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Papaefthymiou, George; Pinson, Pierre

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Modeling of spatial dependence in wind power forecast uncertainty

George Papaefthymiou, *Member, IEEE,* and Pierre Pinson

***Abstract*— It is recognized today that short-term (up to 2-3 days ahead) probabilistic forecasts of wind power provide forecast users with a paramount information on the uncertainty of expected wind generation. When considering different areas covering a region, they are produced independently, and thus neglect the interdependence structure of prediction errors, induced by movement of meteorological fronts, or more generally by inertia of meteorological systems. This issue is addressed here by describing a method that permits to generate interdependent scenarios of wind generation for spatially distributed wind power production for specific look-ahead times. The approach is applied to the case of western Denmark split in 5 zones, for a total capacity of more than 2.1GW. The interest of the methodology for improving the resolution of probabilistic forecasts, for a range of decision-making problems, or simply for better understanding the characteristics of forecast uncertainty, is discussed.**

***Index Terms*— wind power, uncertainty, probabilistic forecasting, Monte-Carlo simulation, stochastic dependence, multivariate Normal, transformation, scenarios.**

I. Introduction

INCREASING the value of wind generation through the improvement of prediction systems' performance is one of the priorities in wind energy research needs for the coming years [1]. Even though today, most of the existing wind power prediction methods provide end-users with point forecasts [2], a large part of the research efforts is concentrated on developing methods for giving the complete information on expected generation, most commonly in the form of probabilistic forecasts of wind generation. Probabilistic predictions can be either derived from meteorological ensembles [3], based on physical considerations [4], or finally produced from one of the numerous statistical methods that have appeared in the literature, see [5]–[8] among others. If appropriately incorporated in decision-making methods, they permit to significantly increase the value of wind generation. Recent developments in that direction concentrate on e.g. dynamic reserve quantification [9] or on the design of optimal trading strategies [10].

Probabilistic forecasts are generated on a per look-ahead time basis and for a specific location. They provide information on the *marginal* uncertainty, in terms of approximation of the probability distribution of the prediction errors for each location and look-ahead time. They do not inform however on the stochastic relationship between these distributions, corresponding either to the temporal development of the prediction errors for a specific location or to the spatial dependence of prediction errors between different locations. This stochastic relationship relates to the interdependence structure between the prediction errors and may be significant for both prediction series at single locations as well as prediction errors between different locations. Information on the interdependence structure of prediction errors may be of particular importance for many decision-making processes. The case of temporal interdependence, which is crucial for operation of energy storage for instance, is dealt by the authors in [11].

The present paper concentrates on the case of spatial interdependence, which is paramount e.g. for the optimal management of power flows in the transmission/distribution networks. Spatial dependence of prediction errors may be induced by the inertia of meteorological systems. For instance, if local effects in a given area are not well predicted, it is unlikely that resulting forecasting error is correlated with that committed in another area located on the other side of a region considered. But, if the prediction error concerns the timing of a large front coming over the whole region, a significant correlation between prediction errors in the various areas would then to be expected. For the spatial modeling of forecast uncertainty, the methodology proposed here is inspired by the works on probabilistic forecasting in [8], those on modeling of stochastic generation in [12], and the method for scenario generation from probabilistic forecasts described in [11].

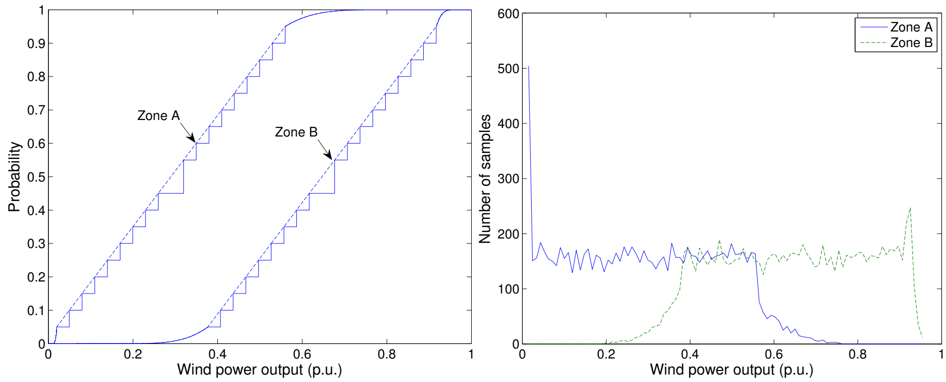
The interest and potential impact of spatial dependence are discussed in Section II. Then, the methodology for spatial modeling of forecast uncertainty is developed in Section III, with emphasis on the principles of Monte-Carlo simulation, nonparametric techniques in probabilistic forecasting and finally on the modeling of interdependence structures. In Section IV, the methodology is applied to the case of forecasting wind generation for western Denmark, split in 5 different zones, over a period of almost 2 years. A static analysis permits to discuss the spatial interdependence structure of prediction errors, and potential explanatory variables or influencing meteorological regimes. In parallel, the methodology for scenario generation of spatially distributed wind capacities is applied to the whole western Denmark. The interest of this approach for improving probabilistic forecasting and its use as input to decision-making methods is discussed. Concluding remarks in Section V end the paper with perspectives on applications and future developments.

