



## Statistical framework for decision making in mine action

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# Statistical framework for decision making in mine action

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# Why do we need statistical models and machine learning?



- Mine action is influenced by many uncertain factors
- The goals of mine action depends on difficult socio-economic and political considerations

Scientist are born sceptical: they don't believe facts unless they see them often enough



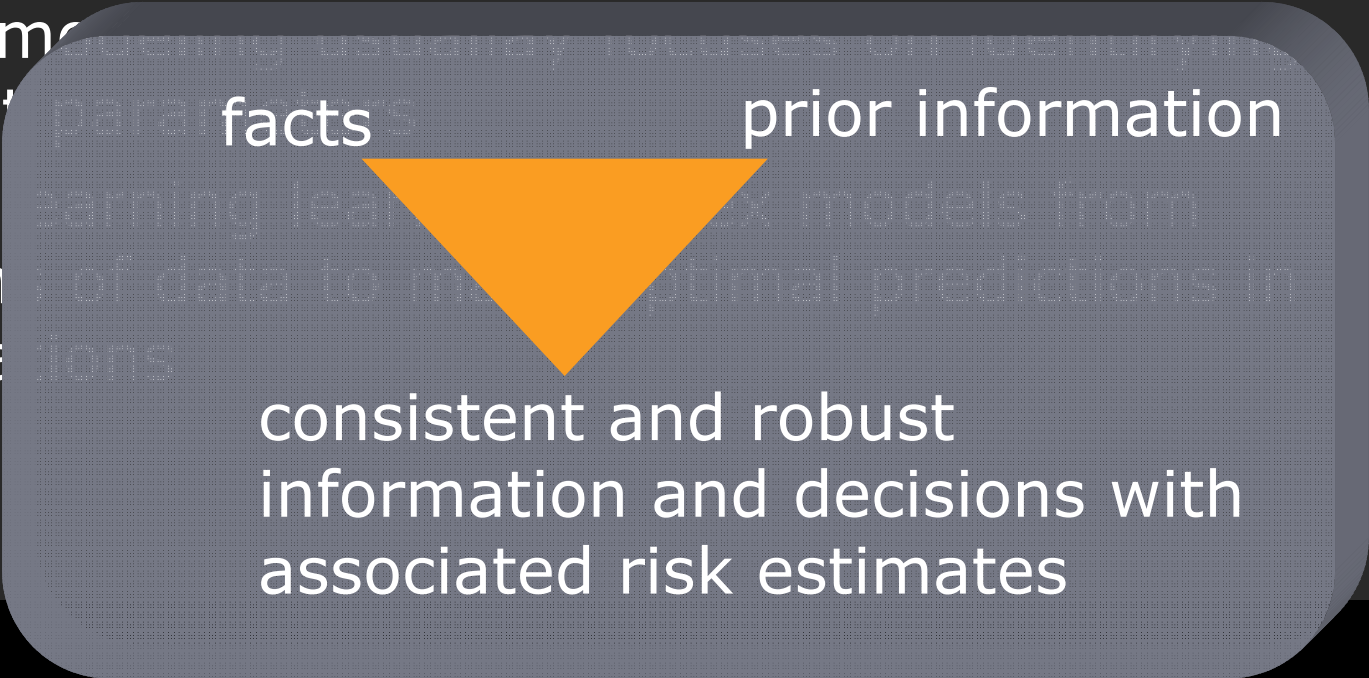
## Why do we need statistical models and machine learning?

- statistical modeling is the **principled framework** to handle uncertainty and complexity
- Statistic modeling usually focuses on identifying important parameters
- machine learning learns complex models from collections of data to make optimal predictions in new situations



# Why do we need statistical models and machine learning?

- statistical modeling is the **principled framework** to handle uncertainty and complexity
- Statistic modeling is an important part of machine learning
- machine learning learns models from collection of data to make optimal predictions in new situations





## There is no such thing as facts to spoil a good explanation!

- Pitfalls and misuse of statistical methods sometimes wrongly leads to the conclusion that they are of little practical use

After the dogs went in we never saw an accident

Most suspected areas have very few mines



# There is no such thing as facts to spoil a good explanation!

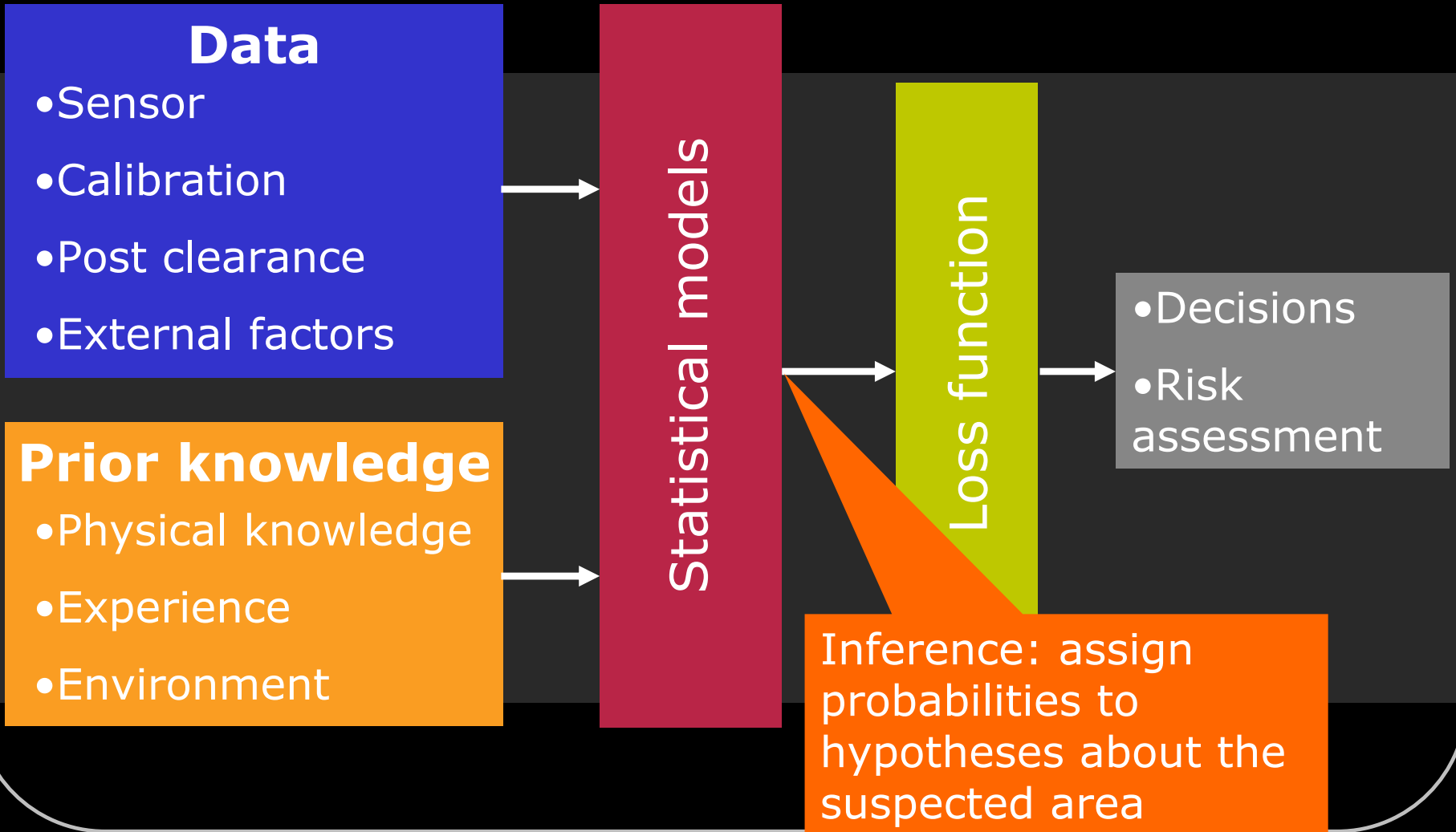
- Pitfalls and misuse of statistical methods sometimes wrongly leads to the conclusion that they are of little practical use

Some data are in the tail of the distribution: generalization from few examples is not possible

Smoking is not dangerous: my granny just turned 95 and has been a heavy smoker all his live



# The elements of statistical decision theory







# Outline

- The design and evaluation of mine clearance equipment – the problem of reliability
  - Detection probability – tossing a coin
  - Requirements in mine action
  - Detection probability and confidence in MA
  - Using statistics in area reduction
- Improving performance by information fusion and combination of methods
  - Advantages
  - Methodology
  - DeFuse project



## Detecting a mine – tossing a coin

$$\textit{Frequency} = \frac{\text{no of heads}}{\text{no of tosses}}$$

*probability = frequency* when infinitely many tosses



## On 99,6% detection probability

$$\textit{Frequency} = \frac{996}{1000} = 99,6\%$$

One more (one less) count will change the frequency a lot!

