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Published in:  
Acoustical Society of America. Journal

Link to article, DOI:  
10.1121/1.1318900

Publication date:  
2000

Document Version  
Publisher's PDF, also known as Version of record

Citation (APA):  
Measurements of anisotropic sound propagation in glass wool

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(Received 30 November 1999; revised 9 August 2000; accepted 24 August 2000)

The attenuation coefficient and phase velocity of plane sound waves propagating in three perpendicular directions in glass wool were measured in the frequency range 50–10 000 Hz. For glass wool of mass density 14 kg/m$^3$ at the frequency 1000 Hz, the attenuation constant for propagation perpendicular to the glass wool sheets was 75 dB/m, and for propagation parallel with the sheets 57 dB/m. For mass density 30 kg/m$^3$, the corresponding numbers were 140 and 100 dB/m. The measured values were compared with calculated ones taking into account the movements of the fiber skeleton. The calculations need the elastic moduli, which were measured. For density 14 kg/m$^3$ and deformation perpendicular to the glass wool sheets, the static modulus was 2.0 kPa, and for parallel deformation 120 kPa. © 2000 Acoustical Society of America. [S0001-4966(00)05611-3]

PACS numbers: 43.58.Vb, 43.20.Jr, 43.55.Ev, 43.35.Mr

I. INTRODUCTION

Sound propagation in fiber materials such as glass wool has been studied intensely, but most studies have been done with plane waves propagating in only one direction with respect to the fiber orientation. Kirkby and Cummings$^1$ reported measurements of propagation constants and characteristic impedance for basalt wool and suggested methods of extrapolation to low frequencies, where the popular formulas of Delany and Bazley$^2$ are less reliable. Tarnow$^3$ measured the propagation constants and characteristic impedance of glass wool for one direction of sound propagation. Lambert$^4$ presented dynamic resistivity data for Kevlar fiber material, where a resonance at 850 Hz was found. Voronina$^5$ gave data for glass, mineral, and basalt wool. These authors used waves propagating in one direction only.

Bokor$^6$ reported measurements of sound attenuation and phase velocity in directions parallel with and perpendicular to fiberglass sheets for frequencies 400–1000 Hz. At 1000 Hz the parallel attenuation coefficient divided by the perpendicular one was 0.59, and the corresponding ratio between the parallel deformation 120 kPa and deformation perpendicular to the glass wool sheets, the static modulus was 2.0 kPa, and for parallel deformation 120 kPa. The corresponding numbers for mass density 30 kg/m$^3$ were 16 and 390 kPa. © 2000 Acoustical Society of America. [S0001-4966(00)05611-3]

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II. CALCULATION OF ATTENUATION COEFFICIENT AND PHASE VELOCITY

Delany and Bazley$^2$ have developed formulas that can be used to compute the attenuation coefficient and phase velocity as a function of frequency. The derivation of these formulas was purely empirical, and they will not be used in the present paper, where we seek to identify the physical mechanism responsible for sound attenuation. The Delany and Bazley formulas do not provide for movements of the fiber skeleton and are not accurate at low frequencies.$^1$

Fiber movement is significant, and was accounted for using a method developed previously.$^8$ The fiber skeleton can carry a stress wave that is coupled to the acoustic pressure wave. The air moves, and the viscous drag on the fibers makes them move. The complex pressure $p$ of a monochromatic plane wave propagating through a layer of glass wool of thickness $x$ can than be computed from

$$p = A e^{ik(x + F/2x)}, \tag{1}$$

where $A$ is an undetermined constant, which value is not used, as only rations of pressures for different values of $x$ will be considered. There are two complex wave numbers $k_1$ and $k_2$, the first of which is mainly associated with the pressure wave in the air, and the second is mainly associated with the stress wave in the fiber skeleton. $F$ is a complex coupling factor given below.