II. Interest and potential impact of spatial dependence

Positive spatial dependence between prediction errors from different locations in a specific area will have a major impact

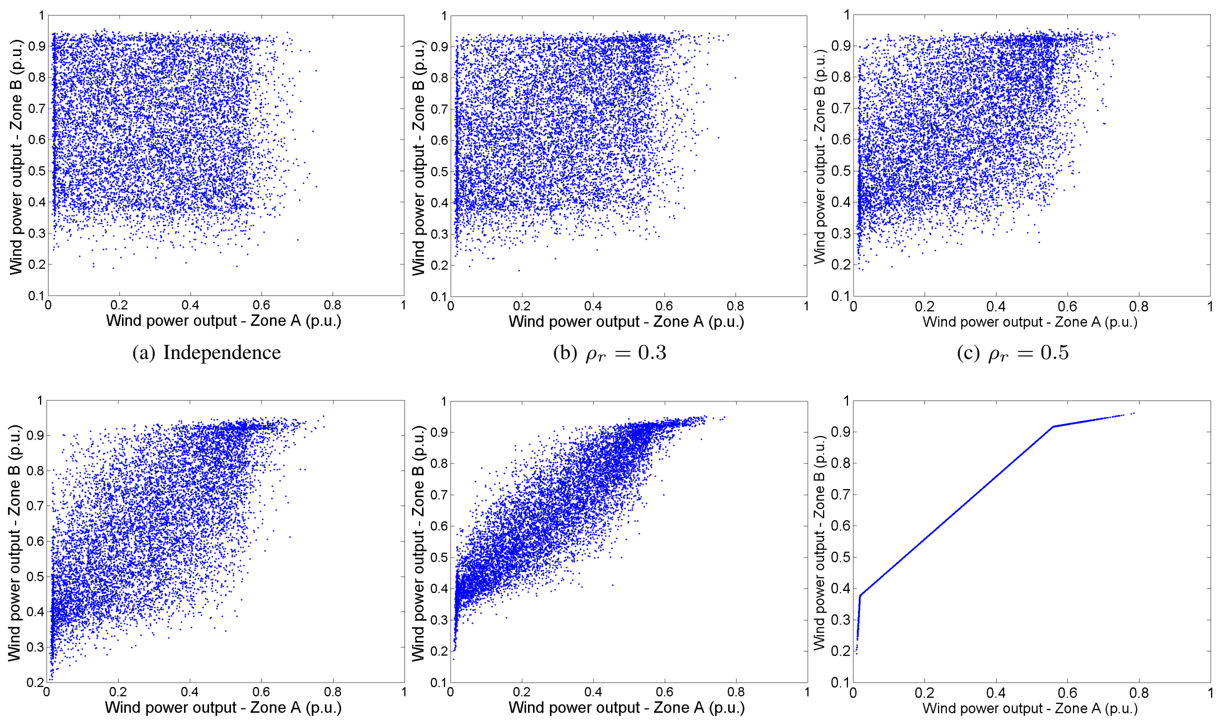
G. Papaefthymiou is with the Electrical Power Systems Laboratory (EPS) of the Delft University of Technology, the Netherlands.

P. Pinson is with the Informatics and Mathematical Modeling department of the Technical University of Denmark.



(a) cdf (b) pdf

Fig. 1: Probabilistic forecasts for two zones for lead time $t + k$.



(a) Independence (b) $\rho_r = 0.3$ (c) $\rho_r = 0.5$

(d) $\rho_r = 0.7$ (e) $\rho_r = 0.9$ (f) $\rho_r = 1$

Fig. 2: Scatter plots for the wind power output uncertainty between zones A and B for different dependence structures.

on the aggregate prediction uncertainty for the whole area. We will illustrate this through a simple example concerning the prediction errors from 2 distinct zones in an area for a specific hour ahead. In particular, in Fig. 1 the probabilistic forecasts issued at time t for lead time $t + k$ are presented. The predictive distributions are given by 19 quantile forecasts with exponential tails, as discussed in Section IV-A.

Probabilistic forecasts are limited to the information presented in Fig. 1. In order to estimate the aggregate prediction uncertainty from the two zones, information on the interdependence structure between the prediction errors is necessary. An infinite number of interdependence structures may correspond to the same predictive distributions; although the shape of the predictive distributions will not change, different interdependence structures will lead to different distributions for the aggregate forecast error. To illustrate this, in Fig. 2, the scatter plots for the two predictive distributions for different correlations are presented. We can see that by increasing the positive dependence from 0 (case of independence) to 1 (perfect dependence), the scatter plots are changed, although they are still consistent to the shape of the marginal predictive distributions. The higher the correlation, the more concentrated the scatter plots are. The case of perfect dependence corresponds to a perfect monotonic relationship, which results from the non-normality of the marginal distributions; if the predictive distributions were normals, this monotonic relationship would deteriorate to a linearity. For a thorough analysis on these issues, one should refer to [12].

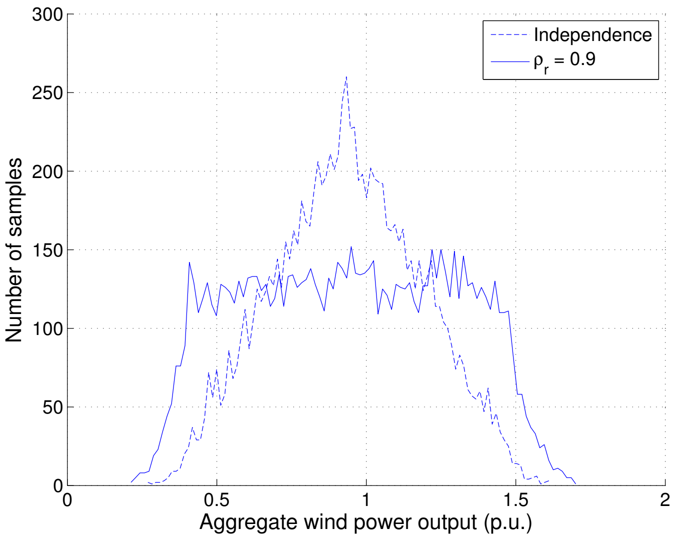


Fig. 3: Aggregate wind power output uncertainty.

