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A hybrid model for optimal decisions within personal finance and retirement

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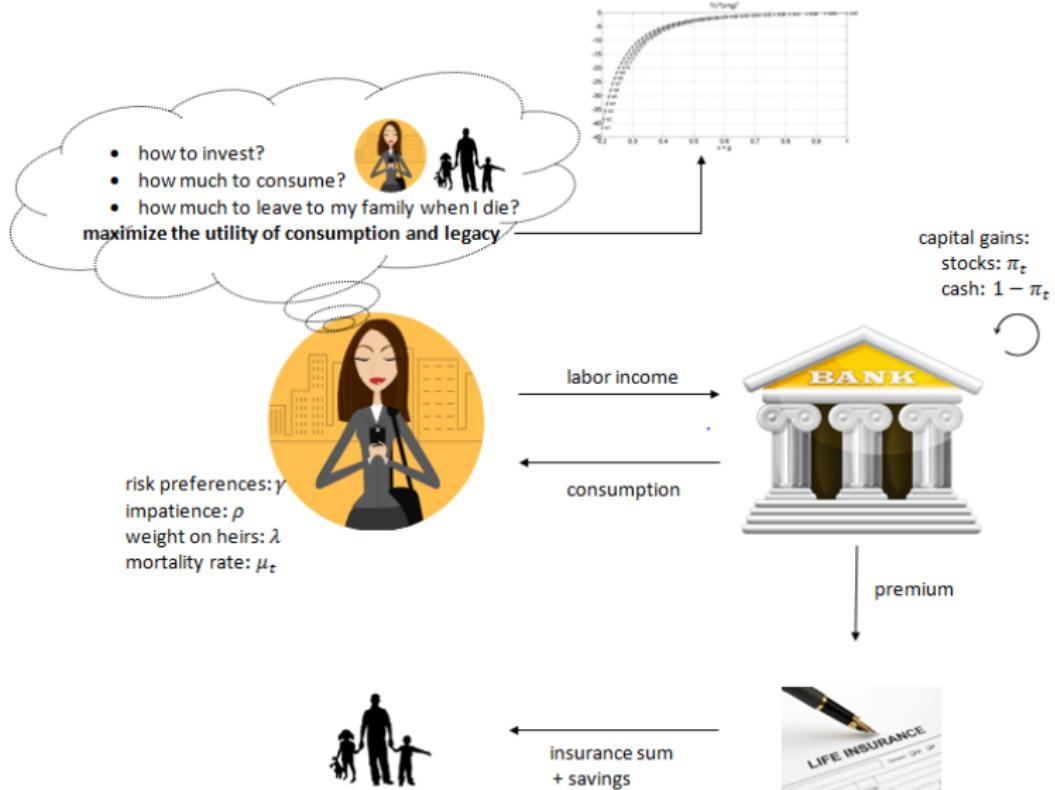
$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$

$\Delta \int_a^b \Theta^{\sqrt{17}} + \Omega \int \delta / e^{i\pi} =$

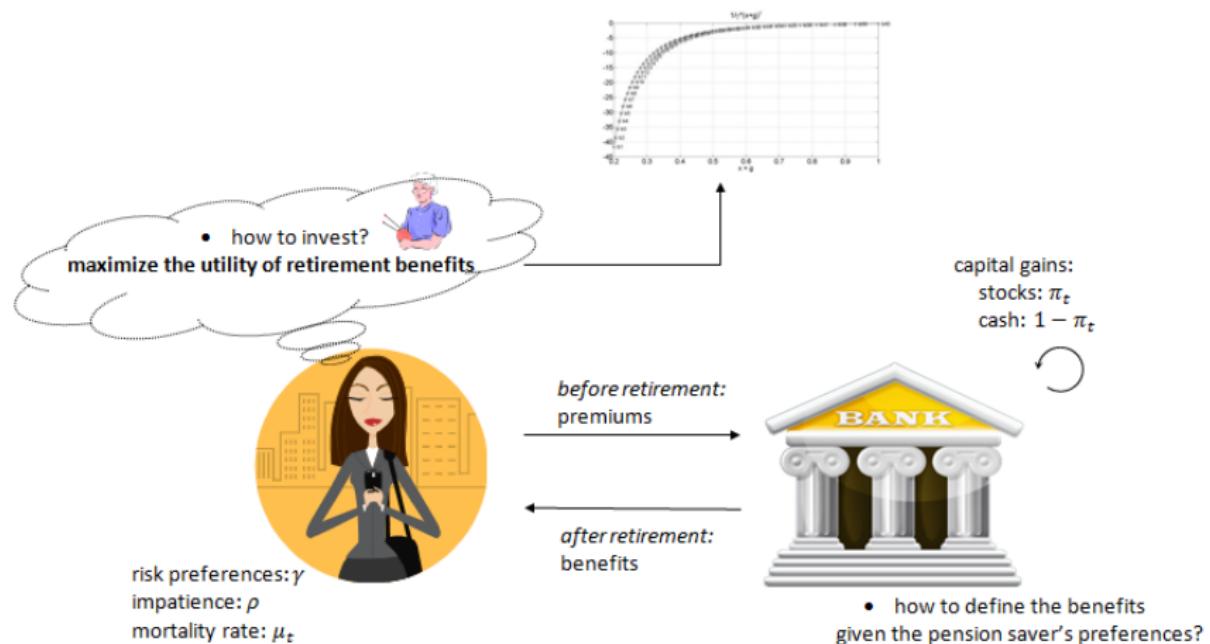
$\infty \oplus \square = \{2.718281828459045\}$

