W}_k$  is limited within a certain interval  $[\underline{W}_k, \overline{W}_k]$ . The rigorous mathematical formulation is described in Section 3.1. As an alternative, the limit can be imposed in the probability space so that the optimal ratio  $\tilde{r}_k$  is limited in a similar interval  $[\underline{r}_k, \overline{r}_k]$ . This is introduced in Section 3.2.

#### 3.1. Constraints in the decision space

In this section we propose the determination of the allowed interval for the bid as a function of the expected value of wind power production  $\widehat{W}_k$ .

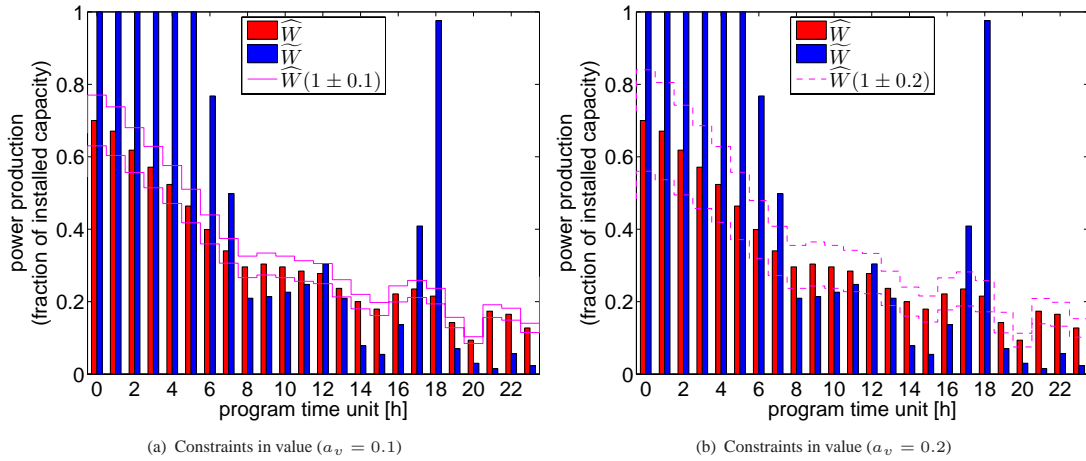
The allowed interval of the decision space is centred around the point forecast

$$\widehat{W}_k = \mathbb{E} \{ W_k \} \quad (32)$$

and has radius equal to a certain percentage of this value itself. Two values for the radius are used in the application case-study, i.e. 10% and 20% of  $\widehat{W}_k$ . Naturally the larger the allowed interval the more risk-neutral the strategy. The suggested bid in this case can be determined as

$$\widetilde{W}_k^{v, a_v} = \min \left\{ \max \left\{ \widehat{W}_k, \widehat{W}_k \cdot (1 - a_v) \right\}, \widehat{W}_k \cdot (1 + a_v) \right\} \quad (33)$$

where  $a_v$  is to be set to either 0.1 or 0.2. Figures 6(a) and 6(b) show the EUM bid and the point forecast  $\widehat{W}_k$  along with the allowed intervals with  $a_v = 0.1$  and  $a_v = 0.2$ .



**Figure 6.** Point forecast ( $\widehat{W}$ ), EUM bid ( $\widetilde{W}$ ) and allowed interval with constraints on the decision space.

### 3.2. Constraints in the probability space

In the second method proposed here, the ratio  $\widetilde{r}_k$  in Equation (21) is allowed to span a certain interval in the probability space. This interval is centred around the value of the cumulative distribution at the point forecast  $\widehat{W}_k$

$$\widehat{r}_k = F_{W_k}(\widehat{W}_k) \quad (34)$$

The radius of the interval is then to be set to a certain fraction of the probability space. In this work the radii 0.1 and 0.2 are used. The constrained bid can then be determined as

$$\widetilde{W}_k^{p,a_p} = F_{W_k}^{-1}(\min\{\max\{\widetilde{r}_k, \widehat{r}_k - a_p\}, \widehat{r}_k + a_p\}) \quad (35)$$

where  $a_p$  is to be set to 0.1 or 0.2 according to the desired risk aversion of the bid. Figures 7(a) and 7(b) show the EUM bid and the point forecast  $\widehat{W}_k$  along with the allowed intervals with  $a_p = 0.1$  and  $a_p = 0.2$ .