$$\textit{Frequency} = \frac{9960}{10000} = 99,60\%$$



## Detection probability - tossing a coin

- $N$  independent tosses number of
- $y$  number of heads observed
- $\theta$  probability of heads

$$\hat{\theta} = \frac{y}{N}$$

$$P(y | \theta) = \text{Binom}(\theta | N) = \binom{N}{y} \theta^y \theta^{N-y}$$

Data likelihood



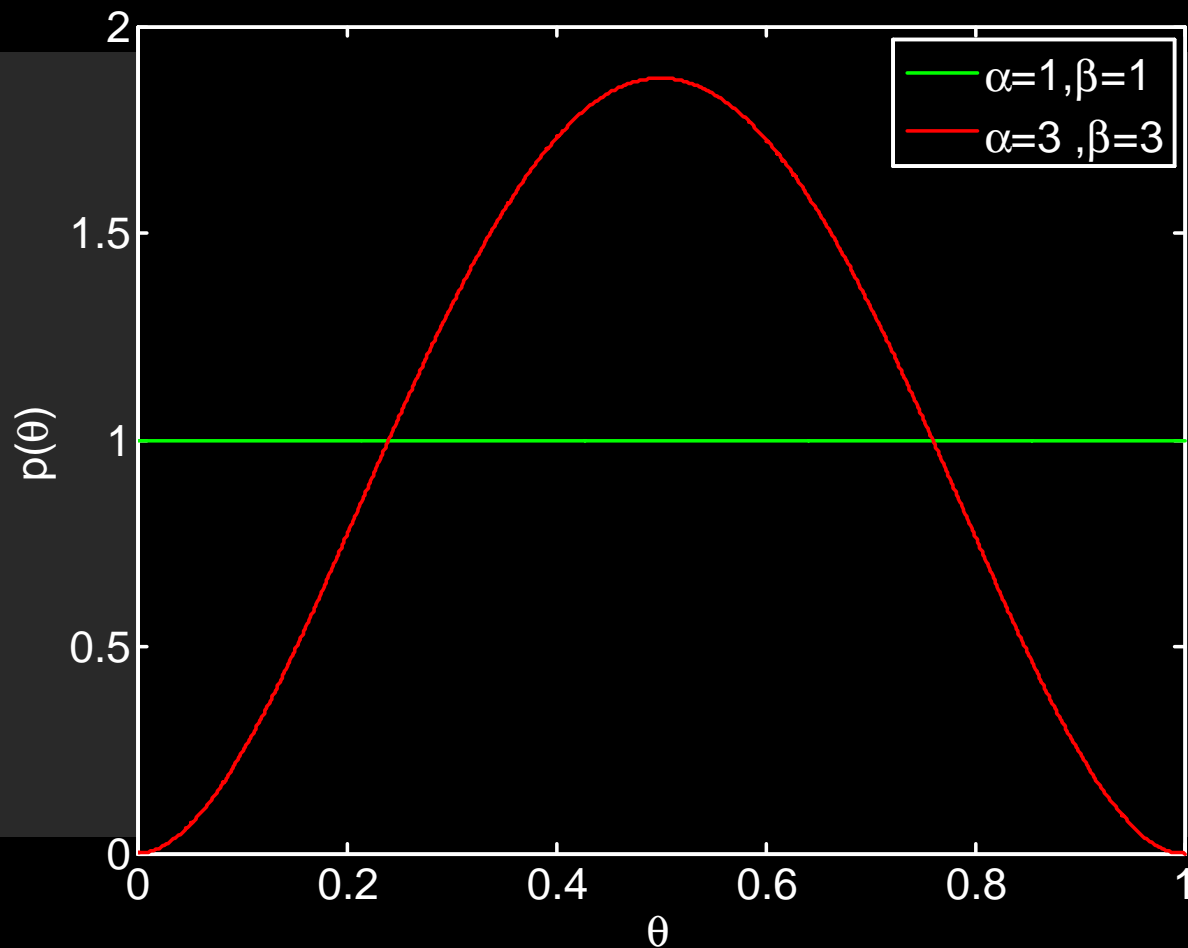
## Prior beliefs and opinions

- Prior 1: the fair coin:  $\theta$  should be close to 0.5
- Prior 2: all values of  $\theta$  are equally plausible

$$p(\theta) = \text{Beta}(\theta \mid \alpha, \beta)$$



## Prior beliefs and opinions





## Bayes rule: combining data likelihood and prior

$$P(\theta | y) = \frac{P(y | \theta) p(\theta)}{P(y)}$$

$$P(\theta | y) = \text{Beta}(\theta | y + \alpha, \beta + n - y) \sim \theta^{y+\alpha} \theta^{n-y+\beta}$$



## Posterior probability is also Beta

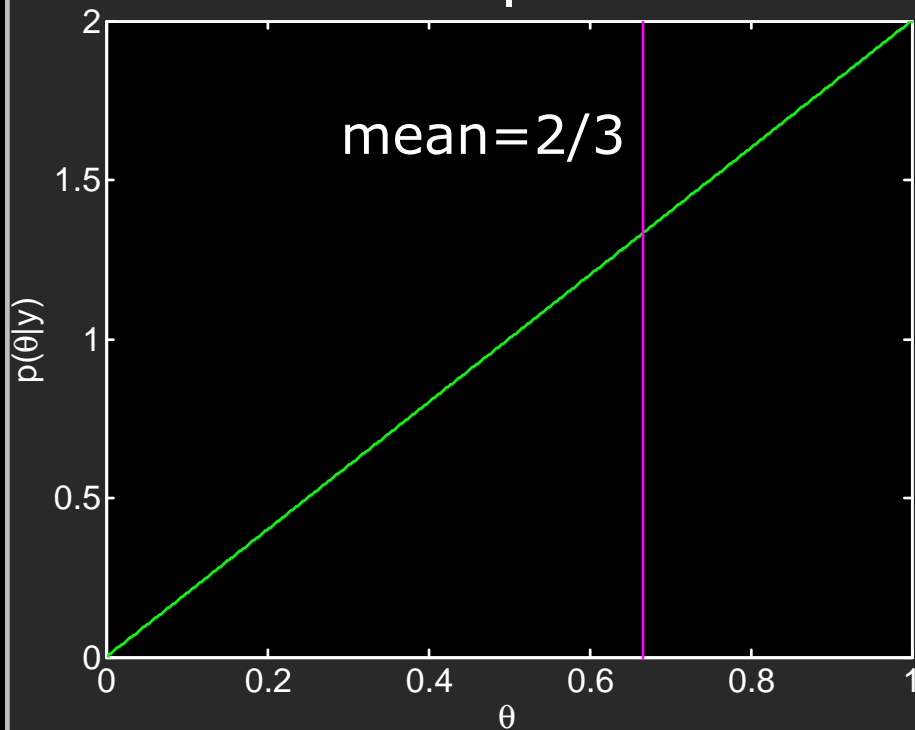
$$P(\theta | y) = \text{Beta}(\theta | y + \alpha, \beta + n - y) \sim \theta^{y+\alpha} \theta^{n-y+\beta}$$





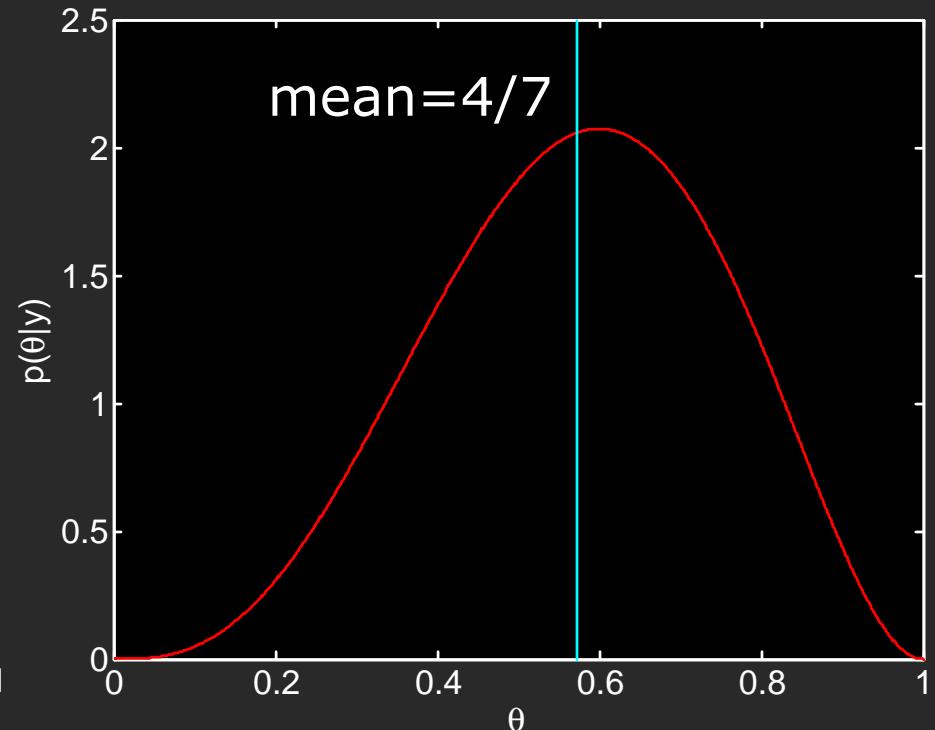
# Posteriors after observing one head

Flat prior



$$Beta(\theta | 2, 1)$$

Fair coin



$$Beta(\theta | 4, 3)$$



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## What are the requirements for mine action risk

- Tolerable risk for individuals comparable to other natural risks
- As high cost efficiency as possible requires detailed risk analysis – e.g. some areas might better be fenced than cleared
- Need for professional risk analysis, communication management and control involving all partners (MAC, NGOs, commercial etc.)



