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Hansen, Thomas Mejer; Mosegaard, Klaus; Schiøtt, Christian

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Kriging interpolation in seismic attribute space applied to the South Arne Field, North Sea

T. M. Hansen¹, K. Mosegaard², and C. R. Schiøtt³

ABSTRACT

Seismic attributes can be used to guide interpolation in-between and extrapolation away from well log locations using for example linear regression, neural networks, and kriging. Kriging-based estimation methods (and most other types of interpolation/extrapolation techniques) are intimately linked to distances in physical space: If two observations are located close to one another, the implicit assumption is that they are highly correlated. This may, however, not be a correct assumption as the two locations can be situated in very different geological settings. An alternative approach to the traditional kriging implementation is suggested that frees the interpolation from the restriction of the physical space. The method is a fundamentally different application of the original kriging formulation where a model of spatial

variability is replaced by a model of variability in an attribute space. To the extent that subsurface geology can be described by a set of seismic attributes, we present an automated multivariate kriging-based interpolation method that is guided by geological similarity rather than by the conventional distance measure in XYZ space. Through a case study, kriging in attribute space is used to estimate 2D porosity maps from a number of well logs and seismic attributes in the Danish North Sea. Cokriging provides uncertainty estimates that are dependent on the primary data locations in space, whereas kriging in attribute space provides uncertainty estimates that reflect subsurface geological variability. The North Sea case study demonstrates that kriging in attribute space performs better than linear regression and cokriging.

INTRODUCTION

Geophysical prospecting data $\mathbf{z}_{obs}$ are often measured at a discrete and limited set of locations $\mathbf{u}$ in space. Interpolation of such data is then often used to provide an area-covering map, which indicates the spatial distribution of the parameter. Several interpolation techniques exist to provide such interpolation. Examples of such techniques, to name a few, are linear regression, inverse distance, spline interpolation, and kriging-based interpolation.

An obvious way to increase the quality of an interpolated map is to increase the density of direct data observations $\mathbf{z}_{obs}$. Often, however, the cost of measuring $\mathbf{z}_{obs}$ is high and therefore prohibitive (e.g., drilling of new exploration wells). An alternative approach is to make use of cheaper information — for instance seismic data — that in some way is linked or sensitive to changes in the primary parameter of interest. Here, we will refer to such data as “attributes.”

Distance in physical space as a guide for interpolation

Most interpolation techniques use a measure of distance in physical space as a measure of similarity and hence as a guide for interpolation. The implicit assumption is that an observed parameter at two locations separated by a small distance have similar values. In some cases, this may be a valid criterion for interpolation. However, in a geological scenario, this approach has some significant drawbacks.

Consider two locations, A and B, and *primary data*, z_A and z_B , measured at A and B, respectively. Assume that A and B are located close to each other in physical space. If no other information is known about A and B, one could expect z_A to be strongly correlated to z_B . However, A and B may be located in very different geological settings, if for example, A and B are separated by a fault. In such a case, one would not necessarily expect z_A to be correlated to z_B .

On the other hand, assume that the locations A and B are located far away from each other in physical space. Intuitively one would

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¹Formerly at Niels Bohr Institute, Copenhagen, Denmark; presently at DTU Informatics, Technical University of Denmark, Richard Petersens Plads, Bygning 321, 2800 Lyngby, Denmark. E-mail: tmeha@imm.dtu.dk, thomas.mejer.hansen@gmail.com.

²Technical University of Denmark, DTU Informatics, Technical University of Denmark, Richard Petersens Plads, Bygning 321, 2800 Lyngby, Denmark. E-mail: kmos@imm.dtu.dk.

³Hess, Copenhagen, Denmark. E-mail: christian.schiott@hess.com.

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perhaps not expect z_A to be similar to z_B , but if A and B are situated in almost identical geological settings, one could argue that z_A and z_B could indeed be strongly correlated.

This simple example illustrates that rather than using the distance in physical space, it may sometimes be desirable to perform interpolation based on geological similarity. It is this viewpoint that we will consider in this paper.

We will assume that we have access to only a limited number of primary data (e.g., well data), but that an exhaustive set of area-covering attributes (e.g., seismic data/attributes) are available. To the extent that the attributes available are sensitive to changes in subsurface geology, we will consider interpolation based on geological similarity, as opposed to the distance in physical space.

Multivariate interpolation

Existing methods for high-dimensional interpolation include linear regression (Hampson et al., 2001; Russell et al., 2002; Hansen et al., 2008), spline interpolation, nearest neighbor, cokriging (Doyen, 1988), and neural networks (Hampson et al., 2001; Russell et al., 2002; Pramanik et al., 2004; Herrera et al., 2006). Some comparative studies of these methods are available. Hampson et al. (2001) compare the use of linear regression, multilayer feed-forward neural networks and probabilistic neural networks. Russell et al. (2002) compare generalized regression neural networks and radial basis function networks for prediction of log properties from seismic attributes. Pramanik et al. (2004) compare the application of linear regression, neural networks, cokriging with impedance, and cokriging of actual porosity estimates with a porosity map obtained using a neural network. Herrera et al. (2006) uses a combination of linear regression and neural networks.

