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Abstract

Liner shipping networks are the backbone of international trade providing low transportation cost, which is a major driver of globalization. These networks are under constant pressure to deliver capacity, cost effectiveness and environmentally conscious transport solutions. This article proposes a new path based **MIP** model for the Liner shipping Network Design Problem minimizing the cost of vessels and their fuel consumption facilitating a green network. The proposed model reduces problem size using a novel aggregation of demands. A decomposition method enabling delayed column generation is presented. The subproblems have similar structure to **Vehicle Routing Problems**, which can be solved using dynamic programming. An algorithm has been implemented for this model, unfortunately with discouraging results due to the structure of the subproblem and the lack of proper dominance criteria in the labeling algorithm.

Keywords liner shipping, network design, mathematical programming, column generation, green logistics

1 Introduction

Global liner shipping companies provide port to port transport of containers, on a network which represents a billion dollar investment in assets and

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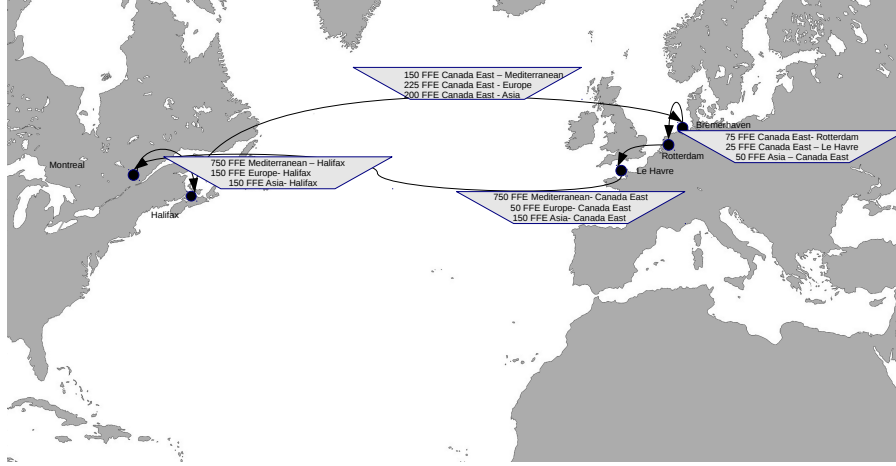
operational costs.

Figure 1: A Canada-Northern Europe service. FFE is Forty Foot Equivalent unit container used to express the volume of containers in each cargo category.

The liner shipping network can be viewed as a transportation system for general cargo not unlike an urban mass transit system for commuters, where each route (service) provides transportation links between ports and the ports allow for transshipment in between routes (services). The liner shipping industry is distinct from other maritime transportation modes primarily due to a fixed public schedule with a given frequency of port calls (Stopford 2009). The network consists of a set of *services*. A *service* connects a sequence of ports in a cycle at a given frequency, usually weekly as an industry standard. In Figure 1 a service connecting Montreal-Halifax and Europe is illustrated. The weekly frequency means that several vessels are committed to the service as illustrated in the figure, where four vessels cover a round trip of 28 days placed with one week in between vessels. This round trip for the vessel is referred to as a *rotation*. Note that the Montreal service carries cargo to North Europe, the Mediterranean and Asia, with the two latter transshipping in Bremerhaven. In a similar way cargo headed for Canada has multiple origins. This illustrates that transshipments to other connecting services is at the core of liner shipping. Therefore, the design of a service is complex, as the set of rotations and their interaction through
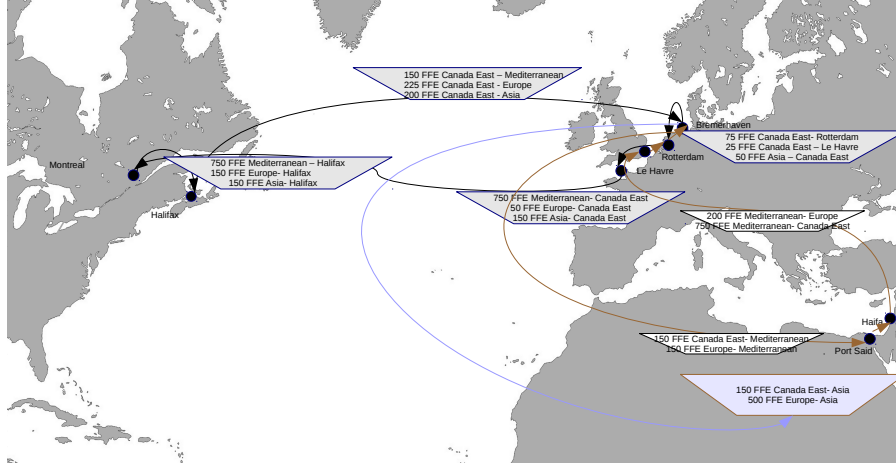
Figure 2: Two connecting services. The Montreal service from Figure 1 and a Europe-Mediterranean service with a round trip time of 2 weeks illustrated by two white vessels. The cargo composition on board vessels illustrate transshipments at the core of the liner shipping network design. The light blue incomplete service illustrates a larger service transporting cargo between Europe and Asia.

transshipment is a transportation system extending the supply chains of a multiplum of businesses. Figure 2 illustrates two services interacting in transporting goods between Montreal-Halifax and the Mediterranean, while individually securing transport between Montreal-Halifax and Northern Europe, and Northern Europe and the Mediterranean respectively. The Montreal service additionally interacts with a service between Europe and Asia, which is partly illustrated.