The wave numbers can be computed from the following determinant equation:

$$\begin{vmatrix}
-\omega^2(\mu - d\rho) + k^2 c_{nn} - i\omega R', & k^2 dK + i\omega R' \\
k^2 dK + i\omega R', & -\omega^2\rho + k^2 K - i\omega R'
\end{vmatrix} = 0, \tag{2}
$$

where $\omega$ is the angular frequency, $\mu$ is the mass density of glass wool, $d$ is the volume density of fibers ($d = 1 - \phi$, $\phi$ is the porosity), $\rho$ is the static mass density of air, $k$ is the unknown wave vector sought, $c_{nn}$ is the elastic modulus of the fiber skeleton for the direction of wave propagation considered. $R'$ is a factor that takes care of the viscous drag of...
the air on the fibers, \( R' = R + i\omega \rho \), where \( R \) is the complex resistivity to air flow through fixed fibers, \( R = -i\omega \rho_x \), and \( \rho_x \) is the complex air mass density function, which is often used. The complex resistivity is preferred, because it is nearly constant and real for frequencies from d.c. to 1000 Hz. \( K \) is the bulk modulus of air, \( K = 1/\phi C \), where \( C \) is the complex compressibility of air itself.

The wave in the glass wool is excited by a plane wave in air that hits the plane surface of glass wool head-on at \( x = 0 \). \( F \) was calculated from the boundary condition, zero stress, on the interface between air and glass wool, \( x = 0 \). The details are described in the author’s paper.\(^8\) One has

\[
F = \frac{(\omega R')^2}{(-\mu \omega^2 + c_{nn} k_1^2 - i\omega R')(-\rho \omega^2 + k_1^2 K - i\omega R')}. \tag{3}
\]

To compute the sound pressure one has to know the complex air compressibility and resistivity of the glass wool with fixed fibers. Formulas for these were given by Attenborough.\(^9\) They were derived by assuming that the glass wool consists of a solid with circular holes. The microstructure of real glass wool is far from this, and, therefore, they are not used.

Allard\(^10\) proposed formulas for the compressibility and resistivity based on work by Johnson et al.,\(^11\) who suggested a formula for interpolating the complex resistivity between static flow and very high frequencies. This formula depends on a parameter, the viscous length, defined from the very high frequency resistivity. It can be shown by direct computation that the suggestion of Johnson is not valid for a material consisting of parallel fibers. The viscous length can be computed for this model, and the complex resistivity can then be computed from Johnson’s formula. But at frequencies about 2000 Hz, the real value of the resistivity computed from Johnson’s formula becomes far smaller than the values computed by finite elements. This means that the Johnson procedure can not be used to compute attenuation of the glass wool samples with the densities considered in this paper; therefore, Allard’s procedure is not useful for these glass wool samples.

We use the formulas proposed by Wilson\(^12\) because they are simple, and do not make any assumption on the microstructure of the glass wool.

The air compressibility was calculated by Eq. (24) of Wilson,\(^12\)

\[
C = \frac{1}{\gamma P} \left[ 1 + \frac{\gamma - 1}{(1 - i\omega \tau_c)^{1/2}} \right]. \tag{4}
\]

where \( \gamma = 1.40 \) for atmospheric air, \( P \) static air pressure, and \( \tau_c \) is a time constant, that was found in the measured compressibility in Ref. 13.

The resistivity can be calculated by Eq. (23) of Wilson\(^\text{12}\)

\[
R = \frac{-i\omega \rho}{1 - (1 - i\omega \tau)^{1/2}}. \tag{5}
\]

where \( \rho \) is the static mass density of air (1.20 kg/m\(^3\) at 1 atm and temperature 20 °C). \( \tau \) is a time constant, which in this paper was found by fitting of the calculated attenuations at high frequencies to the experimental ones. The resistivity calculated from Eq. (5) approaches the value \( 2\rho/\tau \) asymptotically at low frequencies.

III. MEASUREMENT METHOD AND RESULTS

Measurements were done in an anechoic room. Six glass wool slabs each measuring 10×60×90 cm were placed in a rectangular chest, made of 22-mm-thick chipboard and measuring 60×60×90 cm inside. There were no air spaces between the different sheets of glass wool. One side of the chest was open facing a loudspeaker 1.7 m away as shown in Fig. 1. In this way the glass wool pile was exposed to approximately plane sound waves from the loudspeaker. Three different sides of the chest could be opened, so sound waves in three different directions could be sent into the pile of glass wool. The sound pressure was measured with one microphone placed at the open surface of the glass wool and a second inside the glass wool. The sound pressures and their phases were recorded. The attenuation coefficient was found from the attenuation of the sound pressure in dB divided by the thickness of the glass wool between microphones. The phase velocity \( c \) was found from the measured phase shift \( \phi \) and the distance between the microphones \( x \). Thus \( c = \omega x/\phi \).