As opposed to most of the listed methods, kriging-based interpolation produces not only an estimate of the primary parameter, but also an estimate of the local uncertainty. Cokriging has been applied to porosity estimation, using an exhaustive map of acoustic impedance as secondary data (Doyen, 1988). Cokriging provides an improvement compared to traditional kriging in that it allows modeling local variability conditioned by secondary variables. Still, the inference of the covariance model used to perform kriging is intimately related to distance measures in physical space and implies a certain amount of smoothness to the kriging mean and variance results.

As an alternative to the methods described above, we propose to utilize a variant of kriging that is free of constraints from physical distance. We assign all primary data to a point in the high-dimensional space spanned by the considered attributes, assuming that the attributes (secondary data) are available everywhere in the area of investigation. Note that the spatial X-, Y-, and Z components can be regarded as attributes and therefore can be included in the estimation if we wish. We then suggest performing univariate kriging interpolation directly in the attribute space, as opposed to the physical XYZ space.

We apply the method in a case study using data from the South Arne field in the North Sea. We demonstrate how a map of porosity can be estimated from a number of well log measurements and seismic attributes extracted from a 3D seismic data set. This result is compared to cokriging using acoustic impedance as secondary data.

MULTIVARIATE SIMPLE KRIGING

Consider N measurements of a primary parameter of interest, z_i at location $\mathbf{u}_i$, such that $\mathbf{z} = [z(\mathbf{u}_1), z(\mathbf{u}_2), \dots, z(\mathbf{u}_N)]$. Each data z is associated with a point in an M -dimensional attribute space, defined by the vector $\mathbf{u}_i$:

$$\begin{aligned}\mathbf{u}_1 &= [a_{11}, a_{12}, a_{13}, a_{14}, \dots, a_{1M}] \\ &\vdots \\ \mathbf{u}_N &= [a_{N1}, a_{N2}, a_{N3}, a_{N4}, \dots, a_{NM}]\end{aligned}$$

The Euclidean distance between two points, $\mathbf{u}_1$ and $\mathbf{u}_2$, in Euclidean isotropic space is given by

$$\text{dist}(\mathbf{u}_1, \mathbf{u}_2)_{\text{Euclidean}} = V^T V = \|V\|^2, \quad (1)$$

where V is the vector between $\mathbf{u}_1$ and $\mathbf{u}_2$. Assume that the data $z(\mathbf{u}_1), \dots, z(\mathbf{u}_N)$ are a realization of a Gaussian random function $Z(\mathbf{u})$ with covariance model $C(h)$, where h is the separation distance between two locations. Then an anisotropic Euclidean measure of distance, given $C(h)$, is given by

$$\text{dist}(\mathbf{u}_1, \mathbf{u}_2, C)_{\text{Euclidean}} = V^T C(h)^{-1} V. \quad (2)$$

For such data, kriging can be used to estimate the mean and variance of the random function $Z(\mathbf{u})$ at unsampled locations. Kriging is identical to linear least-squares inversion (Hansen et al., 2006) and can be formulated as the least-squares regression problem (Goovaerts, 1997):

$$z^*(\mathbf{u}) = \sum_{\alpha=1}^N \lambda_{\alpha} (z(\mathbf{u}_{\alpha}) - z_{\text{trend}}(\mathbf{u}_{\alpha})) + z_{\text{trend}}(\mathbf{u}). \quad (3)$$

$Z(\mathbf{u})$ can be seen as the sum of a trend, $z_{\text{trend}}(\mathbf{u}) = [z_{\text{trend}}(\mathbf{u}_1), \dots, z_{\text{trend}}(\mathbf{u}_N)]$, and a random residual field, $R(\mathbf{u})$, such that $Z(\mathbf{u}) = R(\mathbf{u}) + z_{\text{trend}}(\mathbf{u})$. Depending on the type of trend considered, three kriging systems may be considered. If the trend is known and constant, simple kriging (SK) can be used. If the trend is constant but unknown, ordinary kriging may be applied (OK). Finally, if the trend is unknown but assumed to follow some polynomial model, kriging with a trend can be used (KT). For generality, we initially consider KT because both SK and OK can be seen as special cases of KT.

In the following, we consider the linear system of equations needed to solve a KT kriging system, which provides the weighting factors, λ , and the trend component z_{trend} for equation 3. The following matrix equation is known as the kriging system (e.g., Journel and Huijbregts, 1978):

$$\mathbf{K}\lambda = \mathbf{k}, \quad (4)$$

where

$$\mathbf{K} = \begin{bmatrix} [C_{dd}(h)] & [\mathbf{f}_k(\mathbf{u}_{\alpha})] \\ [\mathbf{f}_k(\mathbf{u}_{\alpha})]^T & [0] \end{bmatrix} \quad \text{and} \quad \mathbf{k} = \begin{bmatrix} [C_{du}(h)]^T \\ [\mathbf{f}_k(\mathbf{u})]^T \end{bmatrix}.$$