1.1 Modelling the Liner Shipping Network Design Problem (LSNDP)

The Liner Shipping Network Design Problem (LSNDP) aims to optimize the design of the networks to minimize cost, while satisfying customer service requirements and operational constraints. The mathematical formulation of the LSNDP may be very rich as seen in Løfstedt et al. (2011), where a compact formulation along with an extensive set of service requirements and network restrictions is presented. A rich formulation like the one presented

in Løfstedt et al. (2011) serves as a description of the LSNDP domain, but is not computationally tractable as the number of feasible services is exponential in the number of ports. Therefore, a formulation of the LSNDP is typically restricted to an interpretation of the domain along with the core costs and constraint structures of the problem. The LSNDP has been modelled as a rich Vehicle Routing Problem (VRP) (Baldacci et al. 2010), where transhipments are not allowed and vessels can be assumed to return empty to a single main port of a voyage in, e.g., Fagerholt (2004) and Karlaftis et al. (2009). The structure is applicable for regional liner shippers referred to as *feeder* services as opposed to global liner shipping in focus in the present paper. Models where the LSNDP is considered as a specialized capacitated network design problem with multiple commodities are found in Reinhardt and Kallehauge (2007), Agarwal and Ergun (2008), Alvarez (2009), and Plum (2010). The network design problem is complicated by the network consisting of disjoint cycles representing container vessel routes as opposed to individual links. The models allow for transshipments, but transshipment cost is not always part of the objective (e.g.,Agarwal and Ergun (2008)). The vessels are not required to be empty at any time. The works of Agarwal and Ergun (2008), Alvarez (2009) identify a two tier structure of constraint blocks: the first deciding the rotations of a single or a collection of vessels resulting in a capacitated network and the second regarding a standard multicommodity flow problem with a dense commodity matrix. The cost structure of LSNDP places vessel related costs in the first tier and cargo handling cost and revenue in the second tier. The work of Plum (2010) has identified two main issues with solving the LSNDP as a specialized capacitated network design problem:

1. Economy of scale on vessels and the division of cost and revenue on the two tiers results in highly fractional LP solutions.

2. The degeneracy of the multicommodity flow problem results in weak LP bounds.

Furthermore, it is well known that the linear multicommodity flow problem and hence capacitated network design problems are increasingly complex to solve with the number of distinct commodities. Computational results for existing models confirm the hardness of this problem and the scalability issues, struggling to solve instances with 10-15 ports and 50-100 commodities.

The model presented in this paper has a single tier and combines revenue with total cost in the service generation problem. The motivation is to ensure efficient capacity utilization of vessels and avoid highly fractional LP solutions. Service generation is based on pick-up-and-delivery of cargoes transported entirely or partly on the service. The cost of a service reflects asset, operational and port call costs of the vessels on the service, along with the cargo handling cost and revenue of collected cargo on the service.

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The cargo handling cost includes load, unload and transshipment costs. The model is inspired by the Pick-up-and-Delivery VRP problem, but is considerably more complex as we allow transshipments on non-simple cyclic routes, where the vessel is not required to be empty at any point in time.

The degeneracy of the multicommodity flow problem is mitigated both by modeling the flow as assignments to services as opposed to the traditional multicommodity flow formulation, but also by exploiting the liner shipping concept of *trade* lanes to aggregate the number of distinct commodities to a minimum. Trade lanes are based on the geographic distances within a set of ports and their potential to import/export to another region.

Maritime shipping produces an estimated 2.7% of the worlds CO_2 emission, whereof 25% is accounted to container vessels according to the WorldShippingCouncil (2010). Many liner shipping companies focus on the environmental impact of their operation and the concept of *slow steaming* has become a value proposition for some liner shipping companies List (2010). Cariou (2011) estimate that the emissions have decreased by 11 % since 2008 by slow steaming alone. A break down of the cost of a service to each vessel Stopford (2009) state that 35-50% of the cost is for fuel (bunker) whereas capital cost accounts for 30-45%, OPEX (crew, maintenance and insurance) accounts for 6-17% and port cost for 9-14%. Slow steaming minimizes the fuel cost, but comes at an asset cost of additional vessels deployed to maintain weekly frequency (Notteboom and Vernimmen 2009). Slow steaming is not always an option as some cargo may have crucial transit times. Current models of LSNDP assumes fixed speed on a service. The model of Alvarez (2009) explicitly aims at minimizing the fuel cost and consumption in the network by varying the speed of services in the model. The works of Løfstedt et al. (2011), Notteboom and Vernimmen (2009), Fagerholt et al. (2009) state that the speed on a service is variable on each individual voyage between two ports. Calculating fuel consumption based on an average fixed speed on a roundtrip is an approximation, as the fuel consumption is a cubic function of speed (Stopford 2009). As a result the actual fuel consumption of a service cannot be estimated until the schedule is fixed. Tramp shipping companies often model their routing and scheduling problem as rich Pick-up-and-Delivery VRP problems with Time Windows (Fagerholt and Lindstad 2007, J. E. Korsvik and Laporte 2011). Fagerholt et al. (2009) is the first article within tramp shipping with variable speed between each port pair in the routing. The optimization of speed and hence minimizing the fuel consumption and environmental impact is driven by the time windows and the optional revenue of spot cargoes. (Fagerholt et al. 2009, I. Norstad and Laporte 2011) report significant improvements in solution cost using variable speed. Minimizing the fuel consumption of the network can be a post optimization regarding speed of the liner shipping network, when deciding on the schedule in terms of berthing windows or the transit time of individual cargo routings. The path based model presented in this paper

assumes a fixed speed for each vessel class and in the dynamic programming algorithm the number of vessels deployed to a service is rounded up to the nearest integer in order to ensure that a weekly frequency can be maintained on each service.

