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## STEADY STATE CRACK PROPAGATION IN LAYERED MATERIAL SYSTEMS DISPLAYING VISCO-PLASTIC BEHAVIOUR

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*Summary* The steady state fracture toughness of elastic visco-plastic materials is studied numerically, using both a conventional and a higher order model. Focus is on the combined effect of strain hardening, strain gradient hardening and strain rate hardening on cracking in layered material systems, and predictions for the crack tip shielding ratio is brought forward. Included is a novel procedure for extracting information on the rate-independent toughness without approaching this numerically cumbersome limit.

### INTRODUCTION

The fracture toughness of elastic-plastic materials is primarily governed by plastic dissipation, that decomposes into irrecoverable heat energy and cold work associated with the dislocation structure, which is responsible for the phenomenon of stable crack growth [1, 2]. At steady state cracking, under small-scale yielding, the remote regions follow the elastic solution, while an active plastic zone travels with the crack tip and shields it from the surroundings. Trailing behind, is a residual plastic strain wake where elastic unloading takes place, and where a secondary loading zone exists in terms of reverse plastic deformation so that the material remains in yielding close to the free fracture surface. The size and shape of these plastic regions, and thus the macroscopic material toughness, here reflected by the shielding ratio, are controlled by a wide range of parameters, which has been extensively studied in the literature [1, 2, 5, 6]. It is well known that properties such as strain hardening significantly influences on the dissipated energy associated to crack growth, thus any hardening effect should be expected to play a noticeable role on the macroscopic fracture toughness. The present work focuses on the combined effect of strain hardening, strain gradient hardening and strain rate hardening on cracking under stationary conditions in layered material systems. Main focus is on interface crack growth. For the numerical analysis, a steady state finite element formulation [1] is employed to integrate both a conventional and a higher order visco-plastic material model, while a variation of the Suo, Shih and Varias model [3] is used to facilitate a fracture criterion.

### VISCO-PLASTIC MODEL

A visco-plastic formulation, widely used for metals in the range of negligible to moderate viscous behaviour, is employed in both the conventional and the higher order model. The effective plastic strain rate,  $\dot{\varepsilon}_e^p$ , is here governed by the power law

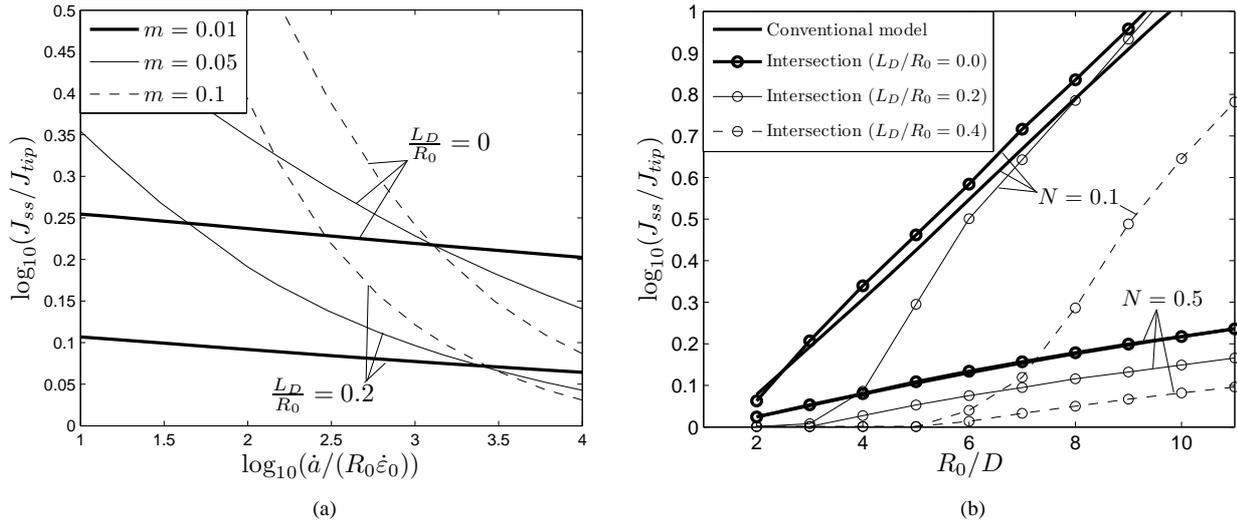
$$\dot{\varepsilon}_e^p = \dot{\varepsilon}_0 \left( \frac{\sigma}{g(\varepsilon_e^p)} \right)^{1/m} \quad \text{with} \quad g(\varepsilon_e^p) = \sigma_y \left( 1 + \frac{E\varepsilon_e^p}{\sigma_y} \right)^N, \quad (1)$$

where  $E$  is Young's modulus,  $\sigma_y$  is the initial yield stress,  $N$  is the hardening exponent,  $\dot{\varepsilon}_0$  is the reference strain rate and  $m$  is the strain rate hardening. The effective stress,  $\sigma$ , is the Mises stress in the conventional model, whereas a corresponding quadratic stress measure that includes the gradient effects is employed in the higher order model [4].