$\mathbf{K}$ consists of the data-to-data covariance values $C_{dd}(h)$, i.e., the covariance between all pairs of observed data separated by distance h , $\mathbf{k}$ consists of the data-to-unknown covariance values $C_{du}(h)$, i.e., the covariance between the location to be kriged and the location of the observed data, and a definition of the trend components $\mathbf{f}_k$ at the loca-

tion to be kriged. If a constant trend is considered, then $\mathbf{K} = \mathbf{C}_{dd}(h)$ and $\mathbf{k} = \mathbf{C}_{du}(h)$ and the kriging system above is reduced to the simple kriging system for which we shall provide the solution below. $\boldsymbol{\lambda}$ is the vector of kriging weights of equation 3, associated with the observed data. Kriging weights are found using

$$\boldsymbol{\lambda} = \mathbf{K}^{-1}\mathbf{k} \quad (5)$$

and used to estimate the local Gaussian probability density of the outcome of the random function at the unsampled location:

$$\mu_{KT} = \mathbf{z}'\boldsymbol{\lambda} + \mu_\phi \quad (6)$$

$$\sigma_{KT}^2 = \mathbf{C}(0) - \mathbf{k}'\boldsymbol{\lambda}, \quad (7)$$

where $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_N]$, μ_ϕ is the a priori mean value, μ_{KT} is mean expected value, and σ_{KT} is the standard deviation. See e.g., [Goovaerts \(1997\)](#) for details on the solution of kriging systems.

As can be seen from the above equations, in order to solve the kriging system, one must choose a model for the trend $\mathbf{f}_k$ and compute the covariance $\mathbf{C}(h)$ using some predefined covariance model from the computed separation distance h between pairs of data locations.

One way of choosing the trend model $\mathbf{f}_k(\mathbf{u})$ can be based on prior knowledge of the system or on observations. Alternatively, $\mathbf{f}_k(\mathbf{u})$ can be chosen as the outcome of another interpolation method, as for example, the trend model presented by [Hansen et al. \(2008\)](#). The method we present here will model the variability around this trend model. In this paper, we focus on the inference of the covariance model and the application to kriging in the high-dimensional attribute space.

Covariance model, shape, and range

$\mathbf{C}_{dd}(h)$ and $\mathbf{C}_{du}(h)$ contain covariance values as a function of separation distance h . To compute these, one needs to select some covariance model to describe the spatial correlations in the residual (non-trend) part of the random function $R(u)$.

The choice of covariance model is very important because it determines the part of the interpolation not accounted for by the trend. Furthermore, it determines the estimated interpolation variance. Therefore we pay special attention to the choice of covariance model.

For lower-dimensional problems (up to three dimensions) a covariance model can be inferred directly from data, using classical semi-variogram analysis (e.g., [Goovaerts, 1997](#)). To provide objective estimates for higher dimensional problems, we suggest defining a generic choice of covariance model and selecting an optimization criterion that can be used to infer the parameters of this model. The choice of optimization criterion should result in both reliable interpolation and uncertainty estimates. In the following, we will discuss in detail how to choose such a generic covariance model and suggest an approach to infer the parameters of this model.

Defining a generic covariance model for multivariate data

The most widely considered analytical covariance model types are the Nugget, Gaussian, Exponential, and Spherical models (e.g.,

[Goovaerts, 1997](#)). They differ mostly in the behavior at small separation distances, except for the Nugget model, which implies no spatial correlation, but only small scale variability below the smallest separation distance. The Gaussian model tends to provide the most smooth interpolation result. A generic definition of a covariance model, given any of these covariance types V with a sill S and a range r is given by

$$\mathbf{C}(h) = SV(r)(h), \quad (8)$$

where h is the separations distance between two locations. We consider the combination of a Nugget model with one of the Gaussian, Spherical, or Exponential covariance models. We will assume that the global sill value S (i.e., the sum of the sills of the selected covariance models) is known and is equal to the variance of the detrended data observations such that a generic covariance model can be defined:

$$\mathbf{C}(\beta, S, r, M, h) = \beta SNug(0) + (1 - \beta)SV(r)(h). \quad (9)$$

The nugget fraction β is a convenient measure to use. If the nugget fraction is 1, the combined covariance model is identical to a pure nugget model (i.e., a covariance model suggesting no spatial correlation). In this case, only the trend model is estimated, and the kriging variance will be constant and equal to the size of the global sill value. A nugget fraction of 0 suggests that there is no small-scale variability. In this way, the nugget fraction controls the relative weight of the trend and the spatial correlation in the kriging estimation. The spatial correlation is controlled by the range of the non-Nugget component of the covariance model. A covariance model with zero range is equivalent to a pure nugget model. An infinitely long range will result in an interpolation result equal to linear interpolation.

Anisotropy

The isotropic formulation of the covariance model in equation 9 is adequate to specify the covariance model for the kriging system with only 1 attribute (i.e., in $\mathcal{R}^1$). For higher dimensions, however, an anisotropic covariance model is usually needed. In the context of the present work, there is little chance that an isotropic covariance model would properly describe data in an attribute space spanned by, for example, seismic impedance and two-way travel time, simply because these two attributes are measured on different scales.

Anisotropic covariance models are well described in the geostatistical literature for up to three dimensions (e.g., [Chiles and Delfiner, 1999](#)). Anisotropy can be defined by a combination of scaling for each dimension and a rotation of the coordinate system axes. In the simplest case, the anisotropy axes coincide with the coordinate axes, and hence only scaling is applied and rotation is not needed ([Chiles and Delfiner, 1999](#)). We will refer to this form of anisotropy as “scaling anisotropy.” Only the range along each coordinate axis needs to be specified. In this case only, the range along each axis needs to be specified. Thus M range values need to be selected in $\mathcal{R}^M$. The more general “elliptic” anisotropy, making use of both scaling and rotation, will be applied if the anisotropy axes do not coincide with the coordinate axes. In this case, the rotation of the coordinate system is performed using a transformation matrix. The number of angles needed to completely define the rotation is 0 in $\mathcal{R}^1$, 1 in $\mathcal{R}^2$, and 3 in $\mathcal{R}^3$. Hence, $(M * (M - 1)) / 2$ angles are needed in $\mathcal{R}^M$. It is unfeasible to infer meaningful values to all angles in higher dimensions, so we will only consider scaling anisotropy in the remainder of this paper.