## What are the requirements for mine action risk

- Tolerable risk for individuals comparable to other natural risks

### Fact

99.6% is not an unrealistic requirement

but... today's methods achieve at most 90% and are hard to evaluate!!!

NGOs, commercial

GICHD and FFI are currently working on such methods [Håvard Bach, Ove Dullum NDRF SC2006]



## A simple inference model – assigning probabilities to data

- The detection system provides the probability of detection a mine in a specific area:  $\text{Prob}(\text{detect})$
- The land area usage behavior pattern provides the probability of encounter:  $\text{Prob}(\text{mine encounter})$

$$\text{Prob}(\text{casualty}) = (1 - \text{Prob}(\text{detect})) * \text{Prob}(\text{mine encounter})$$

For discussion of assumptions and involved factors see  
“Risk Assessment of Minefields in HMA – a Bayesian  
Approach”

PhD Thesis, IMM/DTU 2005 by Jan Vistisen



## A simple loss/risk model

- Minimize the number of casualties
- Under mild assumptions this equivalent to minimizing the probability of casualty



## Requirements on detection probability

$$\text{Prob}(\text{causality}) = (1 - \text{Prob}(\text{detection})) * \text{Prob}(\text{encounter})$$

$$\text{Prob}(\text{detection}) = 1 - \text{Prob}(\text{causality}) / \text{Prob}(\text{encounter})$$

- $\text{Prob}(\text{encounter}) = \rho * a$ 
  - $\rho$  : homogeneous mine density (mines/m<sup>2</sup>),  $a$ : yearly footprint area (m<sup>2</sup>)
- $\text{Prob}(\text{causality}) = 10^{-5}$  per year



## Maximum yearly footprint area in m<sup>2</sup>

P(detection)	$\rho$ : mine density (mines/km <sup>2</sup> )				
	0.1	1	10	100	1000
0.996	25000	2500	250	25	2.5
0.9	1000	100	10	1	0.1

Reference: Bjarne Haugstad, FFI





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# Evaluation and testing in MA

- How do we assess the performance/detection probability?
- What is the confidence?

Changing environment

- mine types, placement
- soil and physical properties
- unmodeled confounds

system design phase

operation phase

## Overfitting

- insufficient coverage of data
- unmodeled confounding factors
- unsufficient model fusion and selection


evaluation phase



# Two types of error in detection of mines

## Sensing error

The system does not sense the presence of the mine object

 decrease in detection probability

## Decision error

The detector misinterprets the sensed signal

 increase in false alarm rate



# Two types of error in detection of mines

**Sensing error**

The system does not sense the presence of the mine



**Example: metal detector**

- Sensing error: the mine has low metal content
- Decision error: scrap metal with too low probability

**Decision error**

**Example: mine detection dog**

- Sensing error: the TNT leakage from the mine was too low
- Decision error: the dog handler misinterpreted the dogs indication



# Confusion matrix in system design and test phase which should lead to certification

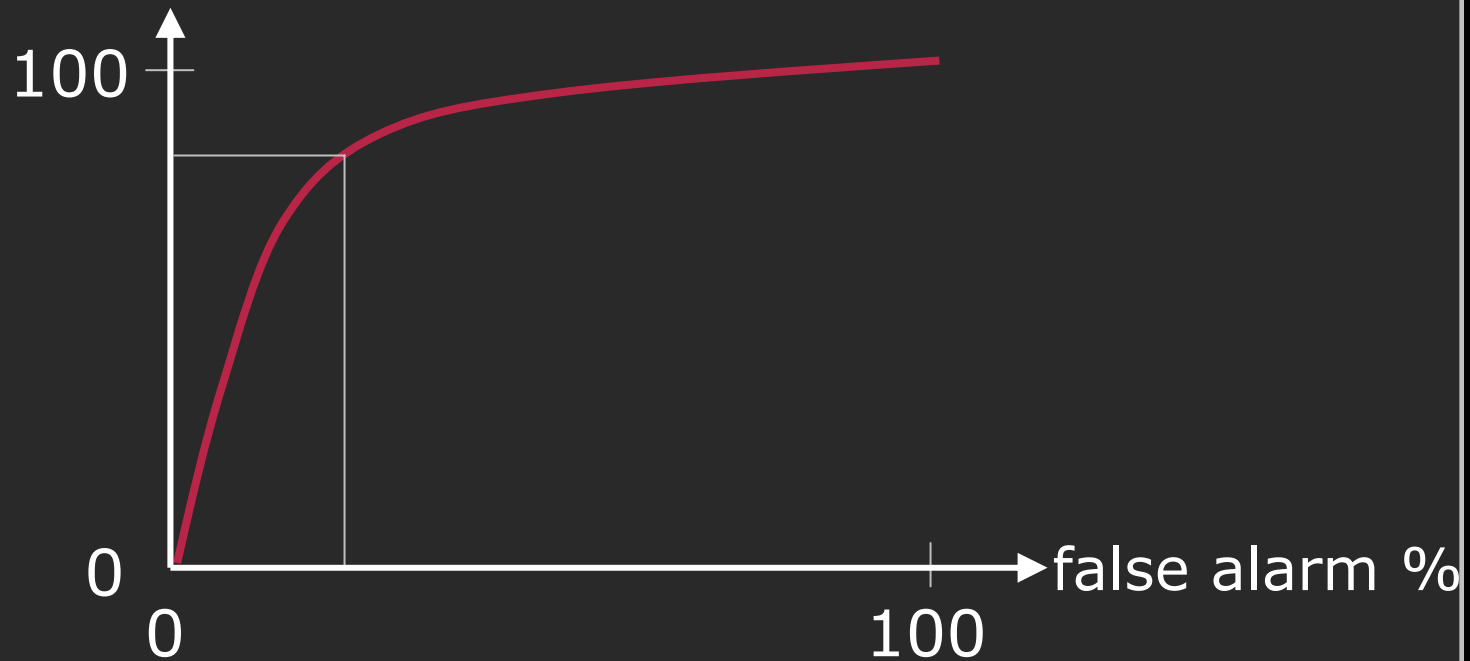
		True	
		yes	no
Estimated	yes	a	b
	no	c	d

- Detection probability (sensitivity):  
 $a/(a+c)$
- False alarm:  
 $b/(a+b)$
- False positive (specificity):  
 $b/(b+d)$



# Receiver operation characteristic (ROC)

detection probability %





## Inferring the detection probability

- $N$  independent mine areas for evaluation
- $y$  detections observed
- true detection probability  $\theta$

$$P(y | \theta) \sim \text{Binom}(\theta | N) = \binom{N}{y} \theta^y \theta^{N-y}$$