### RESULTS

Crack propagation in metals produces plastic dissipation when material in front for the crack tip enters the active plastic zone traveling with the tip, and later ends up being part of the residual plastic strain wake. Thus, the macroscopic work required to advance the crack is typically much larger than the work needed in the near tip fracture process. For rate sensitive materials, the extent of the plastic zone can be strongly dependent on the crack velocity, which in turn affects the macroscopic fracture toughness. In a recent study, Nielsen and Niordson [5] demonstrated this for an interface crack propagating at steady state by using a conventional elastic-viscoplastic material model. When compared to the rate-independent toughness ( $m \rightarrow 0$ ), a significant increase in fracture toughness occurs for slowly growing cracks (see also Fig. 1a), while a decrease was found for fast growing cracks. Similar predictions were later made in [6] for a strain gradient enhanced material, by adapting the model set-up to a higher order plasticity theory [4]. It is shown that elevated strain hardening and strain gradient hardening lowers the toughness. Both models revealed that the macroscopic toughness increases monotonically with increasing rate sensitivity at low crack velocity, while it decreases monotonically with increasing rate sensitivity at high crack velocity. This has to do with the time aspect of the stress build-up/relaxation in the vicinity of the crack tip when  $m > 0$ . For a slowly growing crack, the material has time to relax the stress field through plastic straining, whereby the macroscopic toughness increases. Vice versa, the material has limited time to relax the near tip stress field at high crack velocity, whereby the macroscopic toughness decreases and approaches the near tip toughness,  $J_{tip}$ . Bearing in mind that the shielding ratio is little affected by the crack velocity when  $m$  is small, a velocity for which the shielding ratio equals for two different  $m$ -values must naturally exist, when keeping all other

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**Figure 1.** (a) Steady state fracture toughness vs. crack velocity for a homogeneous material subject to Mode I loading. Here, showing the effect of strain gradients and rate sensitivity ( $N = 0.2$ , where  $R_0/D = 4$ ,  $R_0 = K^2/(3\pi\sigma_y^2)$  is the approximate plastic zone size). (b) Steady State fracture toughness vs. crack tip strength for a homogeneous material subject to Mode I loading with  $m \rightarrow 0$ .

material parameters constant. This is confirmed in Fig. 1a where the shielding ratio is shown as function of the crack tip velocity,  $\dot{a}/(R_0\dot{\epsilon}_0)$ . What is intriguing here is the fact that, for a given model set-up, curves of different  $m$ -values intersect at a single characteristic velocity for which the macroscopic toughness becomes independent of the strain rate hardening. The existence of this characteristic velocity can be argued according to the visco-plastic stress-relaxation mechanism discussed above, and it is found to exist independently of: the strain hardening,  $N$ , the quantity,  $R_0/D$ , the length parameter,  $L_D/R_0$ , the mode mixity  $\Psi = \tan^{-1}(K_{II}/K_I)$ , and the mismatch in plastic properties across the interface. Unfortunately no physical interpretation can be made of this characteristic velocity. However, Nielsen et al. [6] recently exploited this to study the rate-independent limit of a higher order model, for which the current visco-plastic formulation is numerically unstable when  $m \rightarrow 0$ . As discussed, the intersection point is independent of the material rate-sensitivity. Consequently it directly brings out the shielding ratio for the rate-independent limit ( $m = 0$  would be a horizontal line in Fig. 1a). By repeating calculations similar to those in Fig. 1a, this can be used to extract the variation of the rate-independent shielding ratio for parameters of interest for the higher order model. An example is given in Fig. 1b where each point is extracted from the intersection of two "toughness vs. velocity" curves with  $m = 0.1$  and  $m = 0.08$ , respectively. For comparison to the case of  $L_D = 0$ , predictions from the conventional model (with  $m = 0.001$ ) is included in Fig. 1b. Clearly, an impressive accuracy is obtained for the indirect approach to the rate-independent limit.

## CONCLUSIONS

The present study of steady state crack advance in visco-plastic materials reveals that additional hardening owing to plastic straining and strain gradients lower the toughness, whereas the effect of strain rate hardening is dependent of the crack tip velocity. Moreover, a characteristic velocity for which the toughness becomes independent of the rate sensitivity has been identified, and it is shown to facilitate a novel approach to extract the rate-independent response of metals, characterized by the current visco-plastic formulation. In addition, this approach is believed to apply to a much wider range of problems.

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