Although the predictive distributions are kept unchanged, the different interdependence structures result in different aggregate forecast error distributions. In particular, as can be seen from the respective scatter plots, for the case of independence the occurrence of a specific prediction error in the one zone has no impact on the amplitude of the error observed in the other. In parallel, positive dependence induces a functional relationship between the errors in the two zones, meaning that when a large error is observed in the one zone, it is most probable that a large error will occur on the other also. Positive dependence will therefore lead to more extreme aggregate error distributions for the whole area, as a result of the concentration observed in the scatter plots towards monotonicity. In Fig. 3, the aggregate forecast error distribution for the cases of independence and $\rho = 0.9$ is presented. Although the predictive distributions for each zone are kept constant, the different interdependence structures yield different aggregate forecast error distributions. As discussed in [12], these distributions have the same mean value but different dispersion, which increases with increasing positive dependence.

Therefore, in order to evaluate the aggregate error from local uncertainty estimates, it is necessary to assess both the marginal distributions (in terms of probabilistic forecasts) and spatial interdependencies. For real-time applications one should be able to collect information separately for both these factors and employ suitable algorithms for modeling the aggregate forecast uncertainty. This forms the kernel of the approach proposed in this contribution.

III. Methodology for Spatial Modeling of Forecast Uncertainty

A. *Basic theory on Monte-Carlo sampling*

The modeling of forecast uncertainty between different locations leads to a multivariate uncertainty analysis problem. For the modeling of such problems, one should estimate the joint distribution over a number of dependent, non-normal random variables, each corresponding to a probabilistic forecast for a specific location and look-ahead time. The modeling of stochastic dependence has been a cornerstone in the research on multivariate uncertainty analysis in recent years [13], [14], [15]. The related research leads to a main approach, namely the splitting of the modeling effort in two separate tasks:

- *Marginals*: model the one-dimensional marginal distributions.
- *Stochastic dependence*: model the stochastic dependence structure between the inputs.

This splitting is performed by transforming the marginal distributions into ranks, corresponding to uniform distributions. The transition between the rank-uniform domain and the actual domain is performed by applying the cumulative distribution function (cdf) transformation separately to each random variable (r.v.). In particular, by definition for a r.v. X with an *invertible* cdf $F_X(x) = P(X \leq x)$, the r.v. $F_X(X)$ follows a uniform distribution on the interval $[0, 1]$. This relationship forms the basis for the sampling of any r.v. in Monte-Carlo simulation studies. For the sampling of a r.v. X with invertible cdf F_X, first a random realization u from a uniform r.v. U in $[0, 1]$ is generated and then the transformation $x = F_X^{-1}(u)$ is applied. In this case, the samples x follow the distribution F_X.

$$F_X(X) = U \Leftrightarrow F_X^{-1}(U) = X \tag{1}$$

Thus, by applying the cdf-transformation $F_X(X)$ we achieve the transition from the actual domain of each r.v. to the rank-uniform domain. The inverse-cdf transformation further ensures the transition back to the initial domain of the r.v. To ensure the validity of this double transition, the function F_X should be invertible, which is true, since by definition the cdf and therefore the inverse-cdf are increasing functions.

The significance of these increasing transformations lies in the property that their application does not affect the dependence structure. The reason for this is that for dependence modeling we are mainly interested in the relative ranking between the consequent samples of the random variables. By applying increasing transformations as the ones in equation (1), this ranking is not changed, therefore the interdependence structure remains unchanged. By applying these transformations we aim in getting all random variables in the same domain (rank-uniform domain or normal domain) The dependence modeling takes place in that common domain and then the random variables can be transformed back to their original marginals by the inverse-cdf transformation without any loss of information.

The approach presented in this paper uses proposes a transition to the Normal domain for dependence modeling. There, one can take advantage of the properties of the multivariate Normal distribution in order to generate a set of normal random variables correlated according to specific covariance matrix. The inverse-normal cdf transformation is further used for the transition back to the rank-uniform domain. This set of transformations can be summarized as follows:

$$F_X(X) = U \Leftrightarrow \Phi^{-1}(U) = N \Leftrightarrow \Phi(N) = U \Leftrightarrow F_X^{-1}(U) = X \tag{2}$$

where $\Phi(\cdot)$ is the *standard Normal cdf* and N a standard Normal r.v. For the case of modeling of spatial dependence in prediction uncertainty, three parts may be identified in the modeling process: *(i)* the retrieval of marginal distributions from probabilistic forecasts, *(ii)* the adaptive estimation of the covariance matrix and *(iii)* the Monte-Carlo sampling algorithm for the generation of scenarios of spatially correlated wind power production.