$\Sigma !$

case (A) - optimal investment, consumption and life insurance, [Richard, 1975]



case (B) - optimal investment with optimal annuities



stochastic optimal control - explicit solutions

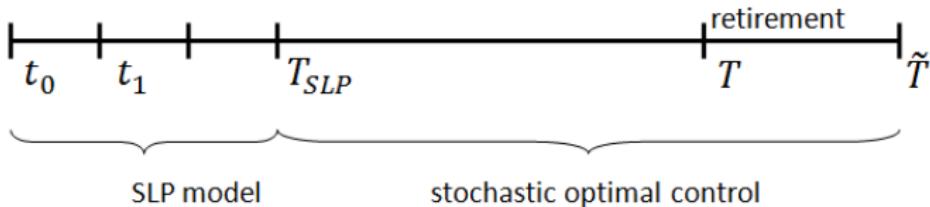
- ✓ ideal framework - produce an optimal policy that is easy to understand and implement
- ✗ explicit solution may not exist
- ✗ can't deal with details

stochastic (linear) programming (SLP)

- ✓ general purpose decision model with an objective function that can take a wide variety of forms
- ✓ can address realistic considerations, such as transaction costs
- ✓ can deal with details
- ✗ problem size grows quickly as a function of number of periods and scenarios
- ✗ challenge to select a representative set of scenarios for the model

Hybrid model

- first years decisions - multi-stage stochastic linear programming (**SLP**)
- decisions for the long steady period - **stochastic optimal control** (dynamic programming)



Dynamic programming I



Wealth dynamics

$$dX_t = (r + \pi_t(\alpha - r))X_t dt + \pi_t \sigma X_t dW_t + I_t dt - c_t dt - \mu_t^* I_t dt,$$
$$X_0 = x_0.$$

Maximize expected utility of consumption and bequest

$$V(t, x) = \sup_{\pi, c, I \in \mathcal{Q}[t, \tilde{T}]} E_{t,x} \left[\int_t^{\tilde{T}} e^{-\int_t^s \mu_\tau d\tau} \left(u(s, c) + \mu_s U(s, X_s + I_s) \right) ds \right],$$

with the utility functions:

$$u(c, t) = \frac{1}{\gamma} w^{1-\gamma}(t) c^\gamma = \frac{1}{\gamma} e^{-\rho t} c^\gamma, \quad U(x) = \frac{1}{\gamma} v^{1-\gamma}(t) x^\gamma = \frac{1}{\gamma} \lambda^{-\gamma} e^{-\rho t} x^\gamma,$$

$1 - \gamma$ - risk aversion, ρ - impatience factor, λ - weight on bequest, μ_t - mortality rate, μ_t^* - pricing mortality rate.