![Image 1](https://example.com/image1)

**FIG. 1.** The measurement setup in the anechoic room. Six slabs of glass wool each with dimension 10×60×90 cm were placed in the chest. The distance from chest to loudspeaker was 170 cm.

![Image 2](https://example.com/image2)

**FIG. 2.** Attenuation coefficient for glass wool of density 14 kg/m\(^3\). The points marked ‘‘x’’ are the measured value for propagation in the X-direction. The points marked ‘‘O’’ are for the Y-direction, and ‘‘a’’ is for the Z-direction. The curves were calculated. The full line is for the X-direction, the dashed for Y-direction, and the dotted for Z-direction.

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\]

where \( \gamma = 1.40 \) for atmospheric air, \( P \) static air pressure, and \( \tau_c \) is a time constant, that was found from the measured compressibility in Ref. 13.

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The glass wool is manufactured as slabs with dimension 10×60×90 cm. The direction of the 10-cm edge is called the X-direction, the Y-direction is the direction of the 60-cm edge, and the Z-direction is in the direction of the 90-cm edge. The glass fibers are mostly oriented in the YZ-plane, and their directions approximately randomly distributed, but the elastic moduli data seem to show that they are not completely randomly distributed. Measurements were done on two types of glass wool, having density 14 and 30 kg/m³. Both types have a fiber diameter of 6.8 μm.

Figure 2 shows the result of measurements of attenuation coefficient of sound waves in glass wool of density 14 kg/m³. Sound pressure was measured at the free surface and in the glass wool 30 cm from the free surface. The points on Fig. 2 for propagation in the X-direction are marked ‘‘x.’’ Over most of the frequency range, the attenuation coefficient is highest in this direction. The measurement uncertainty of the attenuation coefficient is 1.6 dB/m.

To compute the curves one needs experimental data. Tortuosity and porosity are assumed to be 1, because the most dense glass wool, 30 kg/m³, has a relative fiber volume density of 0.016.

The elastic moduli of the glass wool samples were measured by a static method described in the author’s paper. The measured elastic moduli are presented in Table I. The glass wool is much softer in the X-direction than in the other directions.

The compressibility was computed from Eq. (4) with the time constant 0.50 ms, which was found from earlier measurements of the compressibility. These measurements were done on glass wool samples with the same fiber diameters but a little higher density, 16 and 40 kg/m³, where the samples studied here have 14 and 30 kg/m³. Measurements of compressibility are very difficult, and the data are very scattered. Figure 3 shows the experimental data normalized by division with the isentropic compressibility. The full line curve was computed with time constants 0.50 ms. From the scattering of the data one can estimate the accuracy of the time constants to 20%. Fortunately the computation of attenuation of sound wave is not very sensitive to the value of these time constants. The time constant for the two glass wool samples is given in Table II.

The resistivity was computed from Eq. (5). The values of the time constants were found by fitting graphically the curves to attenuation data at high frequencies. The time constants are given in Table II. The attenuation is sensitive to the value of the time constant in contrast to the phase velocity, which is less sensitive.

The calculated attenuation coefficient shown in Fig. 2 was computed from Eq. (1) in the same way as it was measured. The value of the constant A in Eq. (1) has no influence on the attenuation per meter, and it was set to 1. The complex sound pressure was computed at the surface of the glass

<table>
<thead>
<tr>
<th>Direction</th>
<th>Density 14 kg/m³</th>
<th>Density 30 kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>2.0 kPa</td>
<td>16 kPa</td>
</tr>
<tr>
<td>Y</td>
<td>120 kPa</td>
<td>390 kPa</td>
</tr>
<tr>
<td>Z</td>
<td>150 kPa</td>
<td>270 kPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density</th>
<th>Compressibility</th>
<th>Resistivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density 14 kg/m³</td>
<td>Density 30 kg/m³</td>
</tr>
<tr>
<td>X</td>
<td>0.36 ms</td>
<td>0.16 ms</td>
</tr>
<tr>
<td>Y</td>
<td>0.55 ms</td>
<td>0.21 ms</td>
</tr>
<tr>
<td>Z</td>
<td>0.80 ms</td>
<td>0.23 ms</td>
</tr>
</tbody>
</table>

FIG. 3. The complex compressibility of air between fibers. The data are normalized by division with the isentropic compressibility. The ‘‘x’’ points are for density 16 kg/m³, and the full line is computed by Eq. (4) with time constant 0.50 ms. The ‘‘○’’ points and the dashed line are for density 40 kg/m³ with time constant 0.30 ms.