B. Marginals from probabilistic forecasts

Consider being at time t and aiming at having complete information on wind generation for lead time $t+k$. For that purpose, nonparametric probabilistic predictions, i.e. for which no restrictive assumption is made on the shape of predictive densities, may take the form of quantile, interval or density forecasts. If denoting by p_{t+k} the measured power value at time $t+k$, p_{t+k} can be seen as a realization of the r.v. P_{t+k}. Write f_{t+k} the probability density function of P_{t+k}, and let F_{t+k} be the related cdf. Provided that F_{t+k} is a strictly increasing function, the quantile $q_{t+k}^{(\alpha)}$ with proportion $\alpha \in [0,1]$ of the r.v. P_{t+k} is uniquely defined as the value x such that

$$\mathrm{P}(P_{t+k} < x) = \alpha, \quad \text{or} \quad q_{t+k}^{(\alpha)} = F_{t+k}^{-1}(\alpha) \tag{3}$$

A quantile forecast $\hat{q}_{t+k|t}^{(\alpha)}$ with nominal proportion α is an estimate of $q_{t+k}^{(\alpha)}$ produced at time t for lead time $t+k$, given the information set Ω_t at time t. Note that the commonly provided point forecast $\hat{p}_{t+k|t}$ (issued at time t for lead time $t+k$) corresponds to the conditional expectation of P_{t+k}, provided that the parameters of the model employed have been estimated with Least Squares (LS) estimation techniques.

For most decision-making processes, such as power system operation, a single quantile forecast is not sufficient for obtaining an optimal decision. Instead, it is necessary to have the whole information about the r.v. P_{t+k} for horizons ranging from few hours to several days ahead [10]. A nonparametric forecast $\hat{f}_{t+k|t}$ of the density function of the variable of interest at lead time $t+k$ can be produced by gathering a set of m quantile forecasts

$$\hat{f}_{t+k|t} = \{\hat{q}_{t+k|t}^{(\alpha_i)} \mid 0 \leq \alpha_1 < \ldots < \alpha_i < \ldots < \alpha_m \leq 1\} \tag{4}$$

that is, with chosen nominal proportions spread on the unit interval. Let $\hat{F}_{t+k|t}$ denote the cdf related to $\hat{f}_{t+k|t}$.

A requirement for nonparametric probabilistic forecasts is that the nominal probabilities, i.e. the nominal proportions of quantile forecasts, are respected in practice. That required property is commonly referred to as reliability. Consider for instance predictive distributions defined by quantiles whose nominal proportions are uniformly spread between 0 and 1. This property intuitively translates to saying that there is the same probability the measured power value at time $t+k$ falls in any of the interval defined by two neighboring quantile forecasts. Besides this requirement, it is highly desirable that probabilistic predictions provide forecast users with a situation-dependent assessment of the prediction uncertainty. The shape of predictive distributions should then vary depending on various external conditions. For the example of wind power forecasting, it is intuitively expected that predictive densities would have a different shape when predicted wind speed equals zero and when it is near cut-off speed. This desirable property of probabilistic forecasts is commonly referred to as their sharpness of resolution. For a more thorough discussion on these various aspects, see [16], [17].

1) Transformation through marginals: Consider a number of l zones over a region. Let us focus on a single look-ahead time k, and on the j^{th} of the l zones. As explained above, the predictive distributions $\{\hat{f}_{j,t+k|t}\}_t$ for that look-ahead time and for that zone are defined as reliable if the observed proportions for each of the quantiles correspond to the nominal ones [16]. In such a case and according to the transformations given by Eq. (1), the r.v. $Y_{j,k}$ whose realization $Y_{j,k}(t)$ at time t is defined by

$$Y_{j,k}(t) = \hat{F}_{j,t+k|t}(p_{j,t+k}), \quad \forall t \tag{5}$$

where $p_{j,t+k}$ is the measured power production in zone j at time $t+k$, is distributed uniform on the unit interval, i.e. $Y_{j,k} \sim \mathrm{U}[0,1]$. Thus, the r.v. $Y_{j,k}(t)$ will correspond to the distribution on the ranks of the forecast errors if and only if the forecasts are reliable. Note that in practice, a continuous cdf for each look-ahead time is obtained by fitting a smooth curve through the set of m predictive quantiles, as presented in Fig. 1a. Also, the predictive distributions $\{\hat{f}_{j,t+k|t}\}_t$ are different based on the prediction horizon considered, i.e. 6-, 24- or 43-hour ahead.

As discussed in Section III-A, the cdf-transformation enables the separation of the impact of the marginal distributions from the dependence structure, by dealing with ranks of the respective r.v. In order to measure the spatial correlation between the prediction errors, one should apply the transformation in Eq. (5) to the predictive distributions $\{\hat{f}_{j,t+k|t}\}_t$ for all look-ahead times and zones. By this, the transition to a common rank-uniform domain is achieved, where the effect of the shape of different marginals is diminished and the correlation can be measured. Based on this information, one can further estimate the covariance matrix adaptively, to use for scenario generation.

C. Adaptive estimation of the spatial covariance matrix

Given the uniform r.v. $Y_{j,k}$ for look-ahead time k, and for zone $j = 1, \ldots, l$, a straightforward way to obtain a standard Normal r.v. $X_{j,k} \sim \mathcal{N}(0,1)$ is to apply the inverse standard normal cdf $\Phi^{-1}(\cdot)$ transformation to every realization $Y_{j,k}(t)$, according to Eq. (2):

$$X_{j,k}(t) = \Phi^{-1}(Y_{j,k}(t)), \quad \forall t \tag{6}$$

Considering the transformed random variables $X_{j,k}$ it is assumed that the random vector $\mathbf{X}_k = (X_{1,k}\ X_{2,k}\ \ldots\ X_{l,k})$ follows a multivariate standard Normal distribution, $\mathbf{X}_k \sim \mathcal{N}(\mu_0, \Sigma_k)$, with the vector μ_0 of mean values being a vector of zeros. In addition, Σ_k is the spatial covariance matrix that contains the whole information about the covariances of the r.v. $X_{j,k}$, $j = 1, \ldots, l$. It has 1-values on its diagonal, since the diagonal elements give the variance of each of the random variables. Hereafter $\mathbf{X}_k^{(t)}$ denotes the realization of $\mathbf{X}_k$ at time t.