Dynamic programming II

Solution (optimal value function)

$$V(t, x) = \frac{1}{\gamma} f^{1-\gamma}(t) (x + g(t))^\gamma,$$

$$f(t) = \int_t^{\tilde{T}} e^{-\frac{1}{1-\gamma} \int_t^s (\mu_\tau - \gamma(\mu_\tau^* + \varphi)) d\tau} \left[w(s) + \left(\frac{\mu_s}{(\mu_s^*)^\gamma} \right)^{1/(1-\gamma)} v(s) \right] ds,$$

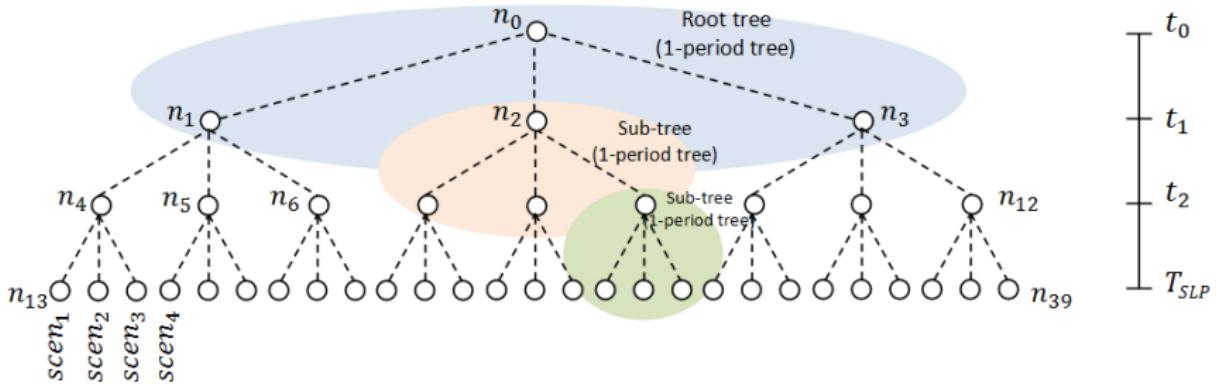
$$g(t) = \int_t^{\tilde{T}} e^{-\int_t^s (r + \mu_\tau^*) d\tau} l(s) ds.$$

The optimal controls

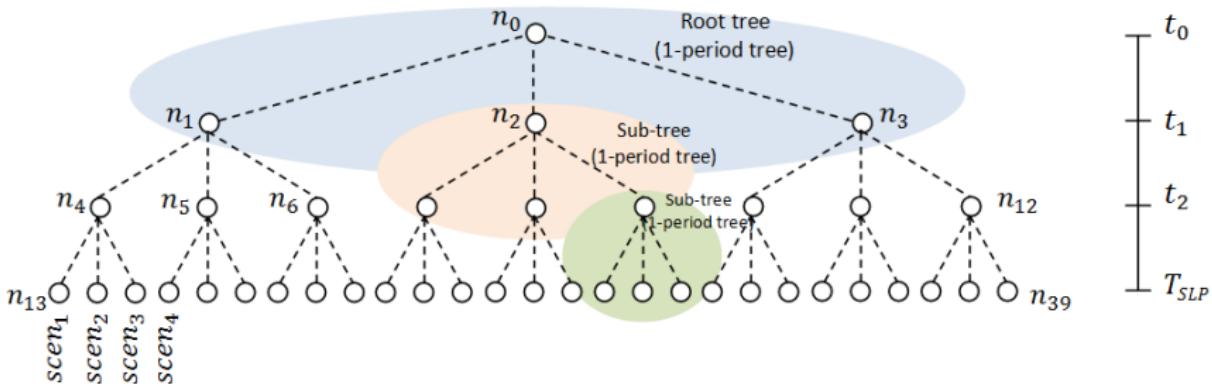
$$\pi_t^* = \frac{\alpha - r}{\sigma^2(1-\gamma)} \frac{X_t + g(t)}{X_t}, \quad c_t^* = \frac{w(t)}{f(t)} (X_t + g(t)),$$

$$l_t^* = \left(\frac{\mu_t}{\mu_t^*} \right)^{1/(1-\gamma)} \frac{v(t)}{f(t)} (X_t + g(t)) - X_t.$$