The path based model is inspired by operations research techniques within the airline industry, where the optimization is divided into faces. Therefore, a solution to the path based model is a generic capacitated network of cyclic services based on a weekly frequency of port calls. The generic network is transformed into an actual network by deciding a specific schedule, deploying vessels and deciding on the speed of the individual voyages and actual flow of all distinct commodities. The slow steaming speed of a vessel is 12 knots and depending on size and age a vessel has a maximal speed of 18 to 25 knots. If the fixed speed is chosen 30-40% above slow steaming speed for each vessel class, rounding up the number of vessels will allow post optimization of the schedule to achieve an energy efficient network with focus on slow steaming, while ensuring the transit time of products. The generic network facilitates the design of a green liner shipping network, while at the same time enabling scalability due to a more general description of the network.

1.2 Demand Aggregation

In models of the LSNDP using a specialized capacitated network design formulation the second tier is a standard multicommodity flow problem. The work of Alvarez (2009) identifies solving the multicommodity flow problem as prohibitive for larger problem instances due to the large number of commodities considered. In Alvarez (2009) the commodities are aggregated by destination, giving a smaller model to solve. This could result in worse LP bounds as identified in Croxton et al. (2007), since the LSNDP will have a concave cost function, due to the economies of scales of deploying larger vessels, and high start up costs, as at least one vessel must be deployed.

A contribution of this paper is to formulate a model that considers aggregated aspects of the demand instead of specific origin-destination (o-d) pairs. This is motivated by the *trade*-centric view of liner-shipping present in the liner shipping industry instead of the o-d-centric view considered in the literature. As seen in Figure 1, the (o-d demand from Halifax to Rotterdam could be considered, but in practice it will be hard to estimate such a specific demand. More realistically one could estimate the volume of exports from Halifax to Northern Europe and reversely the volume of imports from East Coast Canada to Rotterdam (or exports from Mediterranean to Halifax as in Figure 2). Each commodity $k \in K$ will then be characterized by a volume d^{XY} from a region X to a region Y i.e. East Coast Canada or Northern Europe as seen in Figure 1 on the vessels in deep sea. Each set of X,Y will symbolize a *trade*. Each port $p \in X$ will also have an export

and import in the trade: d^{pY}, d^{Xp}, where $\sum_{p \in X} d^{pY} = \sum_{p \in Y} d^{Xp}$ as seen in Figure 1 on the vessels in a region. In effect a port as Halifax will be ensured a volume of export to Mediterranean ports and each of these will be insured a volume of imports from East Coast Canadian ports, without specifying the concrete origin-destination pairs. Note the difference in aggregation approach, compared with the models of Croxton et al. (2007), as we are now aggregating by *trade* origin-region to destination-region, instead of aggregation by destination port. This should give the benefit of fewer variables due to the aggregation, while we still have quite tight LP-relaxations.

The aggregation of demand may be more or less fine grained according to the definition of ports, regions and trade lanes, enabling both detailed networks for a smaller region and coarse network designs for a larger set of ports that may be refined by subsequent optimization methods. We foresee a computational tractability trade-off between the number of ports and the number of distinct commodities when defining regions for ports.

This can also be seen in the light of forecasting accuracy, usually the more detailed the level of forecasting is the more inaccurate it will be. This allows a forecasting to be done at a more natural level, i.e. on total trade volumes and total port import and export volumes.

In the following we will present a path-based formulation of the LNSDP and a column generation approach generating capacitated, cyclic rotations with assigned flow. We will outline a dynamic programming algorithm to solve the pricing problem. Preliminary computational results of an implementation of the algorithm will be given, which reveals poor performance for solving the pricing problem. This leads us to believe that alternative methods must be developed to efficiently solve the pricing problem, for the approach to be able to solve instances of a significant size. This work is an extension of a contribution to the proceedings of IMECS 2011 of Jepsen, Løfstedt, Plum, Pisinger, and Sigurd (2011).

2   Service Based Model

In the following we introduce a model based on a combination of feasible services for each vessel class, into a generic liner shipping network solution. The service based model is based on a Dantzig-Wolfe decomposition of the model presented in Løfstedt et al. (2011). Let S_v denote the set of feasible services for a vessel class $v \in V$ and let $S = \cup_{v \in V} S_v$. Let α_{kps}^{XY} and β_{kps}^{XY} be the amount of respectively load and unload of containers from region X to region Y on the k'th visit to port p on service $s \in S$. We assume that $\alpha_{kps}^{XY} = \beta_{kps}^{XY} = 0, \forall p \notin X \cup Y \cup G^{XY}$, where G^{XY} is the set of ports where transshipments is allowed for *trade XY*. Let M_p be the maximal number of port visits to port p for each service. Furthermore, let γ_{pq} equal the number of times the service sails between ports $p \in P$ and $q \in P$. The move cost