Assume that the range parameters are $\mathbf{r} = [r_1, r_2, \dots, r_M]$ in $\mathcal{R}^M$, the sill is S , and the nugget fraction of the sill is β . The general form of our covariance model will then be

$$C(\beta, S, \mathbf{r}) = \beta S \text{Nug}(0) + (1 - \beta) SV(\mathbf{r})(h). \quad (10)$$

In $\mathcal{R}^M$ this corresponds to $M + 2$ parameters to be inferred. If the of covariance model type is known, the number of unknown parameters is $M + 1$. Figure 1a shows an example of a 1D covariance model ($\beta = 0.3, S = 1, r = 1$) and Figure 1b an example of a 2D covariance model with scaling anisotropy as defined above ($\beta = 0.3, S = 1, \mathbf{r} = [1, 2]$). Note that the relation between the semivariogram (most often used in geostatistics), $\gamma(h)$, and the covariance model is given by $\gamma(h) = C(0) - C(h)$ (see Figure 1a).

Inference of a multivariate covariance model

In a classical geostatistical work flow an experimental covariance model is calculated from observations. Then an analytical covariance model is fitted to the experimental covariance model. This can be done either manually or using an automated approach for covariance fitting, as suggested by for example, Cressie (1985). This method, however, relies on subjective choices and requires relatively many data to compute a reliable experimental covariance model.

Pardo-Igúzquiza (1998) compares a number of different techniques for inferring the range of a covariance models. He demonstrates that the cross-validation-based maximum likelihood method (Samper and Neuman, 1989a, b, c), and the regular maximum likelihood approach (ML) (Kitanidis and Lane, 1985; Pardo-Igúzquiza, 1997; Diggle et al., 2003) provide almost equally good results. For computational efficiency, we consider the ML method of Pardo-Igúzquiza (1997, 1998).

Assume that the subsurface can be seen as a realization of a 2nd order stationary Gaussian random field and that a sample of the realization is available in form of N direct observations of $\mathbf{z} = [z_1, z_2, \dots, z_N]$. Then the negative log-likelihood L_{nllf} that these observations can be seen as a realization of a Gaussian random field with covariance $\mathbf{C}_M$ is given as

$$L_{\text{nllf}}(\mathbf{Q}, \mathbf{z}, \mathbf{z}_{\text{trend}}) = \frac{N}{2} (\ln(2\pi) + 1 - \ln(N)) + \frac{1}{2} \ln|\mathbf{Q}| + \frac{N}{2} \ln((\mathbf{z} - \mathbf{z}_{\text{trend}})^T \mathbf{Q}^{-1} (\mathbf{z} - \mathbf{z}_{\text{trend}})), \quad (11)$$

where $\mathbf{Q} = \mathbf{C}_M / \mathbf{C}_M(0)$ and $\mathbf{z}_{\text{trend}}$ is the trend model at the observation points. In the maximum likelihood approach, the parameters of the subsurface covariance model can be found by minimizing expression 11 with respect to the components of $\mathbf{Q}$, $\mathbf{z}$, and $\mathbf{z}_{\text{trend}}$ (Pardo-Igúzquiza, 1997, 1998). For the purpose of kriging in attribute space, we minimize equation 11 to infer the nugget fraction, ranges for each attribute direction and the covariance model type of equation 10.

We thus suggest to perform univariate kriging in a multivariate attribute space (equations 4–7), based on automatic inference of a generic covariance model (equation 10) using maximum likelihood maximization of equation 11. In the following, we apply the method to estimate a 2D map of porosity using attributes obtained from seismic data.

CASE STUDY AND DISCUSSION

The South Arne Field is a chalk reservoir situated in the Danish North Sea (Mackertich and Goulding, 1999). The oil-bearing chalk is characterized by porosities in the range 20–45 porosity units (PU) as interpreted from well log data. Estimates of porosity are key values in the assessment of in-place oil volumes. This case study utilizes kriging in multiattribute space as a tool for estimating reservoir porosity guided by seismic attributes. For simplicity, we consider porosity variation in 2D. However, the methodology is readily expandable to 3D. In this study, we focus on the main reservoir, the Tor Formation, which is of Maastrichtian age and is located in the chalk section of the South Arne Field in the North Sea.