In the context of wind power forecasting applications, measurements are regularly collected and consequently used for updating the parameters of the prediction methods. A similar adaptive approach based on a recursive formulation can be applied for the adaptive estimation of the spatial covariance matrix Σ_t. This approach has the advantage of accomodating the long-term variations in the characteristics of the interdependence structure of forecast uncertainty between the various zones considered.

Write $\Sigma_{t,k}$ the spatial covariance matrix estimated from observations up to time t for the case of look-ahead time k. An unbiased estimate of $\Sigma_{t,k}$ is commonly given by

$$\Sigma_{t,k} = \frac{1}{t-1} \sum_{i=1}^{t} \mathbf{X}_k(i)\mathbf{X}_k(i)^\top \tag{7}$$

with $.^\top$ the transposition operator and $\mathbf{X}_k(t)$ the vector of observation of $\mathbf{X}_k$ at time t.

By applying an exponential forgetting scheme, one obtains the following formula permitting to update $\Sigma_{t-1,k}$ when new observations become available at time t:

$$\Sigma_{t,k} = \lambda\Sigma_{t-1,k} + (1-\lambda)\,\mathbf{X}_k(t)\mathbf{X}_k(t)^\top \tag{8}$$

where λ is the forgetting factor, $\lambda \in [0,1]$. The spatial covariance matrix is initialized by setting all its off-diagonal elements to 0 and its diagonal elements to 1. For more information on recursive estimation and exponential forgetting, one may refer to [18].

D. Scenario generation

Based on the estimate of the spatial covariance matrix $\Sigma_{t,k}$ at time t for spatially correlated prediction errors for a look-ahead time k and the predictive distributions $\hat{f}_{j,t+k|t}$ for each zone j, $j = 1, \ldots, l$ consistent wind power production scenarios may be generated. The procedure for obtaining a number d of scenarios for the l zones is as following:

- *(i)* one uses a multivariate Normal random number generator with zero mean and covariance matrix $\Sigma_{t,k}$ in order to have d realizations of the r.v. $\mathbf{X}_k$, with $\mathbf{X}_k \sim \mathcal{N}(\mu_0, \Sigma_{t,k})$. Denote by $\mathbf{X}_k^{(i)}$ the i^{th} of these d realizations;
- *(ii)* d realizations $Y_{j,k}^{(i)}$ of the uniform variable $Y_{j,k}^{(i)}$ are obtained by applying the standard normal cdf transformation Φ to each component of $\mathbf{X}_k^{(i)}$, according to Eq. (2):

$$Y_{j,k}^{(i)} \;=\; \Phi(X_{j,k}^{(i)}), \quad \forall j,k,i \tag{9}$$

- *(iii)* the scenarios of wind power production finally result from the application, for each look-ahead time k and for each zone j, of the inverse cdf $\hat{F}_{t+k|t}^{-1}$ to the d realizations $Y_{k,t}^{(i)}$ of $Y_{k,t}$ for that look-ahead time:

$$\hat{p}_{j,t+k|t}^{(i)} \;=\; \hat{F}_{j,t+k|t}^{-1}\left(Y_{j,k}^{(i)}\right), \quad \forall j,k,i \tag{10}$$

As discussed in Section III-A, the interdependence structure is preserved through these increasing transformations, therefore the generated wind power scenarios will have the same interdependence structure as the multivariate Normal $\mathbf{X}_k$.

IV. Application and results

A. Case-study: western Denmark

Western Denmark is one of the regions with the highest wind power penetration in the world ($> 20\%$). Energinet.dk, who is the Transmission System Operator (TSO) in Denmark, is responsible for the optimal planning of power flows and balancing for the whole Danish grid. Denmark is actually split in two, western Denmark being connected to the UCTE system and eastern Denmark to the Nordel one, without any potential energy exchange between these 2 areas. Wind power measurements from western Denmark were used for this case-study. Western Denmark concentrates most of the Danish wind power, with a total installed capacity of more than 2.1GW at the end of 2007. For management purposes, Energinet.dk has split western Denmark into 17 areas. For confidentiality reasons, the area corresponding to the Horns Rev wind farm is omitted. Another area, which consists of a small island in the North East of Jutland, is also disregarded due to the very limited amount of generation and suspicious quality of data. The power measurements for the remaining 15 areas cover a period of almost 2 years, from beginning of January 2006 until mid-November 2007. In order to reduce the size of the case-study, the areas have been grouped as illustrated in Fig. 4, yielding a total number of 5 zones.

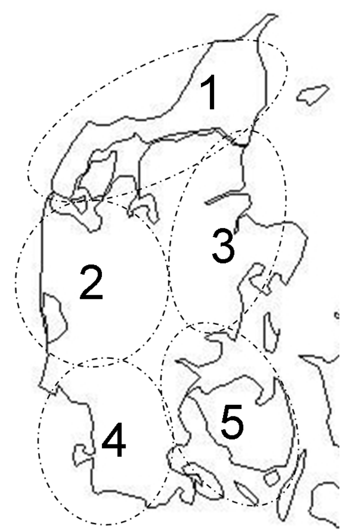


Fig. 4: Test case: western Denmark split into 5 zones.