in a port p for a *trade* $XY \in K$ consist of the unload cost u_p^{XY} and load cost l_p^{XY}. For ports $p \in X$ the transshipment cost is included in the unload cost and the revenue is r_p^{XY}. For ports $p \in P \setminus X$ the transshipment cost is included in the load cost. Each vessel of vessel type $v \in V$ has costs c_v for fuel-, crew- and depreciation of vessel value or time-charter-costs per week. The cost of vessel type v calling a port q is c_q^v. The number of vessels used by the service is the round trip distance of the service divided by W_d^v, the weekly distance covered by vessel type v at the predefined speed. This value is rounded up to ensure the vessels can complete the round trip at the predefined speed. The number of vessels used by the service is given as $n_s = \left\lceil \sum_{p \in P} \sum_{q \in P} \frac{d_{pq} \gamma_{pqs}}{W_d^v} \right\rceil$. The cost of a service $s \in S$ is given as:

$$
\begin{aligned}
c_s = & \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} r_p^{XY} (\alpha_{kps}^{XY} - \beta_{kps}^{XY}) \\
& - \sum_{XY \in K} \sum_{p \in P} \sum_{k \in M_p} (l_p^{XY} \alpha_{kps}^{XY} + u_p^{XY} \beta_{kps}^{XY}) \\
& - c_v n_s - \sum_{p \in P} \sum_{q \in P} c_q^v \gamma_{pqs}
\end{aligned}
$$

The model based on services is as follows:

$$\max \sum_{s\in S} c_s \lambda_s \tag{1}$$

$$\texttt{s.t} \quad 0 \leq \sum_{s\in S}\sum_{k\in M_p}(\alpha_{pks}^{XY}-\beta_{pks}^{XY})\lambda_s \leq d^{pY} \qquad \forall XY \in K, \forall p \in X \tag{2}$$

$$0 \geq \sum_{s\in S}\sum_{k\in M_p}(\alpha_{pks}^{XY}-\beta_{pks}^{XY})\lambda_s \geq -d^{Xp} \qquad \forall XY \in K, \forall p \in Y \tag{3}$$

$$\sum_{s\in S}\sum_{k\in M_p}(\alpha_{pks}^{XY}-\beta_{pks}^{XY})\lambda_s = 0 \qquad \forall p \in G^{XY}, \forall XY \in K \tag{4}$$

$$\sum_{s\in S}\sum_{p\in X\cup Y}\sum_{k\in M_p}(\alpha_{pks}^{XY}-\beta_{pks}^{XY})\lambda_s = 0 \qquad \forall XY \in K \tag{5}$$

$$\sum_{s\in S_v} n_s \lambda_s \leq |v| \qquad \forall v \in V \tag{6}$$

$$\alpha_{kps}^{XY}, \beta_{kps}^{XY} \in \mathbb{Z}^+ \qquad \forall s \in S, \forall XY, \forall p \in X, \forall k \in M_p \tag{7}$$

$$\lambda_s \in \{0,1\} \qquad \forall s \in S \tag{8}$$

The objective (1) maximizes the profit, constraints (2) and (3) ensure that the difference between what is loaded and unloaded (unloaded and loaded) by all services in a port is positive and less than the export capacity (import capacity) of the port for the given *trade*. Constraints (4) ensure that the amount of containers loaded equals the amount of containers unloaded in a transhipment port and constraints (5) ensure that all containers loaded are unloaded for each *trade*. Constraints (6) ensure that the number of available vessels for each vessel class is not exceeded and the binary domain on the variables is defined by (8).

The key issue with the service based model is that the set of feasible services S can be exponential in the number of ports. Therefore, we cannot expect to solve instances of significant size. To overcome this issue we propose to write up the model gradually using delayed column generation and then solve the problem through Branch-and-Cut-and-Price. Branching is done by imposing a limit on the number of times an arc can be used by a given vessel class. We will investigate the possibility of applying an enumeration technique similar to the one used within CVRP (Baldacci et al. 2008).

2.1 Pricing Problem

The pricing problem calculates a non-simple cycle σ centered around any starting node p_s with associated loads and unloads. The cycle respects the capacity of the vessel class, C_v, at every port p, ensures feasibility of a weekly frequency for the vessel class v given the distance of the schedule, and lastly, that port p is visited no more than M_p times $\forall p \in P$. The pricing problem returns a variable representing a load and an unload pattern, which implicitly defines a non-simple cycle starting and ending at the same port $p \in P^v$, deploying n_s vessels to maintain weekly frequency at the fixed speed enforced on the service pattern. The above problem has a similar structure to the pricing problems of Vehicle Routing Problems modelled as a Resource Constrained Shortest Path problem (see Irnich and Desaulniers (2005). The Resource Constrained shortest Path Problem is often solved by label setting algorithms. As it is possible for the demand to be split on different paths, we need to ensure that we allow all possibilities of transshipments. This necessitates that, labels are created for each integral unit of the demand up to the minimum of the available capacity or the demand.

2.1.1 Objective function of the pricing problem

The objective function of the pricing problem is to find the best reduced cost of a master problem variable at the given iteration of the master problem. For each $XY \in K$ a port $p \in P$ is present in at most one of the constraints (2) to (4). Let $\omega_p^{XY}, \forall XY \in K, \forall p \in X \cup Y \cup G^{XY}$ denote the duals from (2) to (4). Let δ^{XY} be the dual variables of constraints (5) and π^v are the duals of constraints (6).