## Bayes rule: combining data likelihood and prior

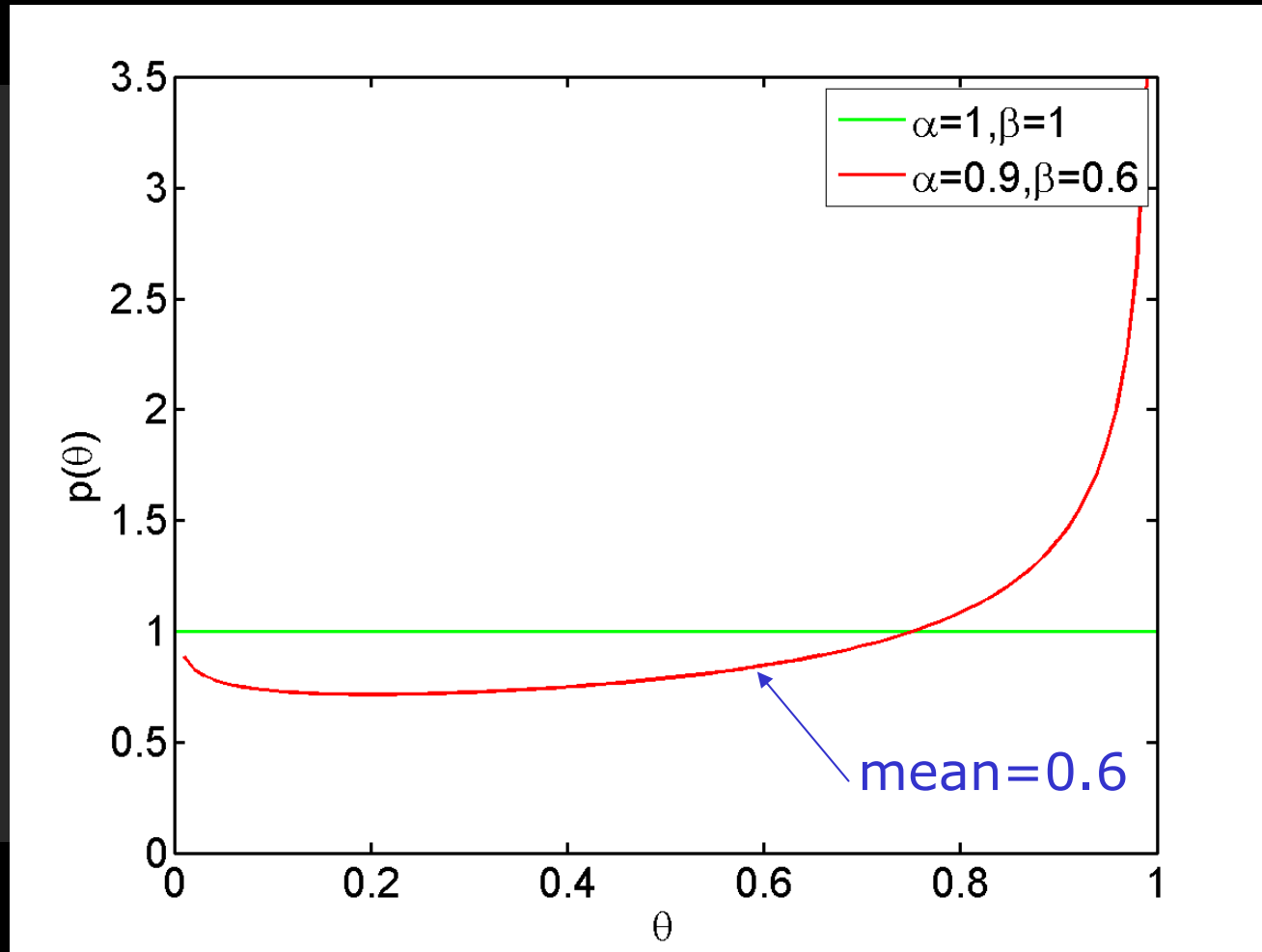
$$P(\theta | y) = \frac{P(y | \theta) p(\theta)}{P(y)}$$

$$P(\theta | y) = \text{Beta}(\theta | y + \alpha, \beta + n - y) \sim \theta^{y+\alpha} \theta^{n-y+\beta}$$





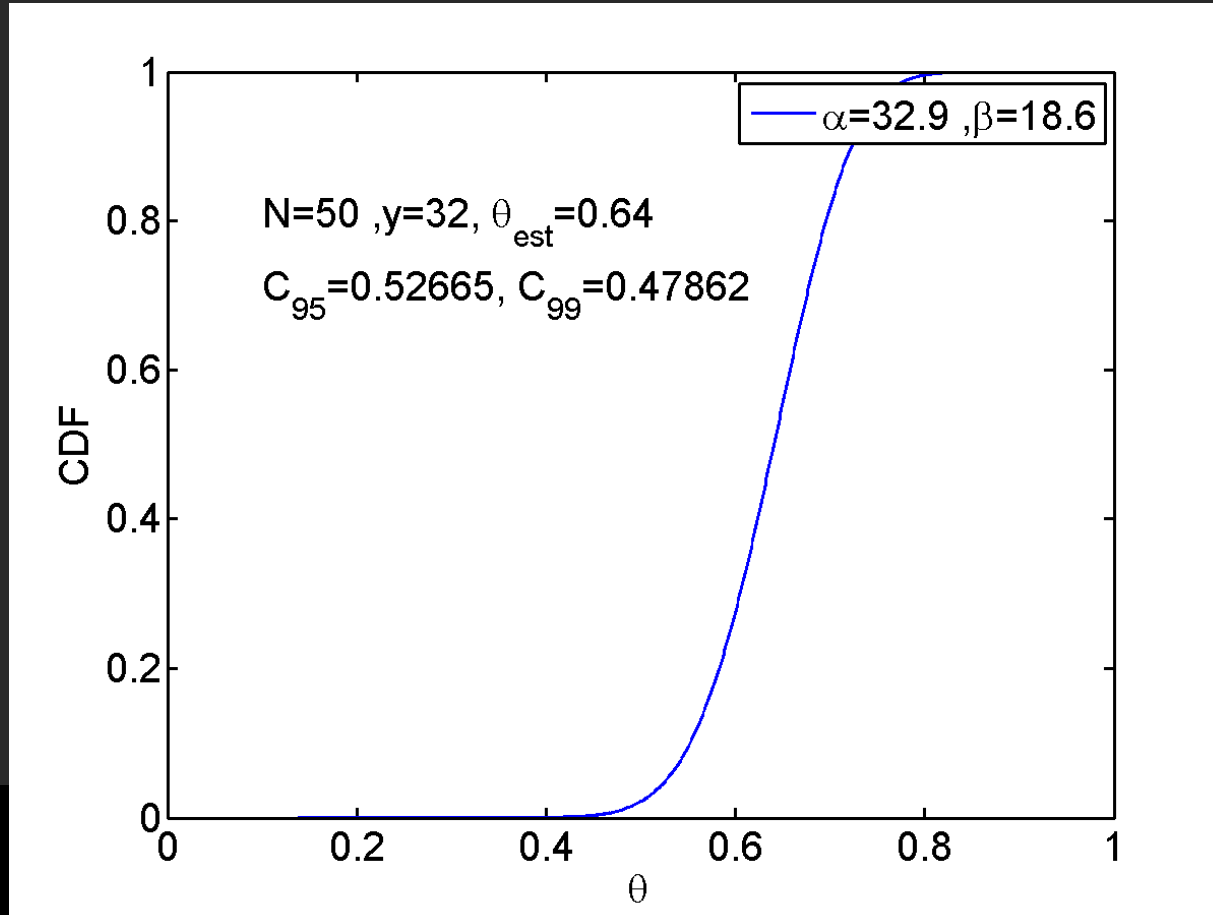
# Prior distribution





# HPD credible sets – the Bayesian confidence interval

interval  $C_{1-\varepsilon} = \{\theta: P(\theta | y) \geq k(\varepsilon)\}, \text{CDF}(\theta | y) > 1 - \varepsilon$





# The required number of samples $N$

- We need to be confident about the estimated detection probability

$$\text{Prob}(\theta > 99.6\%) = C_{1-\epsilon}$$

	$C_{95\%}$	$C_{99\%}$
$\theta_{est} = 99.7\%$	9303	18994
$\theta_{est} = 99.8\%$	2285	3995

	$C_{95\%}$	$C_{99\%}$
$\theta_{est} = 99.7\%$	8317	18301
$\theta_{est} = 99.8\%$	2147	3493

Uniform prior

Informative prior

$$\alpha=0.9, \beta=0.6$$



# Credible sets when detecting 100%

Minimum number of samples  $N$

	Prob( $\theta > 80\%$ )	Prob( $\theta > 99.6\%$ )	Prob( $\theta > 99.9\%$ )
$C_{95\%}$	13	747	2994
$C_{99\%}$	20	1148	4602