As a final remark, it should be pointed out that constraining the EUM bid, although stemming from the need of reducing its risk, also answers some other issues. First of all, this bid is often quite far from the point forecast  $\widehat{W}_k$ . On the other hand market authorities require that the energy bid be representative of the actual (or forecast) production of a generator. Hence an excessive deviation of the bid from the expected production could be seen as a way to take advantage of the market and thus it could be penalised. Secondly, a strategy causing high levels of imbalance might influence the price formation mechanism, especially with respect to the regulation prices. If this happens, the price-taker assumption is violated and, therefore, the model of the market becomes inconsistent. Complying with the need of reducing the risk associated with high imbalances, the constrained versions of this strategy also address these two issues, which in turn increase the practical value of these trading strategies.

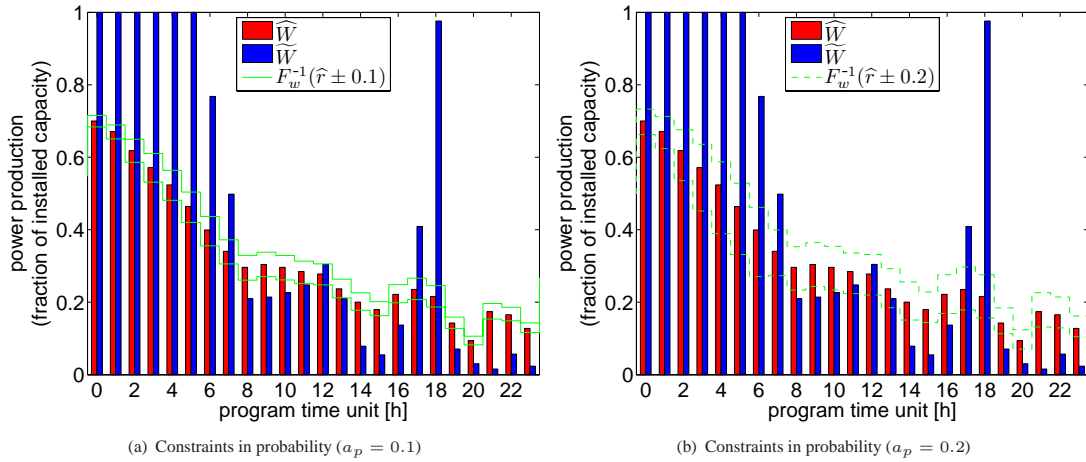


Figure 7. Point forecast ( $\hat{W}$ ), EUM bid ( $\tilde{W}$ ) and allowed interval with constraints on the probability space.

## 4. TEST CASE RESULTS

In this section we discuss the results of a test-case simulating the strategies presented above in a realistic market situation. The setup of the test-case is the same as described in Section 2.1, so the reader should again bear in mind that the economic results may overestimate the performance of the advanced strategies compared to the point forecast bid. Nevertheless, the validity of the strategies is of interest rather than the exact quantification of the possible imbalance cost reduction, which is, anyhow, mostly dependent on the design of the specific market under consideration. Section 4.1 discusses the performance of the bidding strategies from the point of view of the producer and its economic result, while Section 4.2 discusses the implications of the proposed strategies from a system point of view.

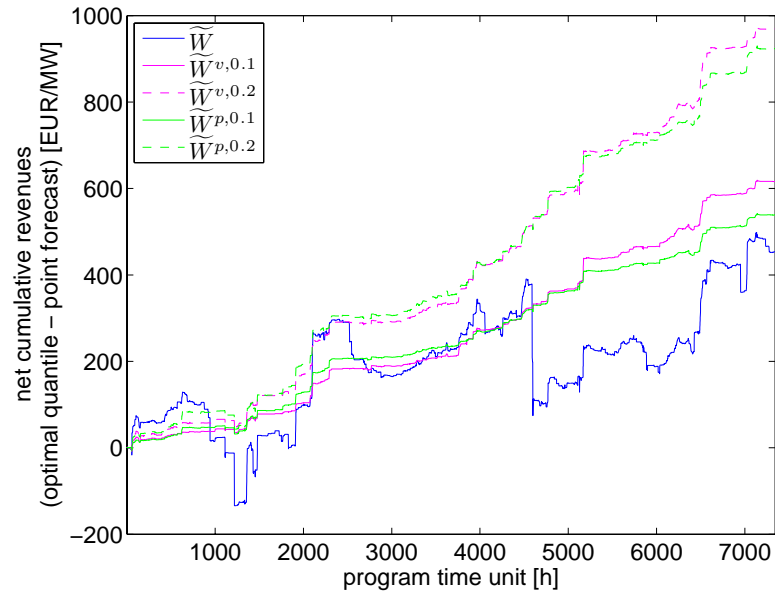
### 4.1. Economic advantage of the strategies