The point predictions used result from the application of the WPPT method [19], which uses meteorological predictions of wind speed and direction as input, as well as historical measurements of power production. These point predictions have an hourly resolution up to 43-hour ahead, and are updated every hour. All predictions and measures are normalized by the installed capacity of the zone considered. The dataset includes in total 15888 point prediction series.

The nonparametric probabilistic forecasts are produced with adapted resampling [8]. They are generated for each of the 5 zones, and for western Denmark as a whole. Predictive distributions are given by 18 quantile forecasts whose nominal proportions range from 0.05 to 0.95% by 0.05 increments (except for the median), as presented in Fig. 1. Following the recommendations about the set up of the adapted resampling method in [8], the range of power values is split in 5 zones, the sample size set to 300, and the number of resampling replications to 100. From a probabilistic point of view, since it is not possible to exclude any possibility, the predictive quantiles with nominal proportions 0 and 1 are always set to normalized power values of 0 and 1, respectively, whatever the look-ahead time. However, the very tails of predictive distributions are modeled with exponential tails, in order to reflect the fact that extreme prediction errors are unlikely to occur. The quality of this probabilistic forecasting method is evaluated and discussed in [16]. It has been shown to have an acceptable level of reliability, and an acceptable overall skill when compared with other nonparametric probabilistic forecasting methods of the state of the art.

B. Static analysis of interdependence structure

In order to investigate the spatial interdependence structure, a statistical analysis of the whole dataset has been performed. By applying the transformation of Eq. (5), one gets all random variables in a common uniform-rank domain where it is possible to investigate their interdependence structure. Different prediction horizons will yield different predictive distributions $\{\hat{f}_{j,t+k|t}\}_t$ and thus different dependence structures may be expected. Three different prediction horizons are investigated here: 6-, 24- and 43-hours ahead. We measured for all cases the resulting rank correlation ρ_r, i.e. the correlation on the ranks of the random variables[1]. The results from this analysis are summarized in Fig. 5 in the form of rank correlations.

In Figs. 5a and 5b, the rank correlations between the different zones for prediction horizons of 24- and 43- hours ahead are presented. In general, high correlations are measured and different trends appear between the different look-ahead times and the different zones. First, a trend of increasing correlation with the look-ahead time is observed. This can be explained by the operation of forecast systems; in particular, the further ahead one predicts, the more the forecast systems produce similar systematic errors which are interpreted as higher correlations. On the other hand, specific geographic trends are also observed, related to the proximity and relative position of the zones. Considering the map of Fig. 4, higher correlations are observed in neighboring zones and especially in zones that are situated in the North-South axis (for instance zones 1-2, 1-3 and 2-4), which can be explained by the prevailing North-South direction of the wind fronts in the region. Non-neighboring zones present lower correlations, for instance between zones 1-4 and 1-5, which can be explained by the traveling time of the North-South fronts. Since correlations are estimated for a temporal resolution of 1 hour, only movements of the fronts in this time scale are detected.

[1]As discussed in [12], in problems involving non-normal marginals, the rank correlation ρ_r should be used to measure dependence. Rank correlation measures the degree of monotonic relationship between random variables and is invariant to increasing transformations.

C. Results from adaptive methodology

All forecasting methodologies involved (i.e. point and probabilistic forecasting and scenario generation) are adaptive. They require a certain batch learning period before reaching a level of performance that would be equivalent to that witnessed in normal operational conditions. In order to ensure that, the first 7000 prediction series are not considered in the following evaluation, which thus relies on the remaining 8888 predictions series. Over this evaluation period, a set of 1000 production scenarios is generated at each time and for every look-ahead time, by applying the method described in Section III.

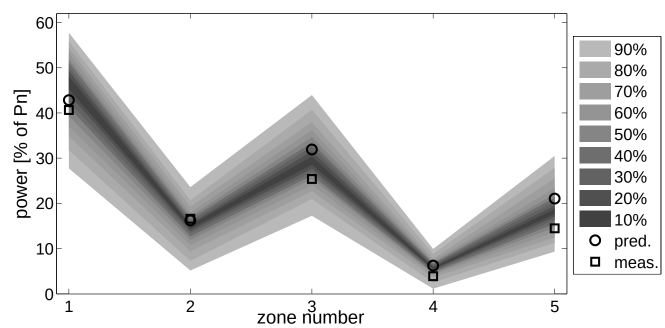
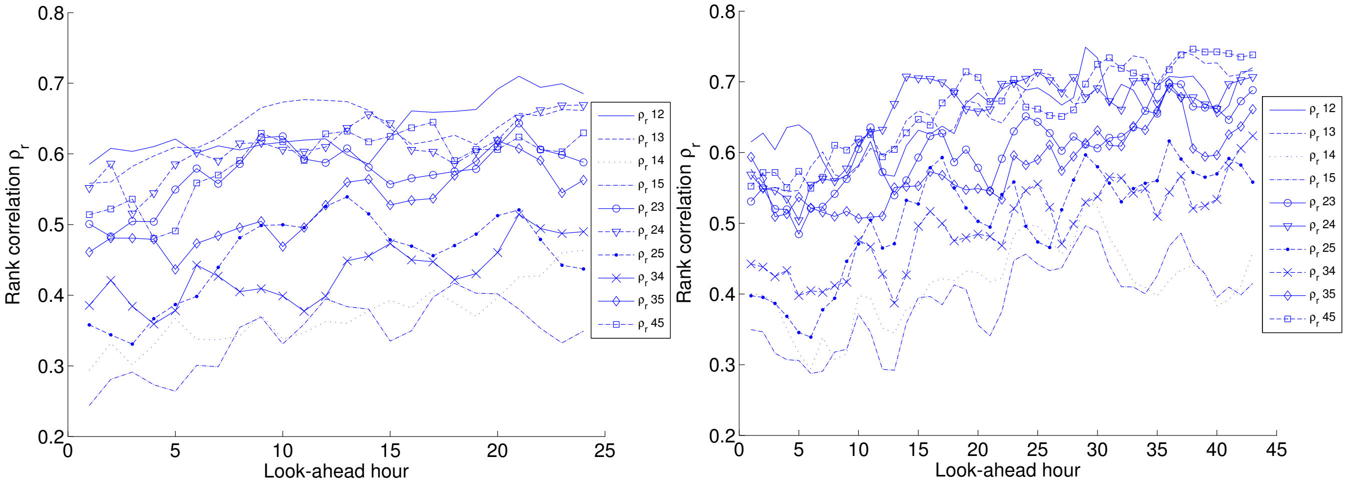