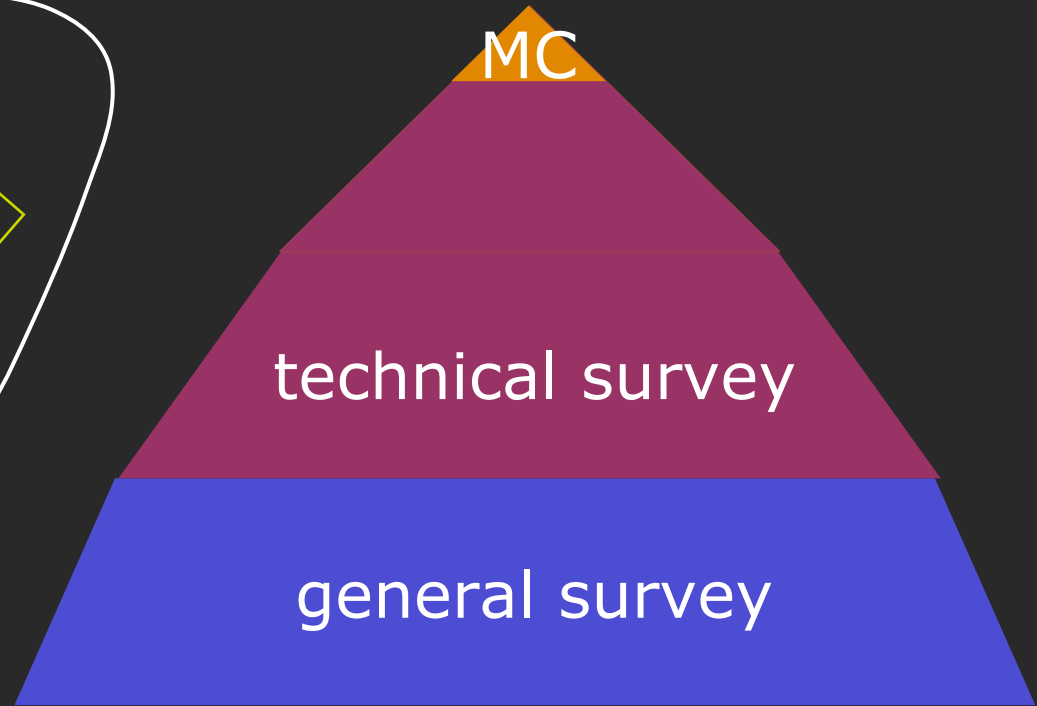
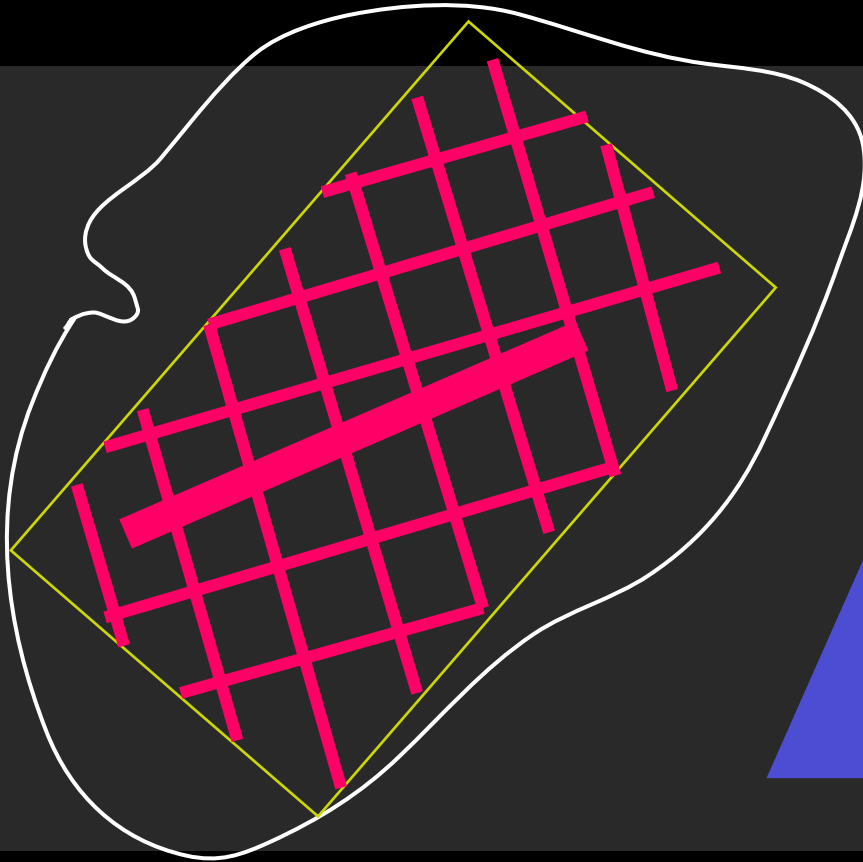


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# Efficient MA by hierarchical approaches



Ref: Håvard Bach, Paul Mackintosh



## Danger maps

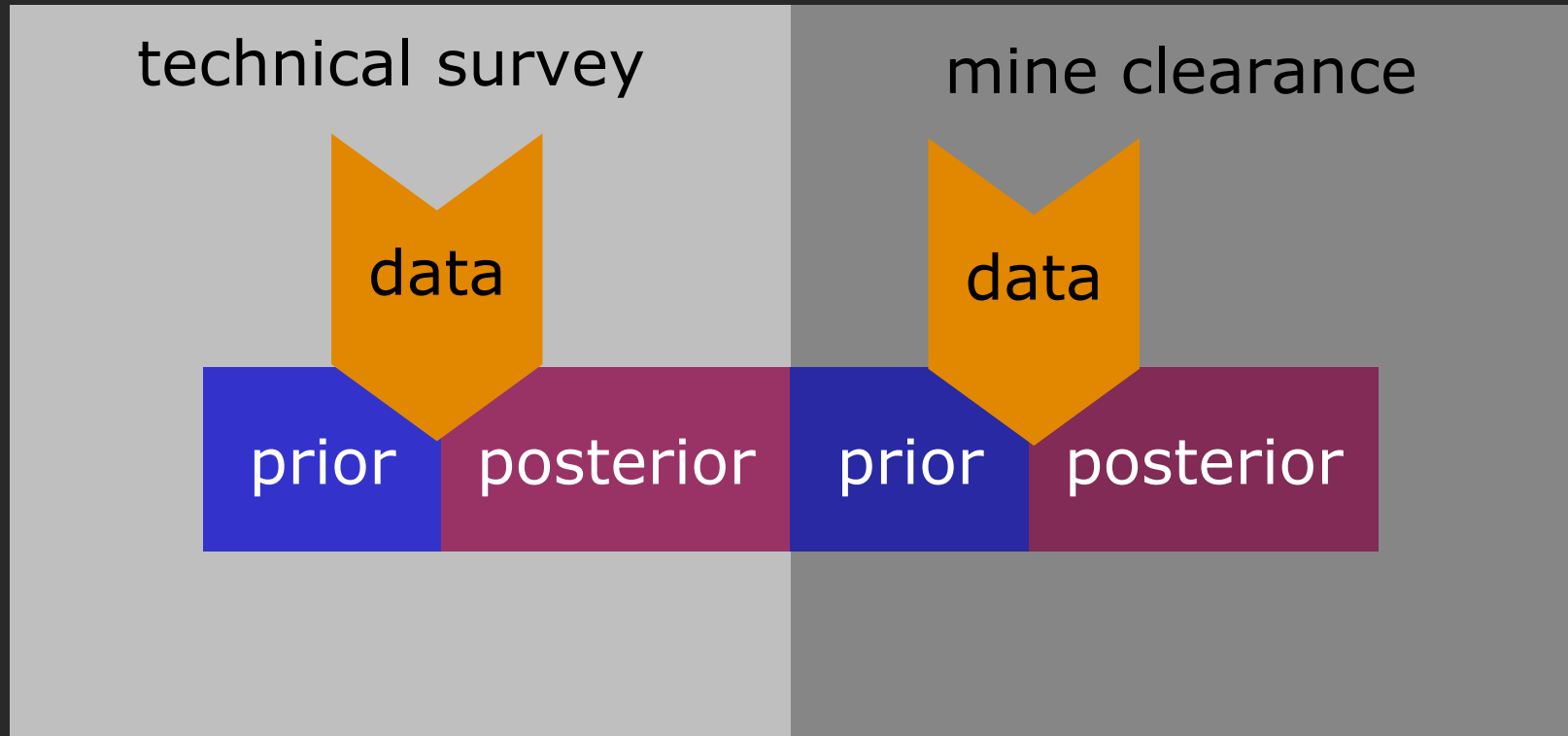
- The outcome of a hierarchical surveys
- Information about mine types, deployment patterns etc. should also be used
- Could be formulated/interpreted as a prior probability of mines



SMART system described in GICHD: Guidebook on Detection Technologies and Systems for Humanitarian Demining, 2006