FIG. 4. Phase velocity for glass wool of density 14 kg/m³. The points marked ‘‘x’’ are the measured value for propagation in the X-direction. The points marked ‘‘○’’ are for the Y-direction, and ‘‘△’’ is for the Z-direction. Measurement uncertainty is shown as vertical error bars. The curves were calculated. The full line is for the X-direction, the dashed for Y-direction, and the dotted for Z-direction.
wool \((x = 0)\) and 30 cm below the surface \((x = 30 \text{ cm})\). From the two amplitudes, the sound attenuation in dB was found and divided by 30 cm to give the attenuation coefficient.

Due to the two wave numbers in Eq. (1) the attenuation coefficient, which is the attenuation per length, is not strictly a constant but depends on the distance in the glass wool. However, the attenuation coefficient was also measured for thicknesses 10, 20 cm, and no dependence on thickness was detected. This was supported by calculations which showed deviations small compared to the experimental scattering of data. This means that for thicknesses 0–30 cm, one can regard the attenuation coefficient as a constant in the frequency range 50–10 000 Hz. It is tempting to neglect the movement of the fiber skeleton, but this will give a high attenuation coefficient in the \(X\)-direction at frequencies about 100 Hz.

At frequencies lower than 100 Hz, the measured points deviate from the calculated curves, because the sound pressure wave, which is reflected from the rear of the glass wool pile, interferes with the direct wave. This is not so at higher frequencies, because the reflected wave is much attenuated before it reaches the microphone inside the glass wool.

Figure 4 shows the phase velocity for glass wool of density 14 kg/m\(^3\), measured with a glass wool layer of thickness 30 cm. The points marked \(\cdot\) are for propagation in the \(X\)-direction. The curves were calculated from the same time constants that were used for calculating the attenuation. At frequencies lower than 100 Hz, the measured phase velocity is smaller than the calculated curve. This is due to reflection of waves from the rear. For propagation in the \(Y\) and \(Z\)-direction at low frequencies, the measurement uncertainty of the phase angles gives a great uncertainty of the phase velocity, because the phase shift is small. The uncertainty is shown as vertical error bars on the graph. The high phase velocity for the \(Z\)-direction (triangular point) in the frequency interval 1000–10 000 Hz is difficult to explain, perhaps the glass wool is not homogenous on the scale of the wavelength. The phase velocity was also measured and computed for thicknesses 10, 20 cm. No dependence of phase velocity on thickness was found.

Figure 5 shows the measured attenuation coefficients for glass wool of density 30 kg/m\(^3\), measured with 10-cm-thickness of glass wool. This thickness was chosen because a thicker layer would attenuate the high frequency wave too much. The measurement uncertainty is 5 dB/m. The attenuation was also measured for thicknesses 20, 30 cm, and it was found to be constant.

Figure 6 shows the phase velocity for glass wool of density 30 kg/m\(^3\), measured with 10-cm-thickness of glass wool. Measurement was also done for thicknesses 20 and 30 cm, and no dependence of phase velocity on thickness was found.

IV. CONCLUSION

The attenuation coefficients and phase velocities for glass wool of densities 14 and 30 kg/m\(^3\) were measured. Attenuation is highest for waves propagating perpendicular to the fibers, the \(X\)-direction. The acoustic properties perpendicular to the \(X\)-direction are approximately isotropic in the frequency range 0–3000 Hz. For density 14 kg/m\(^3\) the attenuation coefficient at 1000 Hz is 75 dB/m in the \(X\)-direction and 57 dB/m perpendicular to this direction. The elastic moduli was 2.0 and 120 kPa in the two respective directions.