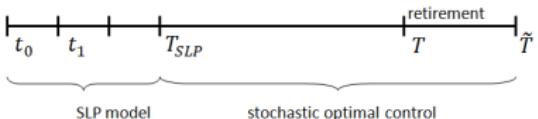
SLP model - objective



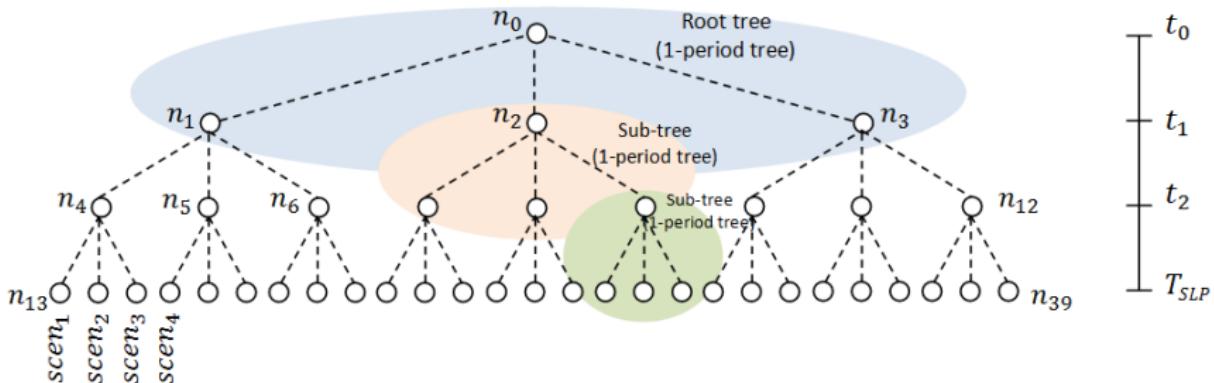
SLP model - objective



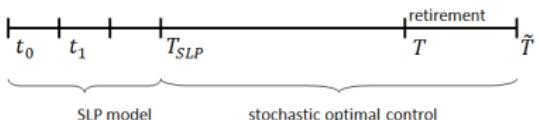
$$\begin{aligned}
 & \sum_{s=t_0}^{T_{SLP}-1} \sum_{n_{\nu(s)}} Pr_{n_{\nu(s)}} \cdot e^{-\int_{t_0}^s \mu_{\tau} d\tau} \left[u(s, \tilde{C}_{n_{\nu(s)}}) + \mu_s U(s, \sum_{i=1}^N \tilde{X}_{n_{\nu(s)}}^i + \tilde{I}_{n_{\nu(s)}}) \right] \\
 & + \sum_{n_{\nu(T_{SLP})}} Pr_{n_{\nu(T_{SLP})}} \cdot e^{-\int_{t_0}^{T_{SLP}} \mu_{\tau} d\tau} \cdot V \left(T_{SLP}, \sum_{i=1}^N \tilde{X}_{n_{\nu(T_{SLP})}}^i \right) \rightarrow \max
 \end{aligned}$$



SLP model - objective



$$\begin{aligned}
 & \sum_{s=t_0}^{T_{SLP}-1} \sum_{n_{\nu(s)}} Pr_{n_{\nu(s)}} \cdot e^{-\int_{t_0}^s \mu_{\tau} d\tau} \left[u(s, \tilde{C}_{n_{\nu(s)}}) + \mu_s U(s, \sum_{i=1}^N \tilde{X}_{n_{\nu(s)}}^i + \tilde{I}_{n_{\nu(s)}}) \right] \\
 & + \sum_{n_{\nu(T_{SLP})}} Pr_{n_{\nu(T_{SLP})}} \cdot e^{-\int_{t_0}^{T_{SLP}} \mu_{\tau} d\tau} \cdot V \left(T_{SLP}, \sum_{i=1}^N \tilde{X}_{n_{\nu(T_{SLP})}}^i \right) \rightarrow \max
 \end{aligned}$$



● Obs! linearize the objective

Constraints

budget equation, $t = t_0, \dots, T_{SLP} - 1$ and $\nu(t) = 1, \dots, K_t$,

$$\sum_{i=1}^N \tilde{P}_{n_\nu(t)}^i + \tilde{C}_{n_\nu(t)} + \mu_t^* \tilde{I}_{n_\nu(t)} = x_0 \mathbb{1}_{\{t=t_0\}} + \sum_{i=1}^N \tilde{S}_{n_\nu(t)}^i + I_t, \quad (1)$$