In total, we have access to nine attributes extracted from seismic data in a dense, geographical 2D XY-grid consisting of 37,791 data locations. The two-way travel time (TWT), “depths” to top (Top Tor), and base (Base Tor) have been established directly from the

seismic data. Based on these data, the thickness of the Tor formation is computed and used as an attribute. At Top Tor, the amplitude (Top Tor Amplitude) and the dip (Top Tor Dip) have been extracted as attributes. The acoustic impedance attribute (AI), derived from the seismic waveforms, is available everywhere. Thus, five seismically related attributes are available (Top Tor, Base Tor, Top Tor Amplitude, Top Tor Dip, AI). In addition, we consider the three (spatial) coordinated UTM X, UTM Y, and UTM Z as attributes. Figure 2a–i shows each attribute (and thereby the areal coverage of the study area), and Figure 2j shows the location and value of porosity measurements. Note that for all 2D plots, we have rotated the original UTM X-UTM Y axes to fit the layout of the data. The original direction to the North is indicated by

Figure 1. (a) 1D ('0.3 Nug + .7 Sph(1)') covariance (thick solid line) and corresponding semivariogram (thin gray line). The nugget is indicated by the dotted line and the range by the dashed line. (b) 2D ('0.3 Nug + 0.7 Sph(1,0.5)') example of the covariance model defined by equation 10.

the arrow on Figure 2j. We consider the well-site porosity measurements as sparsely sampled values of a porosity function, defined over the (up to) nine-dimensional attribute space. We use kriging in attribute space as outlined above to estimate the porosity in form of a probability distribution at every unsampled location in the study area.

Distance in attribute space as a measure of similarity

Figure 3 shows the distance from one location (white) to everywhere else in the study area, calculated in the attribute space defined by X and Y (Figure 3a) and AI (Figure 3c), respectively. The distance in AI attribute space (Figure 3c) describes better the variation in subsurface geology than the spatial distance measure (Figure 3a). Standard kriging-based interpolation is based on distance-dependent covariances. As an example, we consider an exponential covariance model with a range of 20% of the maximum distance in each considered attribute space (3200 m and 7.5×10^5 kg/m² s, respectively). Figure 3b and d shows the corresponding covariance function centered at one location, everywhere in the area. With respect to the AI attribute space, Figure 3d clearly shows that locations correlated to the white circle follow a complex geological (and plausible) pattern, in contrast to the physical-distance-based covariance (Figure 3b). Thus, attribute-guided kriging interpolation tends to follow geological similarity.

Automatic inference and interpolation

The methodology of using kriging in attribute space is similar whether a one-dimensional or a higher-dimensional attribute space is considered:

1 Normal score transformation

Kriging assumes that data can be seen as samples from a continuous Gaussian distribution. Therefore we make use of a normal score transformation of the porosity data, which transform the porosity data to a set of Gaussian distributed data with mean 0 and variance 1. The transformed data are referred to as the normal scores of the original porosity data.

2 Choice of trend model

Using kriging involves choosing or modeling a trend model as discussed previously. The kriging system is then applied to the residuals of the normal score, with the trend removed. Because the covariance model specified for the KT-kriging system describes the covariance of the residuals, we must solve the kriging system in order to find the residuals related to a given trend model. Thus we need to know the trend model to infer the covariance model for the residuals. Further, the esti-

mate of uncertainty obtained by the kriging system relates only to the residuals, and is not affected by the trend model. Therefore we suggest to avoid calculation of the trend model as part of the kriging system and instead use an alternative method (e.g., Russell et al., 1997; Hampson et al., 2001; Russell et al., 2002; Hansen et al., 2008; Pedersen-Tatalovic et al., 2008). Given such a trend model, we suggest using the presented method to improve the trend model by modeling the variations away from the trend.

For this case study, we will consider a very simple trend model. For all directions, except for the AI attribute, we choose a constant trend model, and for the AI attribute, we choose a linear trend model.

Figure 2. (a–i) Available attributes within the test area. Red colors indicate relative high values and blue colors relative low values. (j) Observed porosity from well logs. Solid lines are iso lines for UTM X = 5.785e + 6 m and UTM Y = 6.214e + 6 m.

Figure 3. Distance in (a) X-Y space and (c) AI attribute space to the location denoted by the white circle. Covariance computed in X-Y space using an (b) isotropic covariance model and in (d) AI space.

The latter choice is based on the well-known fact that a strong linear relation exists between acoustic impedance and porosity in the Chalk section of the North Sea.

3 Inference of the covariance model in attribute space

The actual optimization of expression 11 is done using a combination of global and local optimization. A Metropolis-Hastings algorithm is initially used to sample L_{mlf} of equation 11, such as shown in Figure 4. Note that a well-defined maximum can be located. Using the set of covariance model parameters with the location of the highest L_{mlf} as a starting point, a local search is then performed to find the point of maximum likelihood.

4 Kriging in normal score attribute space

Kriging in normal-score attribute space is a matter of solving equations 4–7 to obtain an estimate of the normal scores at unknown locations in attribute space.

Figure 4. Maximum likelihood (ML) distribution of the covariance model parameters in AI attribute space. Darker colors reflects high likelihood and lighter colors low likelihood.

5 Inverse normal score transformation

An actual porosity estimate in original porosity units is obtained by back-transforming the kriging estimates. The result of kriging-based interpolation can now be obtained by visualizing the inverse normal score transform in the 2D UTM X-UTM Y coordinate system. Thus, the kriging is performed in attribute space, but data are visualized in spatial (2D) space.

The kriging result is not just the interpolated values of porosity, but a local probability distribution providing the local distribution of porosity, conditioned to all observations. Because we make use of a normal score transformation of data, the shape of the local probability distribution of porosity is usually not Gaussian. We shall later show examples of how to estimate both confidence intervals and maps showing the probability that the porosity is within certain ranges.