# Sequential information gathering







## Statistical information aggregation

- $e=1$  indicates encounter of a mine in a box at a specific location
- probability of encounter  $P(e = 1)$  from current danger map
- $d=1$  indicates detection by the detection system
- probability of detection  $P(d = 1)$  from current accreditation

$$P(e = 1 \wedge d = 0) = P(e = 1)(1 - P(d = 1))$$

$$P(\text{no mine}) = 1 - P(e = 1 \wedge d = 0)$$



## Statistical information aggregation

Example: flail in a low danger area

$$P(e = 1) = 0.2, P(d = 1) = 0.8$$

$$P(\text{no mine}) = 1 - P(e = 1 \wedge d = 0) = 1 - 0.2 * 0.2 = 0.96$$

Example: manual raking in a high danger area

$$P(e = 1) = 1, P(d = 1) = 0.96$$

$$P(\text{no mine}) = 1 - P(e = 1 \wedge d = 0) = 1 - 1 * 0.04 = 0.96$$



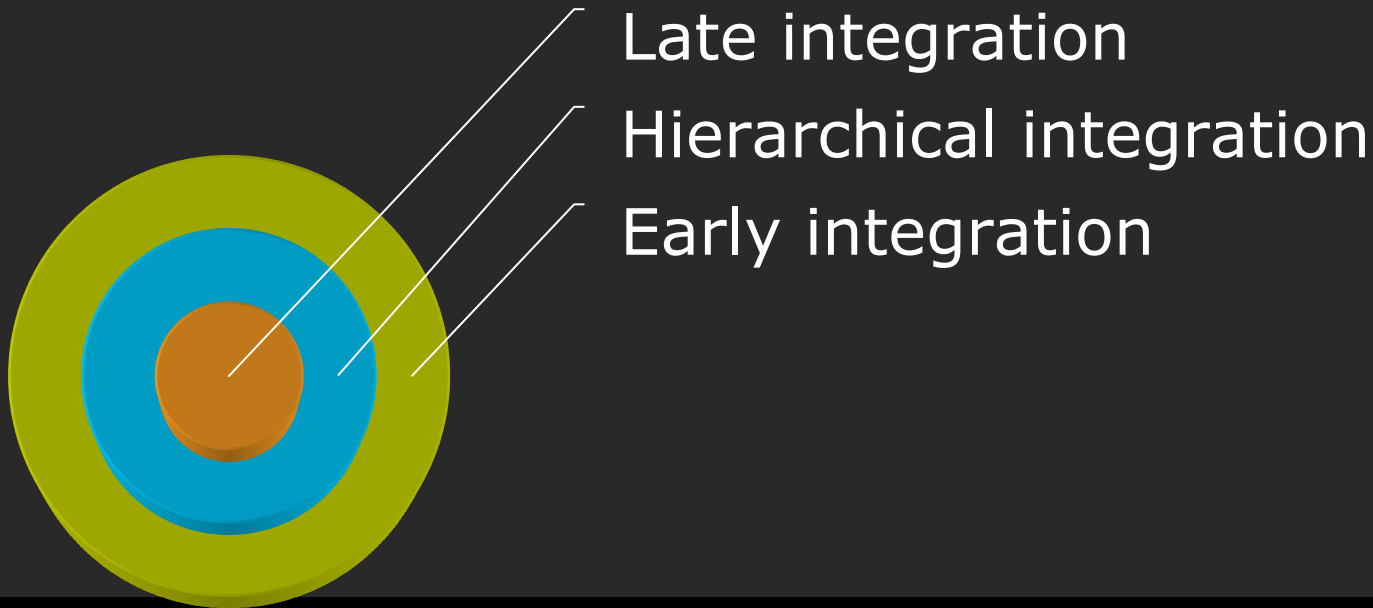
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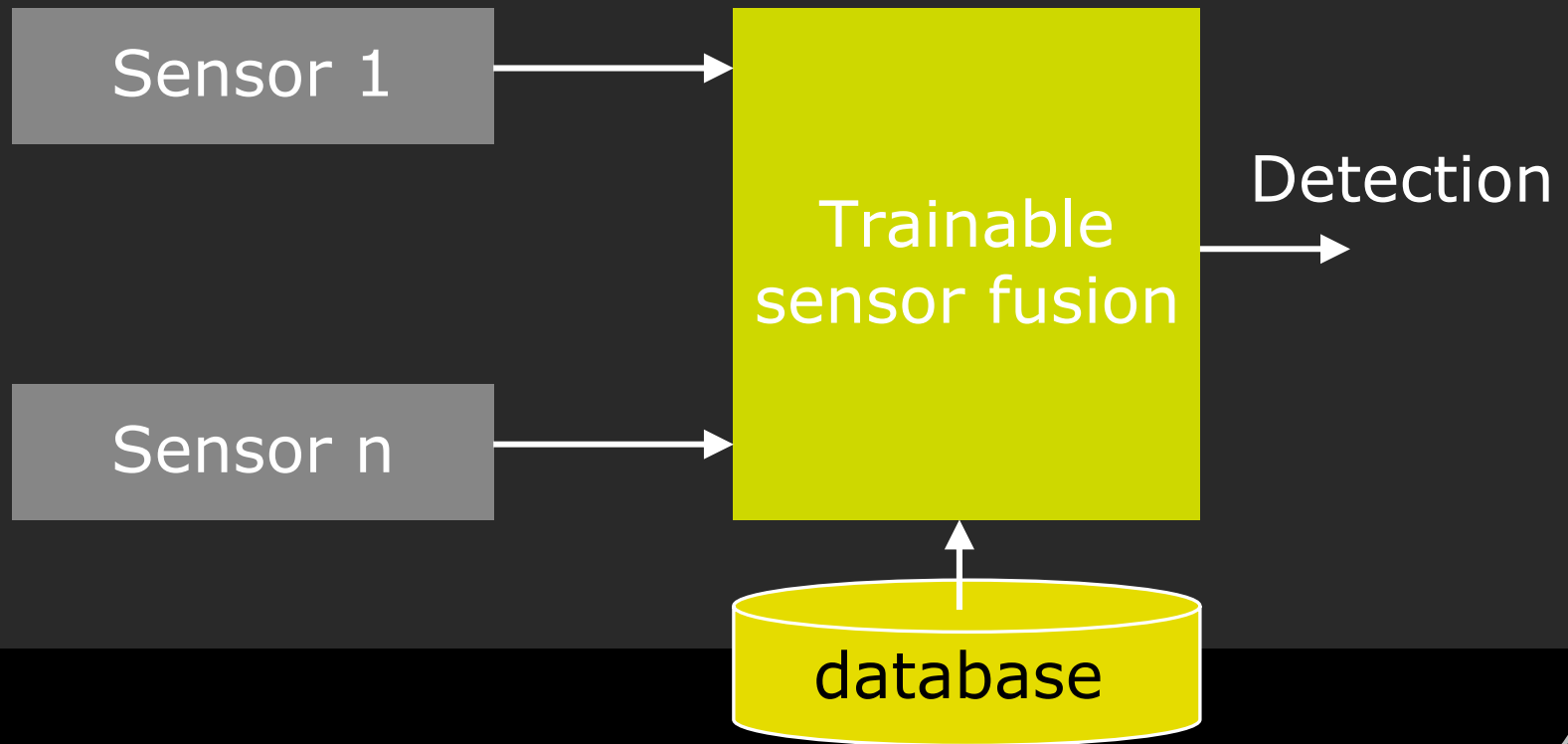
## Improving performance by fusion of methods

- Methods (sensors, mechanical etc.) supplement each other by exploiting different aspect of physical environment



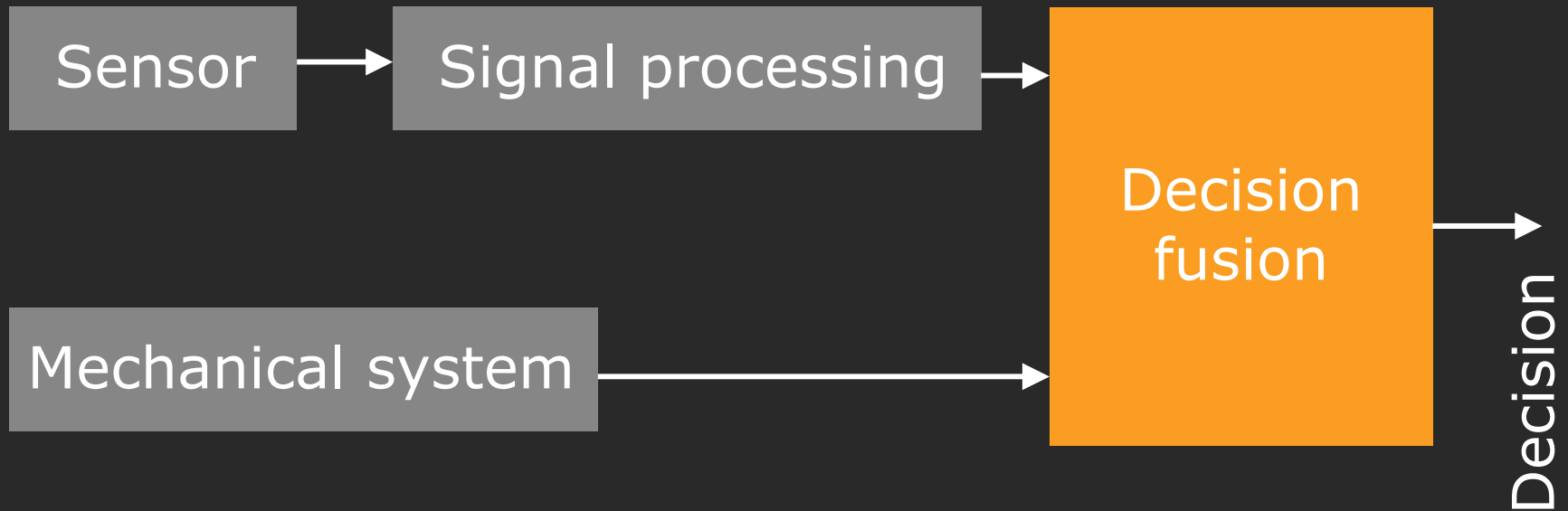


## Early integration – sensor fusion





## Late integration – decision fusion





## Advantages

- Combination leads to a possible exponential increase in detection performance
- Combination leads to better robustness against changes in environmental conditions