Fig. 6: Point predictions, probabilistic forecasts and power measurements for the 5 zones over western. They correspond to the 2849th forecast series of the evaluation set, and relate to 6-hour ahead prediction.

Fig. 6 gives the example of point and probabilistic forecasts for the 5 zones considered, in the form of a fan chart. They correspond to the 2829th forecast series of the evaluation set, and to a 6-hour ahead horizon. In parallel, Fig. 7 depicts a set of 4 scenarios randomly picked out of the 1000 production scenarios generated. They show potential interdependent deviations from forecasts for the 5 zones considered. The related measured power values are also shown. Such alternative production scenarios may then be used in stochastic decision-making algorithms for e.g. optimal power flow management over the transmission system. The covariance matrix that summarizes the interdependence structure of wind power production among the various zones is shown in Fig. 8. Considering also the map of Fig. 4, one clearly sees the level of interdependence between the various zones. For instance at this point in time, and for the 6-hour ahead horizon, the interdependence between zones 1 and 3 (or between zones 2 and 4) is significant, while that between zones 1 and 5 is low.

Finally, a last foreseen benefit of the method for generating scenarios of wind power production for different zones is the possibility of increasing the resolution of probabilistic forecasts for the whole area. Indeed, for each generated production scenario, one may sum production over the various zones, and obtain a production scenario over the whole area. The set of obtained scenarios then comprises a sampling of potential wind generation, which can be transformed into a nonparametric predictive distribution. Predictive distributions obtained by this method are compared here to the ones



(a) 24-hour ahead - ρ_r

(b) 43-hour ahead - ρ_r

Fig. 5: Rank correlations between the areas for different prediction horizons.

produced by directly applying the adapted resampling method to the whole western Denmark area. Evaluation results are given here for the example of 24-hour ahead forecasts. Those for other forecast horizons are qualitatively similar. Evaluation results in terms of reliability are shown in Fig. 9, which consists of a reliability diagram giving deviations from perfect reliability. In parallel, skill score values are collated in Table I, the skill score being that defined in [16]. Higher skill score values translate to higher skill. From the Figure and the Table, one may conclude that the scenario-based predictive distributions are as reliable as the ones generated from adapted resampling, and that they exhibit higher skill. This result from a higher resolution of scenario-based predictive distributions, coming from the use of information on spatial dependence.

V. Conclusions

Owing to the difficulty of producing accurate point forecasts of wind generation, probabilistic predictions are recognized as crucial for optimizing mananagement or trading decisions to be made. However, the fact that probabilistic forecasts do not provide any information on the interdependence structure of prediction errors at the spatial level make them almost useless in certain cases where it is necessary to know the relation between wind generation in various (possibly non-neighboring) areas, e.g. optimal power flow management. In the present paper, a method that permits to generate scenarios of spatially distributed wind generation has been described, based on the modeling of spatial dependence of wind power forecast uncertainty.

TABLE I: Skill score for 24-hour ahead probabilistic forecasts for the whole western Denmark area.

Method	Skill score value
adapted resampling	-0.351
scenario-based	-0.338

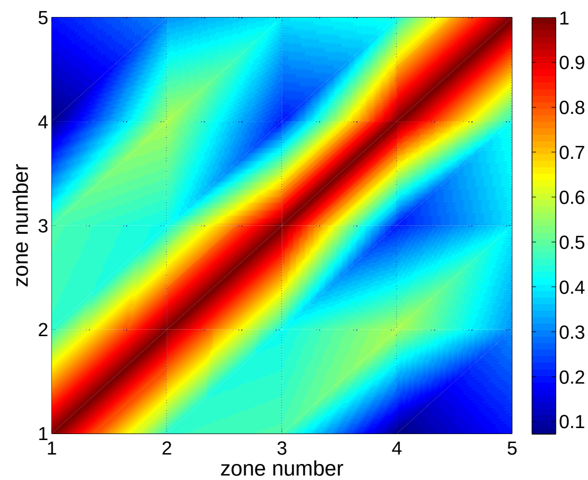


Fig. 8: Covariance matrix of transformed power variable at the end of the dataset, for the 5 zones of western Denmark and for 6-hour ahead predictions (at the time of the 2849 forecast series).