Kriging in high-dimensional attribute space

To investigate the performance of the proposed method, we consider five attribute sets in which we perform kriging in attribute space: [AI], [UTM X, UTM Y], [UTM X, UTM Y, AI], [All but UTM X and UTM Y], and [all attributes]. We also consider six subsets of the original 213 available porosity measurements as observed “known” porosity data: 1) every 2nd, 2) every 5th, and 3) every 10th data point; 4) all data points with UTM X < 5.785e + 5 m; 5) all data points with UTM Y < 6.214e + 6 m; and 6) all data points with UTM Y ≥ 6.214e + 6 m. See Figure 2j for iso lines for UTM X = 5.785e + 5 m and UTM Y = 6.214e + 6 m. Each data subset then considers [106, 53, 21, 178, 99, 114] observations as known data. The rest of the available porosity data are referred to as “blind” data and shall be used for comparison. Kriging in attribute space is now performed for all combinations of data and attribute subsets, by inferring the covariance model in attribute space for each data subset, followed by kriging of the porosity at the blind data locations.

Table 1 reports the average prediction error of the blind data not used in optimization for different choices of selected attributes and data subset. A significant result in Table 1 is that increasing the number of considered attributes generally leads to a decrease in the average prediction error of the data not used in the optimization. Introducing an attribute with no apparent relation to the primary data, as for example, the amplitude attribute, does not reduce the quality of the final prediction result. The maximum likelihood approach for inferring the covariance model therefore seems very robust with re-

Table 1. Mean prediction error of blind data considering the data subsets and different attributes and methods.

Data set	MM1	CoKrig	AI trend	X,Y	X,Y,AI	All but XY	ALL
1 (d = 2)	3.6	3.6	3.5	1.8	1.8	2.0	1.7
2 (d = 5)	2.7	2.7	3.4	2.7	2.5	2.5	2.3
3 (d = 10)	3.6	3.6	3.5	4.1	3.1	3.2	3.2
4 (X ≤ 5.785e + 5 m)	3.5	3.5	4.4	5.6	3.1	3.2	3.2
5 (Y ≤ 6.214e + 6 m)	8.7	8.7	6.7	11.6	8.5	6.4	8.2
6 (Y ≥ 6.212e + 6 m)	5.1	5.0	3.6	4.3	3.3	4.0	3.4

Columns 2 and 3 refer to the use of colocated (MM1) and full (CoKrig) cokriging in XY space, with AI as a secondary attribute. Column 4 refers to the use of a linear trend model (AI trend). The last four columns refer to kriging in attribute space using the given attributes.

spect to noisy/unrelated attributes: If such attributes exist, the inferred covariance model for that specific attribute direction will have either a large nugget or a very short range, indicating little to no correlation for changes in the attribute and therefore little to no effect on the final prediction. Reducing the number of considered attributes based on feature extraction (e.g., [Hampson et al., 2001](#)) may be a valid approach if the number of available attributes becomes very large. However, for the data set used in this study our findings above indicate that feature extraction is not needed because the blind error decreases as attributes (believed a priori to be less significant) are added to the list of attributes considered by the method.

The lowest prediction error of the blind data is at a level of 1.7 PU and is found when considering all the available attributes for data subset 1. This is a large improvement over the simple, traditional approach of using a linear relationship between acoustic impedance and porosity, which results in a prediction error of about 3.5 PU. Figure 5 shows the estimated mean porosity using all attributes available for all data subsets. Figure 6 shows the estimated mean porosity in a smaller region for easier comparison.

Figure 7 shows the estimated mean porosity using all attributes except for the spatial attributes. From Table 1 it is seen that, when considering all attributes but UTM X, UTM Y, and UTM Z, the crossvalidation error is comparable to using all attributes and considerably lower than using simple linear regression. Thus, the presented method provides relatively good results even when discarding the traditional spatial information that controls many interpolation algorithms.

It is clear from Figures 5–7 that the AI attribute plays an important role in predicting the porosity, as should be expected due to the known, strong correlation between acoustic impedance and porosity. Figure 8 shows the estimated mean porosity when not using the AI attribute. For data subsets 1–4 the general trend of high porosity in the northwest and lower porosity towards the southeast is found. However, for the last two data subsets (5 and 6), where data are split into a set of data toward the north and the south, respectively, kriging does not perform very well (neither in physical space, nor in attribute space). This is because there are no trends in the considered data to account for the north-south observed variability in the porosity distribution.

Uncertainty estimates

The result of kriging in attribute space is not simply the mean estimate, but actually an estimate of the local probability distribution of porosity. Because the kriging is performed on a non-linear normal score transform of the data, the kriging estimate of variance cannot be directly back-transformed to an estimate of the variance of the porosity estimate. In fact, the estimated local pdf of porosity is non-Gaussian in back-transformed space. However, the kriging variance estimate in normal score space is useful to visualize the distribution of spatial uncertainty. As an example, Figure 9 shows the estimated variance in normal score space using all attributes for different subsets of data. Using data subset 1 (where the unknown data locations on average are closest to the known data locations), the uncertainty map reflects the location of the known data locations

Figure 5. Estimated porosity (mean) using all attributes for different data subsets as hard data: (a) subset 1, (b) subset 2, (c) subset 3, (d) subset 4, (e) subset 5, and (f) subset 6. Used and blind data locations are indicated using white and black dots, respectively.