For each vessel class $v \in V$ the reduced cost of a service (column) $s \in S_v$

$$
\begin{aligned}
\hat{c}_s = c_s &- \sum_{XY \in K} \sum_{p \in X \cup Y \cup G^{XY}} \sum_{k \in M_p} \omega_p^{XY}(\alpha_{kps}^{XY} - \beta_{kps}^{XY}) \\
&- \sum_{XY \in K} \sum_{p \in X \cup Y} \sum_{k \in M_p} \delta^{XY}(\alpha_{kps}^{XY} - \beta_{kps}^{XY}) - \pi_v n_s
\end{aligned}
$$

Expanding the term $\hat{c}_s$ and rearranging the terms according to load and unload combined with the port belonging to either X, Y or G_k we obtain the following reduced cost:

$$\hat{c}_s = \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} (r_p^{XY} - l_p^{XY} - \omega_p^{XY} - \delta^{XY}) \alpha_{kps}^{XY}$$

$$+ \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} (-r_p^{XY} - u_p^{XY} + \omega_p^{XY} + \delta^{XY}) \beta_{kps}^{XY}$$

$$+ \sum_{XY \in K} \sum_{p \in Y} \sum_{k \in M_p} (-l_p^{XY} - \omega_p^{XY} - \delta^{XY}) \alpha_{kps}^{XY}$$

$$+ \sum_{XY \in K} \sum_{p \in Y} \sum_{k \in M_p} (-u_p^{XY} + \omega_p^{XY} + \delta^{XY}) \beta_{kps}^{XY}$$

$$+ \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M_p} (-l_p^{XY} - \omega_p^{XY}) \alpha_{kps}^{XY}$$

$$+ \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M_p} (-u_p^{XY} + \omega_p^{XY}) \beta_{kps}^{XY}$$

$$- (\pi_v + c_v) n_s - \sum_{p \in P} \sum_{q \in P} c_q^v \gamma_{pqs}$$

The reduced cost can be rewritten as a cost connected to loading, unloading, and sailing in terms of the number of vessels deployed and the cumulative port call cost. The cost of (un)loading a demand from trade XY depends on the region of the port. If the port is from the origin region X a revenue is obtained for loading and subtracted for unloading at the port. This ensures that revenue is only collected at the initial load. The costs are the (un)load cost, and the dual values from constraints (2)-(4) concerning the flow conservation and the dual value from the flow balance constraint for the trade (5). If the port is from the destination region Y the cost is the (un)load cost, and the dual values from constraints (2)-(4) concerning the flow conservation and the dual value from (5). For a transhipment port $p \in G_{XY}$ the cost is only related to (un)load cost and the dual values of (2)-(4).

$$\hat{l}_p^{XY} = \begin{cases} r_p^{XY} - l_p^{XY} - \omega_p^{XY} - \delta^{XY} & \forall p \in X \\ -l_p^{XY} - \omega_p^{XY} - \delta^{XY} & \forall p \in Y \\ -l_p^{XY} - \omega_p^{XY} & \forall p \in G^{XY} \end{cases}$$

$$\hat{u}_p^{XY} = \begin{cases} -r_p^{XY} - u_p^{XY} + \omega_p^{XY} + \delta^{XY} & \forall p \in X \\ -u_p^{XY} + \omega_p^{XY} + \delta^{XY} & \forall p \in Y \\ -u_p^{XY} + \omega_p^{XY} & \forall p \in G^{XY} \end{cases}$$

Finally, the port call cost c_q^v is paid upon each sailing/extension onto a new port $p \in P$ and the cost $\hat{c}_v = \pi_v + c_v$ is inferred each time the distance of W_d^v is traveled.

2.1.2 Label setting algorithm for LSNDP

The $|V|$ pricing problems for each vessel class can be formulated as the following graph problem. Given a directed graph $G^v = (N^v, A^v)$ where the node set is $N^v = P^v \cup L^v \cup U^v$. P^v is the set of ports $\in P$ compatible with vessel class v, $L^v = \bigcup_{w \in P^v} L_w$ the set of load nodes. The sets $L_w = \{\rho_w^{XY} | \forall XY \in K, w \in X \vee Y \vee G^{XY}\}$ represents all possible loads at port w, $U^v = \bigcup_{w \in P^v} U_w$ is the set of unload nodes. The sets $U_w = \{\mu_w^{XY} | \forall XY \in K, w \in X \vee Y \vee G^{XY}\}$ represents all possible unloads at port w. In order to correctly identify transshipments and unloads of a *trade* each demand $XY \in K$ is associated with a set of load nodes $L^{XY} \subseteq L^v$ and a set of unload nodes $U^{XY} \subseteq U^v$, where $L^{XY} = \{\rho_w^{XY} | w \in X \cup Y \cup G^{XY}\}$ and $U^{XY} = \{\mu_w^{XY} | w \in X \cup Y \cup G^{XY}\}$.