The method has been applied to the test case of western Denmark over a period of more than 2 years. The possibility of generating a number of scenarios from probabilistic forecasts over the 5 different zones has been illustrated, as well as the way to adaptively capture the interdependence structure of prediction errors. In addition, the probabilistic forecasts that can be consequently obtained for the whole Denmark have been shown to reach an acceptable reliability, and a higher skill than if they were directly produced from the statistical probabilistic forecasting method considered.

The static analysis performed has shown the importance of having a conditional view of the modeling of the interdependence structure of wind generation at the spatial level. Therefore, future plans should concentrate on capturing the influence of e.g. wind direction or of the hour of the day on the

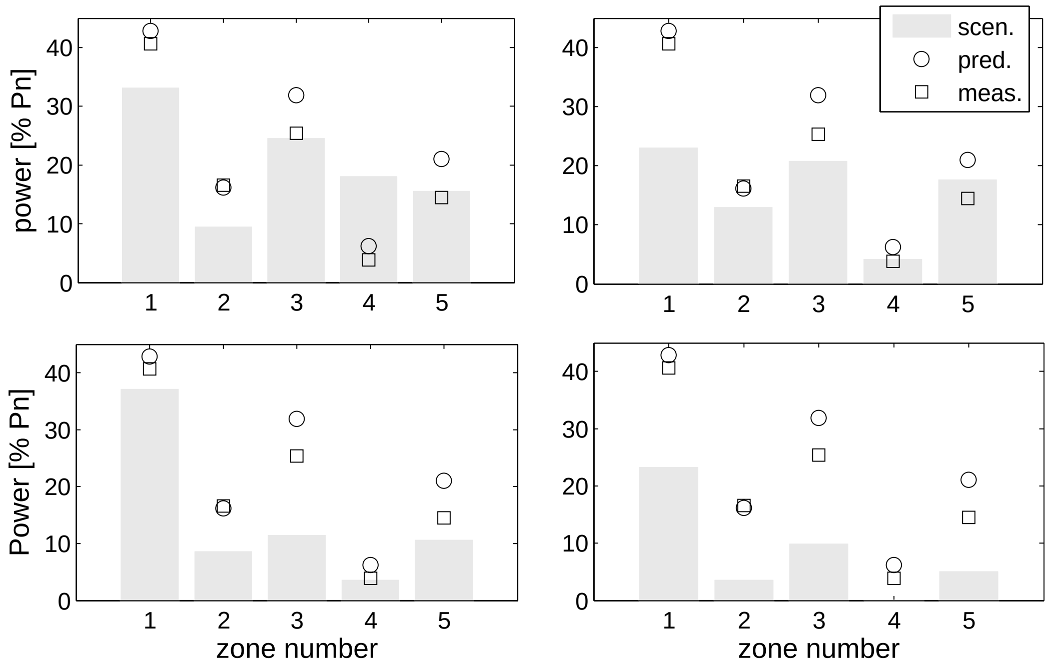


Fig. 7: Set of randomly picked scenarios of power production for the 5 zones of western Denmark (out of the 1000 generated). They correspond to the 2849th forecast series of the evaluation set, and relate to 6-hour ahead prediction.

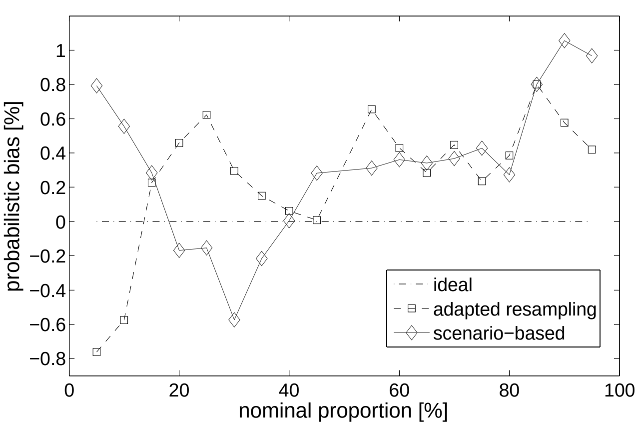


Fig. 9: Reliability diagram for 24-hour ahead probabilistic forecasts over the whole western Denmark area. They are produced either by directly using the adapted resampling method for the whole area, or by generating scenarios for each areas and then deducing probabilistic forecasts for the whole.

spatial dependence of forecast uncertainty. For that purpose, it may be envisaged to consider some regime-switching (or conditional parametric) methods. Broader perspectives relates to the merging of the temporal and spatial aspects when modeling the interdependence structure of wind power forecast uncertainty. And finally, it will be of particular importance to demonstrate the additional value of the proposed scenario generation methodology, via the use of such forecast products for a large range of decision-making problems.

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George Papaefthymiou obtained his Dipl-Eng. degree in Electrical and Computer Engineering from the University of Patras, Greece in 1999 and the Ph.D. degree from the Delft University of Technology in 2007. He is currently with the Electrical Power Systems Laboratory of the Delft University of Technology, as a research associate. His current research interests include modeling of uncertainty in power systems and design of energy systems with large-scale penetration of distributed and stochastic generation.

Pierre Pinson received the M.Sc. degree in Applied Mathematics from the National Institute for Applied Sciences (INSA Toulouse, France) in 2002 and the Ph.D. degree in Energetic from Ecole des Mines de Paris in 2006. He is currently with the Informatics and Mathematical Modeling department of the Technical University of Denmark as an Assistant Professor. His research interests include among others forecasting, uncertainty estimation, optimization under uncertainty, decision sciences, and renewable energies.