Figure 6. As Figure 5 (with same colorscale), but zooming in on a smaller area. Hard data subsets 1–6 in (a–f).

in 2D (Figure 9a). This is similar to the result when using kriging in physical space, where the uncertainty map is related in a simple way to the data locations. Because less data are available, Figure 9b and c clearly shows that the uncertainty variance map tends to reflect the subsurface variation as given by the attributes available.

Actual uncertainties of porosity estimates can be obtained by back-transforming certain quantiles in the normal score space. In this way, probability maps of the porosity can be obtained. We believe that, because the uncertainty estimates using kriging on attributes space better resemble known subsurface variability, these maps are superior to maps based on conventional, spatial kriging-based estimation. For data subset 1, Figure 10 shows five maps of the probability showing that the porosity is above 20, 25, 30, 35, and 40 PU, respectively. Such maps are crucial for risk assessment associated with porosity estimates.

An alternative way to visualize the uncertainty estimates is by plotting the porosity levels corresponding to the lower and upper limit of the 95% confidence interval. These are shown for data subset 3 using all attributes available in Figure 11.

The uncertainty estimates given above are correct for each estimation location independent of other estimation locations. If a joint un-

certainty model for several data locations is needed, we suggest making use of sequential simulation. The result would be a number of realizations from which higher order statistics could be inferred. Note that lower-order statistics such as the posterior mean and variance can be found directly, as shown above. The application of sequential simulation based on kriging in attribute space would be relatively trivial because the method we use is but a utilization of kriging and therefore is readily applicable for use with a sequential simulation framework.

Effect of choice of covariance model type

Figure 12 compares the result of kriging estimation in a 1D attribute space spanned by the AI attribute using a covariance model of types: Gaussian (a), Spherical (b) and Exponential (c). The average estimation error of the 159 points (of 213 available) not used for inference is 3.6, 3.6, and 3.5 PU, respectively. The maximum log-likelihood associated with each choice of covariance model is $L = [-56.9, -56.8, -57.3]$. Normalizing by the maximum log likelihood, the relative probability of the choice of C_m becomes $[0.9, 1.0, 0.6]$. This indicates that there is very little difference in the maximum likelihood estimates using any of the three considered covariance model types.

Figure 7. Estimated porosity (mean) using all attributes except for the spatial attributes (UTM X, UTM Y, UTM Z) for different choices of data subset as hard data. Hard data subsets 1–6 in (a–f).

Figure 8. Estimate porosity (mean) using all attributes except for the AI attribute for different choices of data subset as hard data. Hard data subsets 1–6 in (a–f).

Note that in order to increase the effect of using different covariance model, the results in Figure 12 are based on an assumption of a constant trend model, as opposed to the previously considered linear trend model for the AI attribute. It is seen that there is no visual difference between using the three types of covariance model.

Both in terms of blind prediction accuracy, likelihood of the inferred covariance model, and estimation of the probability maps in Figure 12, there seems to be little difference between using the different covariance models. See for example, Goovaerts (1997) for a discussion on the behavior of different types of covariance models.

Cokriging

Cokriging is an extension of kriging that takes into account the cross correlation between the primary and secondary data such as seismic attribute data. Cokriging has previously been used for porosity estimation using acoustic impedance obtained from seismic data as secondary data (Doyen, 1988; Xu et al., 1992). For comparison, we therefore compare our results from kriging in seismic attribute space to cokriging of porosity with acoustic impedance as secondary data. As discussed previously, the inference of a coregionalization requires the inference of a covariance model for the acoustic impedance C_{AI} , porosity C_{PU} , and a cross-covariance model between acoustic impedance and porosity, $C_{AI,PU}$. Even for this simple model, with only one secondary data set, it proves nontrivial to infer a permissible model of coregionalization because a permissible set of C_{AI} , C_{PU} , and $C_{AI,PU}$ cannot be inferred independently from one another. We use the linear model of coregionalization as described by Goovaerts (1997) to infer a permissible coregionalization. C_{AI} is the most robust covariance model to infer, because it is based on all the AI attribute data available (37,971 data). We find $C_{AI} = 7.510^{11}$ Sph(800). This is valid when considering all data subsets. Inference of C_{PU} and $C_{AI,PU}$ relies on the number of available primary data $\mathbf{x}$ which is significantly smaller than the number of attributes/secondary data. Table 2 summarizes the inferred covariance models for C_{PU} and $C_{AI,PU}$. Note that although C_{AI} may be relatively robust to infer, the inference of C_{PU} and $C_{AI,PU}$ will be highly subjective due to the limited data available.

In cases where secondary data is available everywhere, one can make use of colocated cokriging, using a Markov model approximation to avoid the nontrivial inference of the full model of coregionalization (Almeida and Journel, 1994; Journel, 1999). Here we use the Markov Model 1 for modeling the coregionalization between porosity and acoustic impedance. This requires one to infer the covariance model for the primary variable (acoustic impedance) and the variance of the secondary variable, as well as the cross covariance between the primary and secondary variable, which can be obtained from Table 2. GSTAT was used to perform full and colocated cokriging (Pebesma and Wesseling, 1998).