The main economic results for the power producer are shown in Table II. This shows the total revenues of the producer and its imbalance losses per MW of installed capacity, the percentage reduction in imbalance losses obtained by the strategy compared to the case of point forecast bidding and the average price per MWh paid to the producer.

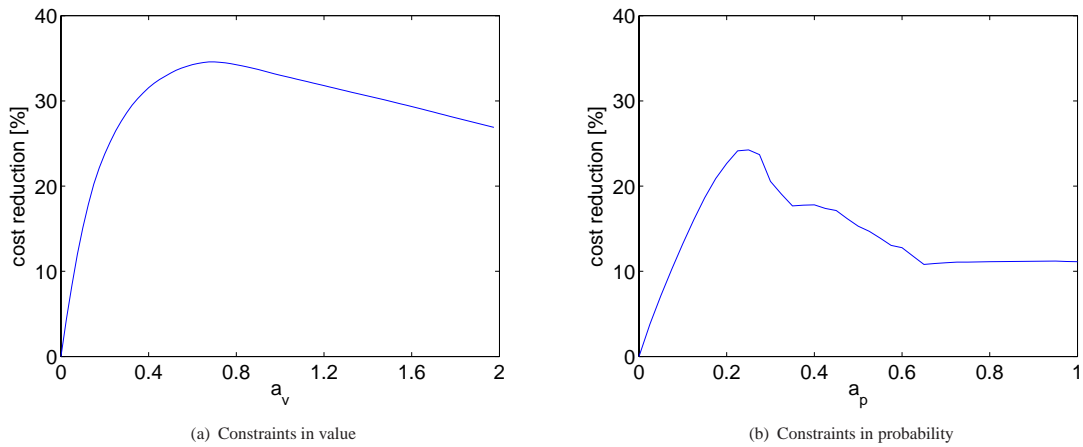
As one can see, the constrained strategies introduced in the previous section produce better results than the plain EUM one. The reduction in imbalance costs amounts to around 15% when the constraint limit is set to 10% (both in value and in probability) and to almost 25% when it is set to 20%. A slightly better performance is obtained by constraining in value than in probability. As far as the last column of Table II is concerned it should be mentioned that with perfect information on wind power production the energy would have been sold at an average price of €56.83 in the considered period.

The improved profits obtained with these strategies, compared to that of using the point forecast bidding, are illustrated in Figure 8. Indeed, this figure displays the difference between the cumulative revenues obtained by using the EUM strategy and its constrained versions and the revenues obtained by bidding the point forecast. All the cumulative revenues in this plot are expressed in € per MW of installed wind power capacity. It can be seen how the EUM bid ( $\tilde{W}$ ) is the least efficient strategy, apart from the point forecast bidding. The constrained strategies, besides performing better than the EUM, are also less exposed to significant isolated losses.

In view of the results above, there is clearly a relationship between range of the constraint and net revenues. Intuitively, there is also a relationship with risk, since as pointed out in Section 3 an increase in the allowed bid range results in a higher risk of a large imbalance, and therefore a higher risk of large losses. Figure 9 shows the imbalance cost reduction obtained in the test-case as a function of the parameters  $a_v$  and  $a_p$ . The trend is increasing in both cases up to a certain value of the parameter (approximately 0.7 and 0.2 for  $a_v$  and  $a_p$  respectively). In other words, the producer can achieve



**Figure 8.** Improvement of the cumulative revenues for the strategies described in Sections 2 and 3 with respect to the point forecast bidding strategy.