## Challenges

- Need for **certification procedure** of equipment under well-specified conditions (ala ISO)
- Need for new procedures which estimate **statistical dependences** between existing methods
- Need for new procedures for statistically **optimal combination**





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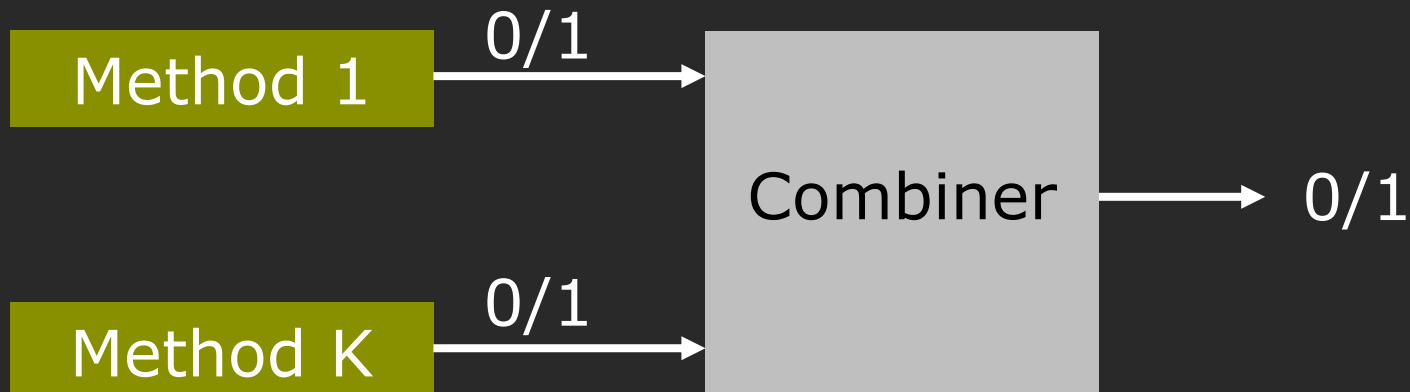
# Dependencies between methods

Contingency tables

		Method j	
		yes	no
Method i	yes	c11	c10
	no	c01	c00



## Optimal combination



Optimal combiner depends on contingency tables



# Optimal combiner

OR rule is optimal for independent methods

Method		Combiner						
1	2	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	1	1	1
1	0	0	1	1	0	0	1	1
1	1	1	0	1	0	1	0	1

$2^{2^{K-1}} - 1$  possible combiners



# OR rule is optimal for independent methods

Method 1: 1 0 0 1 0 0 1 0 1 0

Method 2: 0 1 0 0 1 0 1 1 1 0

Combined: 1 1 0 1 1 0 1 1 1 0

$$\begin{aligned}
 P_d(OR) &= P(\hat{y}_1 \vee \hat{y}_2 = 1 \mid y = 1) \\
 &= 1 - P(\hat{y}_1 = 0 \wedge \hat{y}_2 = 0 \mid y = 1) \\
 &= 1 - P(\hat{y}_1 = 0 \mid y = 1) \cdot P(\hat{y}_2 = 0 \mid y = 1) \\
 &= 1 - (1 - P_{d1}) \cdot (1 - P_{d2})
 \end{aligned}$$

independence



## False alarm follows a similar rule

$$\begin{aligned} P_{fa}(OR) &= \\ P(\hat{y}_1 \vee \hat{y}_2 = 1 \mid y = 0) &= \\ = 1 - P(\hat{y}_1 = 0 \wedge \hat{y}_2 = 0 \mid y = 0) &= \\ = 1 - P(\hat{y}_1 = 0 \mid y = 0) \cdot P(\hat{y}_2 = 0 \mid y = 0) &= \\ = 1 - (1 - P_{fa1}) \cdot (1 - P_{fa2}) & \end{aligned}$$



## Example

$$p_{d1} = 0.8, p_{fa1} = 0.1 \quad p_{d2} = 0.7, p_{fa2} = 0.1$$

$$p_d = 1 - (1 - 0.8) \cdot (1 - 0.7) = 0.94$$

$$p_{fa} = 1 - (1 - 0.1) \cdot (1 - 0.1) = 0.19$$

Exponential increase in detection rate  
Linear increase in false alarm rate

Joint discussions with: Bjarne Haugstad



# Testing independence – Fisher's exact test

		Method j	
		yes	no
Method i	yes	c11	c10
	no	c01	c00

- **Hypothesis:** Method i and j are independent
- **Alternatives:** Dependent or positively (negatively) correlated

$$H : P(\hat{y}_i = 0, \hat{y}_j = 0) = P(\hat{y}_i = 0) \cdot P(\hat{y}_j = 0)$$

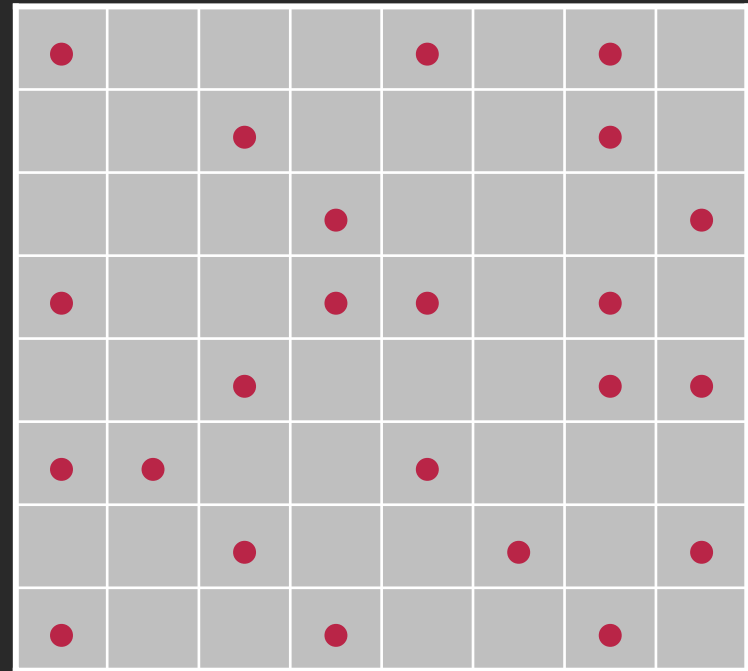
$$A : P(\hat{y}_i = 0, \hat{y}_j = 0) > P(\hat{y}_i = 0) \cdot P(\hat{y}_j = 0)$$





## Artificial example

- $N=23$  mines
- Method 1:  $P(\text{detection})=0.8$ ,  
 $P(\text{false alarm})=0.1$
- Method 2:  $P(\text{detection})=0.7$ ,  
 $P(\text{false alarm})=0.1$
- Resolution: 64 cells



How does detection and false alarm rate influence the possibility of gaining by combining methods?



# Confusion matrix for method 1

		True	
		yes	no
Estimated	yes	19	5
	no	4	36



## Confidence of estimated detection rate

- With  $N=23$  mines 95%-credible intervals for detection rates are extremely large!!!!