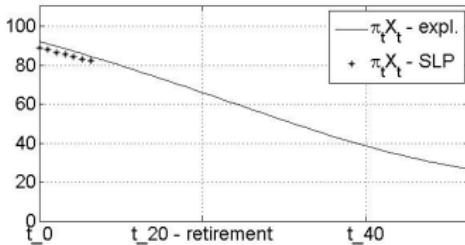
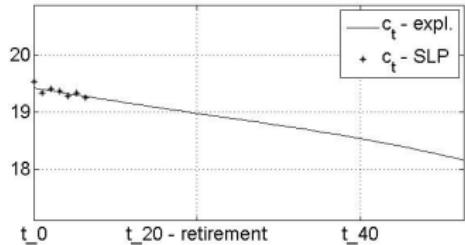
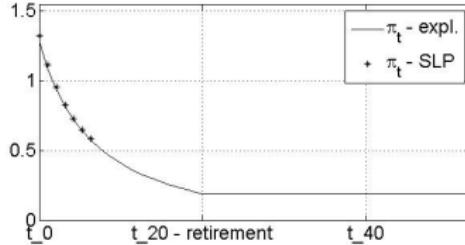
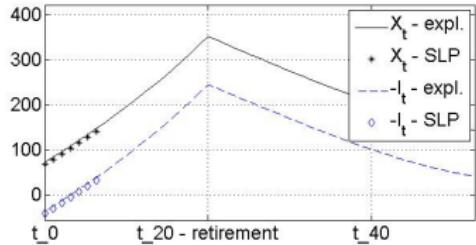
asset inventory balance, $t = t_0, \dots, T_{SLP}$, $\nu(t) = 1, \dots, K_t$, $i = 1, \dots, N$:

$$\tilde{X}_{n_\nu(t)}^i = (1 + R_{n_\nu(t)}^i) \tilde{X}_{n_\nu(t-1)}^i \mathbb{1}_{\{t > t_0\}} + \tilde{P}_{n_\nu(t)}^i \mathbb{1}_{\{t < T_{SLP}\}} - \tilde{S}_{n_\nu(t)}^i \mathbb{1}_{\{t < T_{SLP}\}}, \quad (2)$$

non-negativity, $t = t_0, \dots, T_{SLP} - 1$, $\nu(t) = 1, \dots, K_t$:

$$\tilde{C}_{n_\nu(t)} > 0, \quad \sum_{i=1}^N \tilde{X}_{n_\nu(t)}^i + \tilde{I}_{n_\nu(t)} > 0, \quad \tilde{P}_{n_\nu(t)}^i \geq 0, \quad \tilde{S}_{n_\nu(t)}^i \geq 0, \quad (3)$$

Results I - case (A)



- payments: $I_t = 27.000$ EUR, $x_0 = 60.000$ EUR,
- market: $N = 2$, $r = 0.02$, $\alpha = 0.04$, $\sigma = 0.2$,
- utility function: $\gamma = -3$, $\rho = 0.04$, $\lambda = 10$,
- age₀ = 45, age_T = 65,
- life uncertainty: $\theta = 0.0$, $\beta = 4.59364$,
 $\delta = 0.05032$,

- scenario tree: $T_{SLP} = 8$, $bf = 3$, number of trees = 10,
- linearization: $m = 40$,
- $bp_1^c = 0.7E[c_{T_{SLP}}^*]$, $bp_m^c = 2E[c_{T_{SLP}}^*]$,
- $bp_1^{x+g} = 0.5E[X_{T_{SLP}}^* + g_{T_{SLP}}]$, $bp_m^{x+g} = 2E[X_{T_{SLP}}^* + g_{T_{SLP}}]$,
- $bp_1^{x+ins} = 0.5E[X_{T_{SLP}}^* + I_{T_{SLP}}^*]$, $bp_m^{x+ins} = 1.5E[X_{T_{SLP}}^* + I_{T_{SLP}}^*]$.