**Figure 9.** Producer's revenues in the test-case as a function of the risk parameters.

higher cumulative revenues if he/she is willing to accept higher risk of losses from an individual contract. Increasing the risk parameter further beyond these critical values results in less profits. This is because the distribution of the producer's hourly revenues is bounded on the upper side by  $\pi_k^{(s)} W_k$ . By allowing larger deviations from the point forecasts, this maximum value of the revenues is reached during more and more PTUs. In this way the rate of growth of the revenues slows down, as fewer PTUs offer possible improvements. Meanwhile, when forecasts are not perfect the risk of losses increases. When the critical level of the risk parameter is reached, the increased losses exceed the revenue growth, resulting in the negative slopes on the right sides of Figures 9(a) and 9(b). This decline is only stopped when the allowed bid interval is large enough to contain the optimal quantiles for all PTUs, as in the flat part of the curve on the right side of Figure 9(b). At that point the constrained strategy is in practice equal to the original EUM strategy. Finally, it is to be noted that the increasing parts of the curves on the left hand sides of Figures 9(a) and 9(b) are *efficient frontiers*, as they yield maximum revenues with minimum risk.

Furthermore, Table II and Figures 9(a) and 9(b) indicate that the EUM strategy does not achieve the best performance among the considered strategies in the simulated market period. One would expect that a 10-month period is long enough for considering the incidence of isolated losses on the cumulative revenues negligible, so that the EUM strategy achieves the optimal performance. On the contrary, this study seems to suggest that the EUM strategy is not optimal in practice. Indeed, even from a theoretical point of view the EUM bid is optimal only under the assumption that probabilistic forecasts of wind power production and of market prices are correct. In practice, errors in the probabilistic forecasts might cause the loss of optimality that is observable in this test-case. On the other hand the constrained strategies seem to limit the negative effects of forecast errors both by reducing the risk of losses stemming from single hourly-contracts and by achieving a better performance in the long run.

#### 4.2. Interaction with the system

This section sheds some light on the effects of the strategies presented in Sections 2 and 3 in terms of energy imbalance introduced in the system.

Table III shows the simulation results in terms of imbalance direction. The first three columns show the energy imbalance brought to the system by the wind producer in the considered 10 months, in total and divided between positive, i.e. producer being long (second column), and negative imbalance, i.e. producer being short (third column). All the values are expressed in hours of operation at nominal capacity. It can be seen how the more risk-neutral the strategy, the higher the overall energy imbalance. In this sense, the EUM strategy appears to have an extreme behaviour, pushing the total imbalance from less than 500 hours of operation, obtained with the point forecast bid, to over 1200 hours. The four constrained strategies appear to have a limited effect on the overall imbalance. The strategies with tighter bounds ( $\pm 10\%$  in value and  $\pm 0.1$  in probability) cause only a negligible increase, while when the ones with the less restrictive bounds ( $\pm 20\%$  in value and  $\pm 0.2$  in probability) are used the total imbalance rises by 40 hours.

Furthermore, an evaluation of the second and the third columns shows that generally more advanced strategies tend to bid above the actual production. This means that the producer is more often short rather than long. In fact, one can see that the difference between the values in the second and the third columns, which is almost zero with the point forecast bidding, tends to spread markedly when other strategies are used. This result might at a first sight look counterintuitive, since penalties are on average higher for down-regulation than for up-regulation. Nevertheless other factors, i.e. skewness of wind power production distribution, have an influence on this. According to expectations, the prevalence of down-regulation power is more evident when less risk-averse strategies are used.

The fourth and the fifth columns of Table III show the percentage of market hours during which the producer is long and short respectively. It can be seen that the variation in number of regulation hours, despite the significant variation in the imbalance volumes, does not exceed 3%. This indicates that the proposed strategies change the volumes of the energy imbalance but not the general trend in the number of up or down-regulation periods. Finally, the last two columns show the maximum value of energy imbalance, again expressed in hours of operation at nominal capacity, during a single hour. Interestingly only the row corresponding to the EUM bid shows a considerable increase in the maximum value of energy imbalance. This underlines the fact that constraining the EUM bid is an effective method to limit the maximum value of imbalance.