The arc set is $A^v = A_s \cup A_u \cup A_l$. Define the function $h : U^v \cup L^v \to P^v, L_q \mapsto q, U_q \mapsto q$ for mapping between the load and unload nodes and the actual port $q \in p^v$ of the (un)load. The set of sailing arcs is defined as follows $A_s = \{(i,j) | i \in L^v \cup U^v, j \in P^v \setminus \{h(i)\}\}$, the set of unload arcs $A_u = \{(i,j) | i \in P^v, j \in U_i\} \cup \{(i,j) | i \in U^v, j \in U_{h(i)}\}$ and the set of load arcs $A_l = \{(i,j) | i \in P^v, j \in L_i\} \cup \{(i,j) | i \in U^v = \mu_{h(i)}^{XY}, j \in L_{h(i)} \setminus \{\rho_{h(i)}^{XY}\}\} \cup \{(i,j) | i \in L^v, j \in L_{h(i)}\}$. The graph topology is illustrated in Figure 3. The distance of an arc depends on the arc type:

$$d_{ij} = \begin{cases} d_{h(i)j} & (i,j) \in A_s \\ 0 & (i,j) \in A_l \cup A_u \end{cases} \quad (9)$$

In a label setting algorithm, a label E_i is associated with a node i and represents a (partial) path with a (reduced) cost C of the service and a number of resources θ accumulated along the path. A resource may be associated with lower and upper bounds often referred to as a *resource window*. The proposed pricing problem differs significantly from the Elementary Shortest Path Problem with Resource Constraints (ESPPRC) known from VRP:

- The path is not elementary as $M_p \geq 1$.

- The path represents a cycle, σ.

- It is a longest cycle problem as the reduced cost $\hat{c}_s \geq 0$.

- We do not have a designated starting node and hence will have to start the algorithm in every possible port $p \in P^v$.

- The ability to perform a load on the partial path, which can be unloaded at a previous node of the cycle σ. A second pass of all ports in the cycle σ must be performed only allowing the unload extension function to check for load balance.
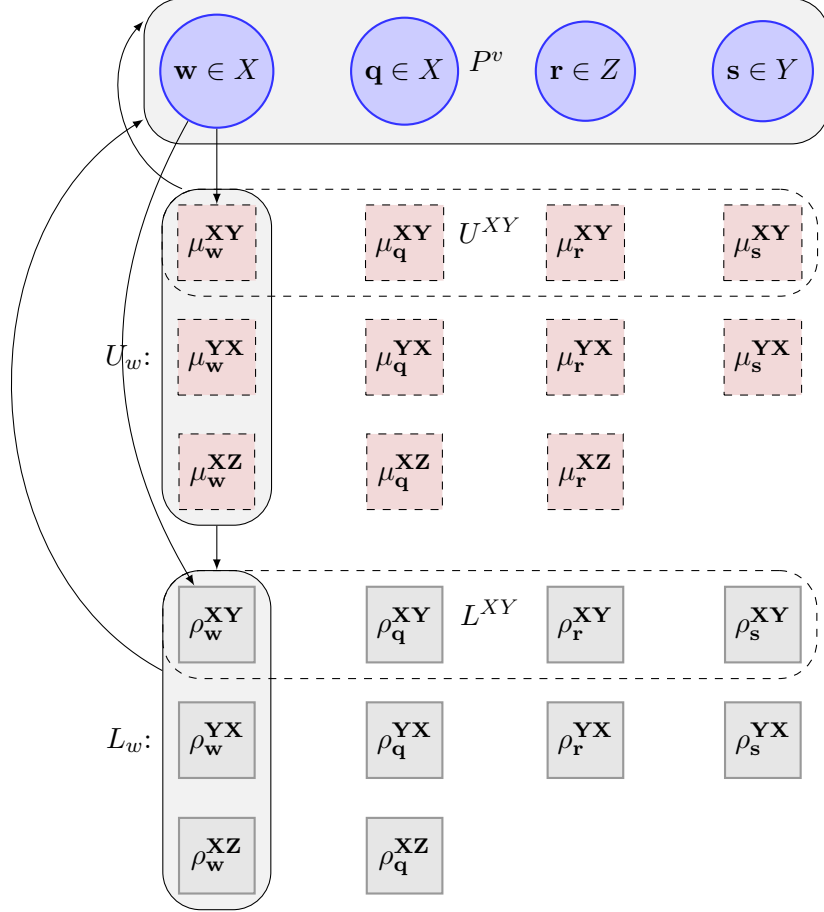
- There are multiple commodities.

Figure 3: A network representation of a graph associated with the label setting algorithm. The set of port call nodes P^v(blue nodes) form a clique. For port $w \in P^v$ the sets U_w (light red nodes), L_w(grey nodes) are illustrated. They represent possible loads and unloads at port w. The sets U_w, L_w form a cliques. A path in the network will follow sequences of $n \in P_v \rightarrow U_n \rightarrow L_n \rightarrow m \in P_v$. It is possible to only unload or load. The load set of a port w is not connected to the unload set of w. Each trade $XY \in K$ is associated with a loadset L^{XY} and an unloadset U^{XY} as illustrated.

- The route is combined with a loading/unloading pattern not unlike the labelling algorithm for the SDVRPTW in Desaulniers (2010).