Modified constraints

budget equation with transaction costs, $t = t_0, \dots, T_{SLP} - 1$, $\nu(t) = 1, \dots, K_t$

$$\sum_{i=1}^N \tilde{P}_{n_\nu(t)}^i (1 + q^i) + \tilde{C}_{n_\nu(t)} + \mu_t^* \tilde{I}_{n_\nu(t)} = x_0 \mathbb{1}_{\{t=t_0\}} + \sum_{i=1}^N \tilde{S}_{n_\nu(t)}^i (1 - q^i) + l_t, \quad (1')$$

asset inventory balance with taxes on capital gains, $t = t_0, \dots, T_{SLP}$, $\nu(t) = 1, \dots, K_t$,
 $i = 1, \dots, N$:

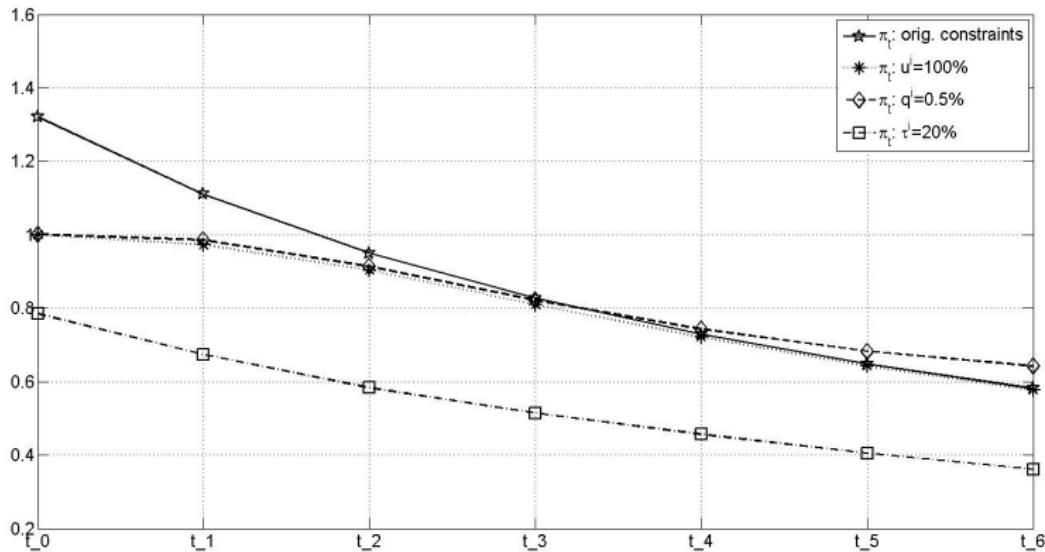
$$\tilde{X}_{n_\nu(t)}^i = (1 + \text{net } R_{n_\nu(t)}^i) \tilde{X}_{n_\nu(t-1)}^i \mathbb{1}_{\{t > t_0\}} + \tilde{P}_{n_\nu(t)}^i \mathbb{1}_{\{t < T_{SLP}\}} - \tilde{S}_{n_\nu(t)}^i \mathbb{1}_{\{t < T_{SLP}\}}, \quad (2')$$

limits on portfolio composition, $t = t_0, \dots, T_{SLP}$, $\nu(t) = 1, \dots, K_t$:

$$\tilde{X}_{n_\nu(t)}^i \geq d_i \sum_{i=1}^N \tilde{X}_{n_\nu(t)}^i, \quad \tilde{X}_{n_\nu(t)}^i \leq u_i \sum_{i=1}^N \tilde{X}_{n_\nu(t)}^i. \quad (4)$$

Results II - case (A) with modifications

Optimal investment: a) original constraints, b) limit on portfolio composition, $u = 100\%$, c) transaction costs, $q = 0.5\%$, d) taxes on capital gains, $\tau = 20\%$.



Bibliography

-  Bruhn, K. and Steffensen, M. (2011).
Household Consumption, Investment and Life Insurance.
Insurance: Mathematics and Economics, 48(3):315–325.
-  Geyer, A., Hanke, M., and Weissensteiner, A. (2009).
Life-cycle Asset Allocation and Consumption Using Stochastic Linear Programming.
The Journal of Computational Finance, 12(4):29–50.
-  Høyland, K. and Wallace, S. W. (2001).
Generating Scenario Trees for Multistage Decision Problems.
Management Science, 48(11):1512–1516.
-  Klaasen, P. (2002).
Comment on: "Generating Scenario Trees for Multistage Decision Problems".
Management Science, 47(2):295–307.
-  Richard, S. F. (1975).
Optimal Consumption, Portfolio and Life Insurance Rules for an Uncertain Lived Individual in a Continuous Time Model.
Journal of Financial Economics, 2(2):187–203.