Table IV looks at the producer's imbalance from a different perspective. This table separates the results for the imbalance into two components: the component opposite to the overall system imbalance, which is paid at the spot market price and is shown in the second and the fourth columns, and the component in the same direction, which is paid at the spot price minus the imbalance cost and is shown in the third and fifth columns. Except for the case of the EUM bid, the third column shows a general diminution when the producer switches from the point forecast bidding to a more advanced strategy. In turn, the second column increases by a significant amount in most cases. These two facts indicate that the increase in energy imbalance caused by the use of more advanced strategies, which has been discussed above, actually involves only the direction in which the producer is not penalised, i.e. the one paid at the spot price. There are two implications of this. On one hand, part of the energy imbalance is shifted from being in the same direction as the system (third column in Table IV)

to the opposite (second column in the same table), thus contributing to restoring the overall balance - yet on a marginal level due to the price-taker assumption. In other words, the proposed constrained strategies are able to better “read” the feedback signal sent by the regulation prices and adapt to it, reducing the system imbalance and therefore being rewarded for it. On the other hand, the variation in imbalance could become significant if the proposed strategies become common practice for producers. As a result, this could influence the formation of the regulation prices as well as possibly change the direction of the system imbalance. Indeed, the trading behaviour of wind power producers is capable of affecting prices at Nord Pool even at the current level of market penetration, see [32, 33]. In the event that the trading strategies presented above became common practice, they might no longer be optimal, as they are based on the assumption of the producer being a price-taker. If the strategies became commonplace, they could possibly destabilise the system. Then, the market power of wind power producers should be accounted for if efficient bidding strategies are to be designed for producers with a large total capacity or for combined producers. This can be achieved by modelling energy markets as closed-loop systems, see for instance [24, 25].

## 5. CONCLUSIONS

In this work, the optimal quantile strategy for trading wind power in liberalised energy market is revisited. It is shown that this strategy maximises the expected value of the market revenues (utility), under the assumption that the wind power producer is a price-taker, i.e. its market strategy is not capable of influencing price formation. The use of the Expected Utility Maximisation (EUM) strategy in practice requires probabilistic forecasts of wind power production, point forecasts of spot and regulation market prices and of the imbalance sign probabilities. All these forecasts can be provided by state-of-the-art forecasting techniques.

An evaluation of the EUM strategy in a realistic test-case in Nord Pool highlights both its improved performance and its risk-neutral nature. The former is underlined by an 11% reduction of the imbalance costs. As far as the latter is concerned, the test-case shows that this strategy is exposed to a number of significant losses that take place in short periods of time. These losses are caused by the use of inaccurate forecasts which cause the bid to differ significantly from the actual wind power production.

As a way to reduce the exposure to risk of the wind power producer, constraining of the bid is proposed. Two different versions are introduced: with constraints in the decision space and in the probability space. The main idea is that bounding the bid to a certain interval around the point forecast of wind power production can help reduce the distance of the bid from the actual wind power production, and therefore the risk of incurring high regulation costs. In parallel, it is pointed out that the bid constraints can also solve some other issues, associated with the control of market authorities of the producer's bid as well as with its influence on the price formation mechanism.

Furthermore, the test-case is extended in order to assess the performance of the constrained strategies. The results of the simulation show that the constrained strategies outperform both the point forecast and the EUM strategies. The latter fact shows that constraining the EUM bid is also an effective way for reducing the impact of forecast errors on long-term revenues. At a second stage in the test-case, the interactions between a producer employing this strategy and the overall system are analysed. It is shown that only the EUM bid causes a significant increase in the total energy imbalance compared to the point forecast bid. The more risk-averse strategies increase the amount of regulated energy at most by about 10% in the case of less restrictive bounds, while the increase is negligible when the strategies with tighter bounds are adopted. Moreover, it is pointed out that this increase in the regulated power involves only the component in the opposite direction compared to the overall system imbalance. As a result, the constrained strategies might be able to reduce the overall imbalance, thus marginally benefiting the system, at least as long as they do not become common practice.