In the label setting algorithm for LSNDP a label E contains the following information:

- Current port, p_c

- Start port, p_s

- (reduced) cost, t

- Accumulated distance, d_s

- The load of each *trade*, $F^{XY} \quad \forall XY \in K$

- Current load, $F_c = \sum_{XY \in K} F^{XY}$

- Visit number, $k_p \quad \forall p \in P^v$

The resources are $d_s, (F^{XY})_{XY \in K}, F_c, (k_p)_{p \in P^v}$ i.e. we have $2 + |K| + |p^v|$ resources. The extension function (Irnich and Desaulniers 2005) of the distance is defined as $e^d_{(ij)}(E_i) = d(E_i) + d_{ij}$. The feasibility and resource consumption of extending label E_i along an arc depends on the arc type:

- *Case 1: extending along a sail arc $(i, j) \in A_s$*
  A feasible extension of label E_i to node j along a sail arc $(i, j) \in A_s$ must satisfy the following conditions:

$$\left\lceil \frac{e^d_{(ij)}(E_i)}{W^v_d} \right\rceil \leq |v| \tag{10}$$

$$k^i_j + 1 \leq M_j \tag{11}$$

Here, (10) ensures the feasibility of the number of vessels deployed to the service and (11) ensures the number of port calls to port j does not exceed M_j. If the extension is feasible a new label E_j is created. Define

$$\varpi = \left\lceil \frac{e^d_{(ij)}(E_i)}{W^v_d} \right\rceil - \left\lceil \frac{d(E_i)}{W^v_d} \right\rceil \tag{12}$$

ϖ expresses whether the label extension will require an additional vessel on the service to maintain weekly frequency. The following extension functions are applied to create label E_j: $p^j_c = j, p^j_s = p^i_s, t^j = t^i - c^v_j - \hat{c}_v \cdot \varpi, d = e^d_{(ij)}(E_i), F^j_C = F^i_C, F^{XY}_j = F^{XY}_i, k^j_j = k^i_j + 1, k^j_p = k^i_p \quad \forall p \in P^v \setminus \{j\})$

- *Case 2: extending along an unload arc $(i,j) \in A_u, j = \mu_p^{XY}$*
  A feasible extension of label E_i to node j along unload arc $(i,j) \in A_u$ must satisfy the following conditions:

$$F^{XY} > 0 \tag{13}$$

  where (13) ensures that the commodity XY is currently loaded on the vessel i.e. that a previous visit to a node in L^{XY} has been performed. To ensure that all possible transhipment and unload patterns are considered all integral unloads in $o \in \{1, \ldots, max\{d^{XY}, F_i^{XY}\}\}$ are created with separate labels.

  If the extension is feasible a new label E_j^o is created using the extension functions:
  $p_c^j = h(j), p_s^j = p_s^i, t^j = t^i + \hat{u}_p^{XY} \cdot o, d = e_{(ij)}^d(E_i), F_C^j = F_C^i - o, F_j^{XY} = F_i^{XY} - o, F_j^{ZW} = F_i^{ZW} \forall ZW \in K \setminus \{XY\}, k_p^j = k_p^i \quad \forall p \in P^v$.

- *Case 3: extending along a load arc $(i,j) \in A_l, j = \rho_p^{XY}$*
  A feasible extension of label E_i to node j along a load arc $(i,j) \in A_l$ must satisfy the following conditions:

$$F_c^i < C_v \tag{14}$$

  (14) ensures that the vessel has excess capacity for loading. To ensure that all possible transhipment and load patterns are considered all integral loads in $o \in \{1, \ldots, max\{d^{XY}, C_v - F_C^i\}\}$ are created with separate labels. If the extension is feasible a new label E_j^o is created with the following extension function: $p_c^j = h(j), p_s^j = p_s^i, t^j = t^i + \hat{l} \cdot o, d = e_{(ij)}^d(E_i), F_C^j = F_C^i + o, F_j^{XY} = F_i^{XY} + o, F_j^{ZW} = F_i^{ZW} \forall ZW \in K \setminus \{XY\}, k_p^j = k_p^i \quad \forall p \in P^v$.

A state is feasible when the start node is reached ($p_c = p_s$) and the containers are balanced for all trades ($F^{XY} = 0 \; \forall XY \in K$) by applying unload extensions to the cycle starting from p_s ending in p_s. To obtain the solution to a service the auxiliary data of what has actually been loaded and unloaded has to be stored and a mapping from L to α and from U to β creates the column entries for (un)load in the master problem. For an exact solution to the pricing problem the service with the best reduced cost (max $\hat{c}_s$) is added to the master problem. However, the label setting algorithm may find several services, where the cost t is greater than 0 and add several columns in an iteration to accelerate convergence of the column generation algorithm.

2.2 Dominance

In order to dominate a label it must hold that the dominating label has the same possibilities for extensions and that no extension of the dominated label can yield a better reduced cost than the dominating label.

A label E_1 dominates a label E_2 if the following holds

- $p_c^1 = p_c^2$

- $p_s^1 = p_s^2$

- $t_1 \geq t_2$

- $d_s^1 \leq d_s^2$

- $k_p^1 \leq k_p^2 \quad \forall p \in P^v$

- $F_c^1 \leq F_c^2$

- $F_1^{XY} = F_2^{XY} \ \forall XY \in K$

Requiring the cargo loads to be identical gives rise to a weak dominance criteria. This means that the labelling algorithm resorts to being practically brute force and a vast number of labels are generated even for relatively small instances. In recent work on dominance criteria for the Pick-up-and-Delivery problem (Ropke and Cordeau 2009) the dominance criteria for the cargo loads are strengthened by relaxing such that in our case we would have

- $F_1^{XY} \leq F_2^{XY} \ \forall XY \in K$

if the delivery triangle inequality defined by Ropke and Cordeau (2009) as $d_{ij} + d_{jk} \geq d_{ik}$ holds $\forall i, j, k \in V$. Here j is a delivery node. It is however not trivial to see, whether this relaxation holds for the pricing problem in this paper as each commodity may have several delivery nodes attached and there are no precedence relation between pickup and delivery nodes, due to the cyclic nature of a route.