Method1 (flail): [64.5%    82.6%    93.8%]

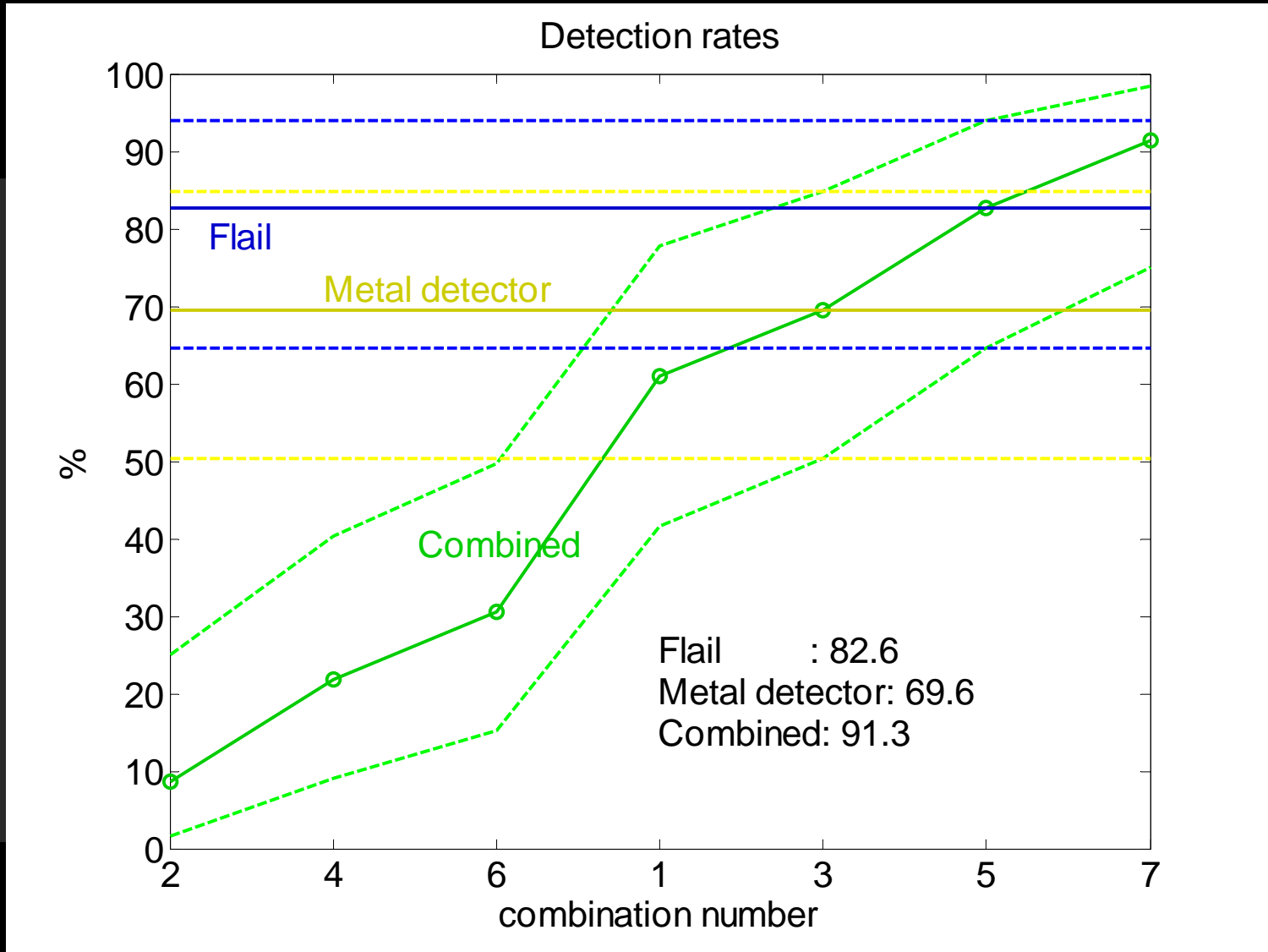
Method2 (MD): [50.4%    69.6%    84.8%]

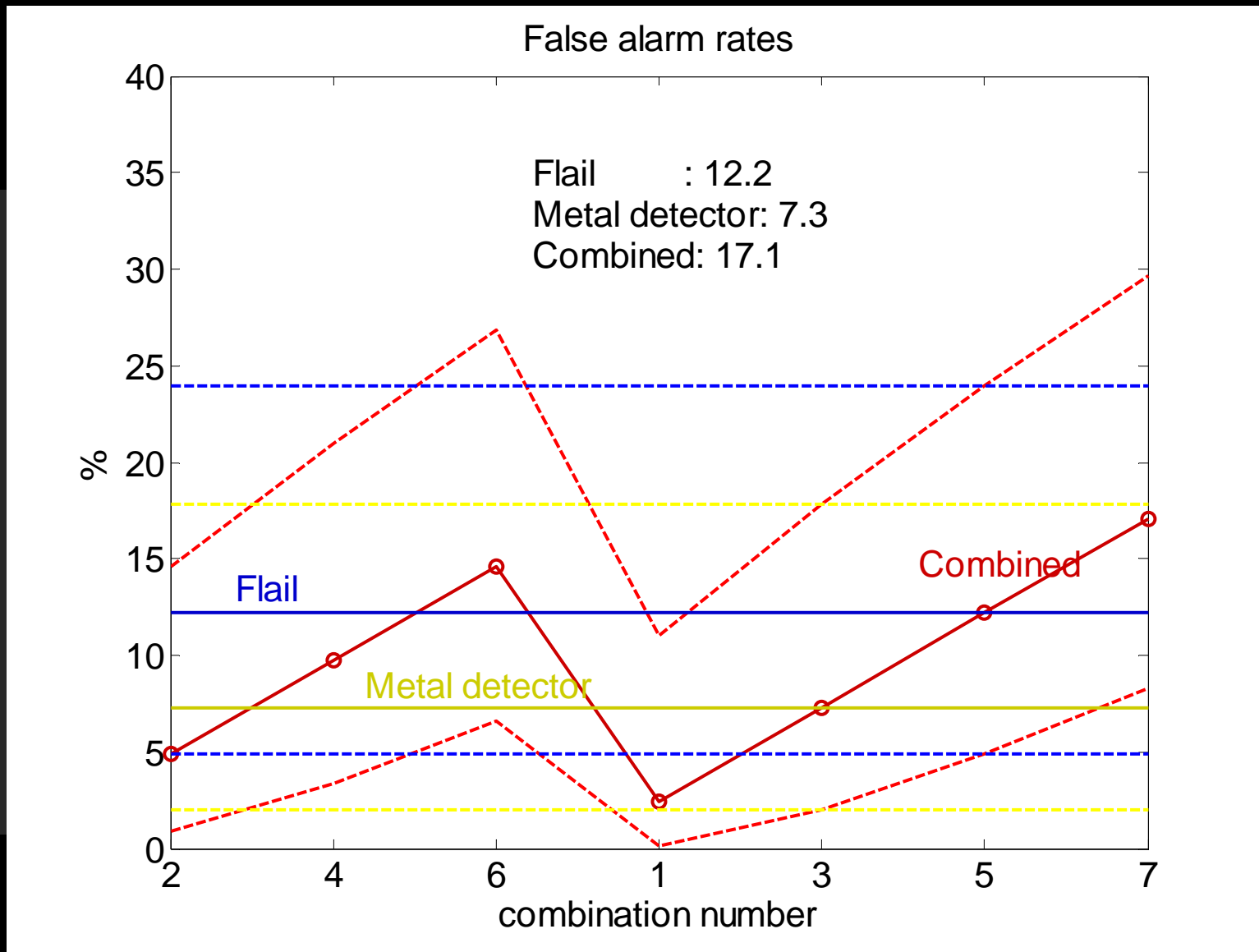


## Confidence for false alarm rates

- Determined by deployed resolution
- Large resolution - many cells gives many possibilities to evaluate false alarm.
- In present case:  $64-23=41$  non-mine cells

Method1 (flail): [4.9%    12.2%    24.0%]









## Comparing methods

- Is the combined method better than any of the two original?
- Since methods are evaluated on **same data** a paired statistical McNemar with improved power is useful

Method1 (flail): 82.6% < 91.3% Combined 

Method2 (MD): 69.6% < 91.3% Combined 



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## They keys to a successful mine clearance system

- Use statistical learning which combines all available information in an optimal way
  - informal knowledge
  - data from design test phase
  - confounding parameters (environment, target, operational)
- Combine many different methods using statistical fusion

MineHunt System and HOSA concepts have been presented at NDRF summer conferences (98,99,01)



# DeFuse

## scientific objectives

- Obtain **general scientific knowledge** about the advantages of deploying a combined approach
- Eliminate confounding factors through **careful experimental design** and specific scientific hypotheses
- Test the general **scientific hypothesis is that there is little dependence** between missed detections in successive runs of the same or different methods
- To accept the hypothesis under **varying detection/clearance** probability levels
- To lay the foundation for new practices for mine action, but it is **not within scope** of the pilot project

Systems: ALIS dual sensor, MD, MDD, Hydrema flail



## Conclusions

danger  
map

clearance

update  
danger  
map

- Statistical decision theory and modeling is essential for optimal use of prior information and empirical evidence
- It is very hard to assess the necessary high performance which is required to have a tolerable risk of casualty
- The use of sequential information aggregation is promising for developing new hierarchical survey schemes (SOPs)
- Combination of methods is a promising avenue to overcome current problems

certify  
methods

DeFuse  
results

combine