We underline that the obtained results hold as long as the wind power producer does not own a significant share of the overall production capacity. When this hypothesis is not true, the power producer cannot be considered a price-taker. It is expected that in this case the performance of the proposed strategies decreases. In addition, the assertion that these

strategies may be beneficial to the system by reducing the overall imbalance might prove incorrect. This is because such a large producer -or many smaller producers using the same bidding policy- might change the direction of the system imbalance, thus contributing positively to it rather than reducing it. For these reasons, an interesting future development of this work could be to study the relationship between the bid of a large wind power producer and the formation of the regulation prices in the real-time market. This could then lead to the formulation of optimal bidding strategies of practical use for large wind power producers, as well as more stable from a system point of view.

Similarly, modelling explanatory variables influencing wind power production and energy market prices at the same time is of clear interest for future research. This would account for the situation where a high penetration of wind power in the system is able to influence the prices, although the considered wind power producer is too small to have any sort of market power on its own.

Besides, trading on the intra-day market could also be included in the problem under the assumption of sufficient liquidity of this market. As shown in [20], this trading floor gives market participants further possibilities for reducing the risk of losses. Indeed, producers can employ forecasts with a shorter lead-time (typically one hour) with clear advantages in terms of accuracy. Therefore, an assessment of the advantages both for the producers and the system obtained by increasing the liquidity of balancing markets would be particularly interesting.

Finally, another direction for further research could be to account for the dynamic aspects of the market. In this way the assumption of independence of decisions in different PTUs would be overcome. The dynamic view of the market could include, for instance, modelling competition among producers as well as the market participation of mixed portfolios. In the latter case a typical situation could be the coupling of wind power with hydro power or energy storage, both of which allow for shifts in the trade of power between different PTUs. This research could lead to the determination of more advanced bidding strategies in competitive market environments, possibly for producers with a diversified portfolio of energy sources.

## ACKNOWLEDGEMENTS

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Strategy	Net revenue per installed MW (€/MW)	Imbalance cost per installed MW (€/MW)	Imbalance cost reduction (%)	Price per MWh (€/MWh)
Point forecast	94436.40	4076.51	0.00	54.48
EUM	94890.16	3622.75	11.13	54.74
Constrained ( $\pm 10\%$ value)	95052.90	3460.01	15.12	54.84
Constrained ( $\pm 20\%$ value)	95405.26	3107.65	23.77	55.04
Constrained ( $\pm 10\%$ probability)	94975.51	3537.40	13.22	54.79
Constrained ( $\pm 20\%$ probability)	95360.14	3152.77	22.66	55.02

**Table II.** Economic results for the wind power producer in the test-case.

Strategy	Energy imbalance (h)			Imbalance hours (%)		Max value (h/h)	
	Total	> 0	< 0	> 0	< 0	> 0	< 0
Point forecast	484.92	235.94	248.98	45.45	54.47	0.54	0.70
EUM	1271.81	286.76	985.05	43.47	56.53	0.74	1.00
Constrained ( $\pm 10\%$ value)	490.89	219.70	271.19	42.97	57.02	0.56	0.74
Constrained ( $\pm 20\%$ value)	526.39	221.73	304.67	41.69	58.28	0.58	0.74
Constrained ( $\pm 10\%$ probability)	488.27	225.48	262.80	45.45	54.55	0.57	0.71
Constrained ( $\pm 20\%$ probability)	528.60	224.85	303.75	44.47	55.53	0.59	0.81

**Table III.** Energy imbalance of the wind power producer in the test-case.

Strategy	Energy imbalance (h)			Imbalance hours (%)	
	Total	Spot price	Penalty	Spot price	Penalty
Point forecast	484.92	277.71	207.21	62.81	37.19
EUM	1271.81	1002.92	268.89	75.38	24.62
Constrained ( $\pm 10\%$ value)	490.89	307.86	183.03	65.41	34.59
Constrained ( $\pm 20\%$ value)	526.39	353.47	172.92	67.78	32.22
Constrained ( $\pm 10\%$ probability)	488.27	302.46	185.82	65.03	34.97
Constrained ( $\pm 20\%$ probability)	528.60	353.89	174.71	67.85	32.15

**Table IV.** Energy imbalance of the wind power producer in the test-case.