2.3 Complexity

Let T denote an upper bound on the distance of a service. The running time of the label setting algorithm can be shown to be $O((T|P|C^{|K|} \prod_{p \in X} d^{pY} \prod_{p \in G^{XY}} C)^2)$. Increasing the number of *trades* and the number of transshipment ports will increase the number of states in the Dynamic Programming algorithm. To solve practical problem instances it is therefore important to make a careful choice of the *trades* and the ports, where transshipment is allowed.

2.4 Relaxation of pricing problem

In CVRP a pseudo polynomial relaxation is used when solving the strongly NP-hard pricing problem (Baldacci et al. 2008) to reduce the practical running time of the algorithm. The method has proven to be very powerful for the CVRP. A pseudo polynomial relaxation of our pricing problem can be obtained as follows: Each port is assigned the minimal load and unload cost and the bounds on the load are removed. In each port the number of different states will then be limited to $T|P||C|$ and a running time of $O(T|P|^2|C|)$ can be obtained. However, defining a strong bound for the minimal load and unload cost for each port is not trivial as several commodities may origin or transship at a given node and further research must be conducted in order to achieve a relaxation with a good bound.

As the pricing problem is very complex, we need not solve the pricing problem to optimality in each iteration, but one could stop once a sufficient amount of columns with positive reduced cost has been found. An easy way to do this is to run the dynamic programming algorithm using a greedy variant adding any reduced cost column instead of the best reduced cost column.

3   Preliminary computational Results

The described algorithm has been implemented using CPLEX to solve the master problem and a labelling algorithm to solve the pricing problem. The results are currently not satisfactory for solving the pricing problem. The structure of the labelling algorithm, the lack of proper dominance criteria and especially the need to generate labels for all integral steps of load and unloads ($o \in \{1, \ldots, max\{d^{XY}, F_i^{XY}\}\}$ respectively $o \in \{1, \ldots, max\{d^{XY}, C_v - F_C^i\}\}$) creates a huge number of very similar labels. The combinations of these causes the labelling algorithm to effectively be a brute force algorithm in an extremely large search space. Even for very small graphs ($n = 4$) the number of considered labels are in the 10'ths of millions.

A simplification of the model is to only consider demand paths, which are fully loaded or unloaded either regarding the demand or capacity, i.e. $o = \{F_i^{XY}, d^{XY}\}$ respectively $o = max\{d^{XY}, C_v - F_C^i\}$. Unfortunately, this approach is inconsistent with the idea of aggregated demands, as these *will* need to split to reach their respective origins / destinations, discouraging this direction.

As a result we have not pursued methods such as bounding to improve upon the current algorithm of the pricing problem as we believe alternative solution methods must be applied for an efficient algorithm to solve the pricing problem. Another alternative is the design and implementation of efficient heuristics to generate variables and subsequently solve a heuristic implementation of a Branch-and-Price algorithm similar to the one seen in

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Agarwal and Ergun (2008).

4 Conclusion

We have presented a new model for LSNDP. Among the benefits of the proposed model is a novel view of demands in liner-shipping, which are considered on a *trade* basis. This has the advantage of giving a natural understanding, and requiring fewer variables. The model assigns cargo to routes, which may result in a tighter search space for a branch-and-bound algorithm.

A solution approach using delayed column generation has been presented, where the proposed subproblem is related to the pricing problems in VRP, where Branch-&-Cut-&-Price has been used with great success. We have discussed a pseudo polynomial relaxation to be used as bounding function, when solving the pricing problem in combination with heuristics and other techniques that have been effective in solving VRP problems. In the VRP context resource limitations have proven to be effective for the dynamic programming algorithms in reducing the state space. In the dynamic programming algorithm presented in this paper these resource limitations do not reduce complexity of the subproblem sufficiently, because dominance criterions are different. The proposed algorithm has been implemented but showed disappointing results, due to the lack of dominance criteria and a large search space for the label setting algorithm. We still believe that the main ideas in this paper can be useful to solve the LSNDP, i.e. the thoughts of combining cost and revenue in a single pricing problem and especially the notion of demand aggregation, which lends to a natural understanding in Liner Shipping. However, we must conclude that alternative methods or extensions of the current dynamic programming algorithm will be needed to solve a pricing problem, where cargo load patterns for multiple commodities are combined with a routing.

Further work with richer formulations of LSNDP, considering aspects as transit time limits on paths, and other operational constraints from liner shipping will tighten the search space of the pricing problems. However, it is uncertain whether additional real-life complexity in the pricing problem will allow for effective dominance criteria in a label setting algorithm.

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Liner shipping networks are the backbone of international trade providing low transportation cost, which is a major driver of globalization. These networks are under constant pressure to deliver capacity, cost effectiveness and environmentally conscious transport solutions.

This article proposes a new path based MIP model for the Liner shipping Network Design Problem minimizing the cost of vessels and their fuel consumption facilitating a green network. The proposed model reduces problem size using a novel aggregation of demands. A decomposition method enabling delayed column generation is presented. The subproblems have similar structure to Vehicle Routing Problems, which can be solved using dynamic programming. An algorithm has been implemented for this model, unfortunately with discouraging results due to the structure of the subproblem and the lack of proper dominance criteria in the labeling algorithm.