Table 1 summarizes the average prediction error using full and colocated cokriging for the six considered data subsets. Figure 13a and b shows the porosity estimates obtained using colocated and full cokriging, which reveals there is little difference using colocated cokriging and full cokriging. This is probably due to the primary data screening the influence of the secondary data, as discussed by Almeida and Journel

(1994). Kriging in attribute space performs better than cokriging for all data subsets. This is to be expected considering all attributes as kriging in attributes space then rely on a much larger data set than

Figure 9. Estimated relative uncertainty in normal score space for different data subsets. Blue reflects relative low variance and red relative high variance. Hard data subsets 1–6 in (a–f).

Figure 10. Probability that porosity levels is above (a) 20, (b) 25, (c) 30, (d) 35, and (e) 40 PU, using kriging interpolation in the full eight-dimensional attribute space for data subset 1.

cokriging in UTM X-UTM Y space with AI as secondary data. However, even considering only the same attributes as available for the cokriging, UTM X, UTM Y, and AI, kriging in attribute space performs consistently better than cokriging. Figure 13c shows the po-

rosity estimate applying kriging in seismic attribute space using UTM X, UTM Y, and AI as attributes for data subset 4.

Figure 14 show the kriging uncertainty estimate (in normal score space) for the same data subset and kriging algorithms as for the mean estimate shown in Figure 13. Full cokriging reduce the estimation uncertainty (Figure 14b) compared to using colocated cokriging (Figure 14a). For both colocated and full cokriging, the uncertainty of the cokriging estimate is simply a mapping of the distance to the data locations in XY space. In other words, the kriging uncertainty estimate is independent of the secondary attribute(s) values.

Using kriging in attributes space the uncertainty is linked to attribute similarities, (see Figures 14c and 9 as considered previously). Therefore the variation of uncertainty tends to follow patterns in the attribute. If the attributes reflect geological variability then so will the uncertainty estimates. Hence, the uncertainty estimate will reflect geological similarity.

Figure 11. Maps of the (a) lower and (b) upper bound for the 95% confidence interval of the porosity estimate for data subset 3.

Figure 12. Effect of choice of covariance model type, (a) Gaussian, (b) spherical, and (c) exponential.

Table 2. Inferred covariance and cross-covariance models for use with cokriging.

Data set	C_{PU}	$C_{AI,PU}$
1	0.30 Nug + 0.35 Sph(1500)	− 6.98e + 5 Sph(2500)
2	0.15 Nug + 0.90 Sph(1500)	− 7.37e + 5 Sph(2500)
3	0.16 Nug + 0.52 Sph(150)	− 6.85e + 5 Sph(2500)
4	0.20 Nug + 0.55 Sph(2000)	− 6.88e + 5 Sph(2500)
5	0.16 Nug + 0.55 Sph(200)	− 4.15e + 5 Sph(2500)
6	0.16 Nug + 0.85 Sph(1500)	− 6.83e + 5 Sph(2500)

Figure 13. Estimated porosity (mean) using data subset 4 considering (a) colocated cokriging (Markov Model 1), (b) full cokriging, and (c) kriging in attribute space (UTM X, UTM Y, AI).

Figure 14. Estimated kriging variance (estimation uncertainty) in normal score space using data subset 4 considering (a) colocated cokriging (Markov Model 1), (b) full cokriging, and (c) kriging in attribute space (UTM X, UTM Y, AI). Blue indicate relative small uncertainty, red relative high variance.

A comparison to full cokriging using all attributes is not feasible and therefore not considered. The simultaneous inference of all direct and cross-covariance models is tedious and highly subjective. In addition, such a full model of coregionalization may lead to a kriging system that is unstable (Almeida and Journel, 1994).

Empirical confirmation of our results

At the same time as this study was carried out, an appraisal drilling campaign was executed to confirm the extension of commercial reservoir properties to the north of the main field. The four well bores largely confirmed the results of the presented study, with Tor porosities in the range 18–23 PU.

CONCLUSION

We have proposed a kriging-based interpolation method for data in a multivariate attribute space. The inference of the multivariate covariance model needed by the kriging algorithm is done by (a) assuming a basic covariance structure consisting of a Nugget model and one of either the Gaussian, Spherical, or the Exponential model, (b) by applying scaling anisotropy, and (c) using maximum likelihood optimization to obtain a covariance model, which describes the correlations in the attribute space. With the exception of the selection of a trend model for the kriging system, the process is completely automated.

The method is robust for inclusion of attributes that are uncorrelated to the parameters being estimated, in that adding such an attribute as an axis in the attribute space will in general have little to no effect on kriging results. The method provides an uncertainty estimate of the porosity in form of a local probability distribution describing the estimated variability of the property considered.

An application of the suggested approach has been demonstrated on porosity data from the South Arne field, Danish North Sea. Numerous blind tests show that the approach provides a good estimate of the porosity at unsampled locations and useful uncertainty estimates. The average prediction error was reduced from 3.5 PU, using linear regression in multiattribute space, to 1.7 PU, using kriging in attribute space for data subset 1. The methodology was verified by data kept out of the optimization process and by real data obtained while this work was carried out.

For the six considered data subsets a case study demonstrates that kriging in attribute space provides better porosity estimates at unsampled locations than colocated and full cokriging. Traditional cokriging produce uncertainty estimates that are only dependent on distances in physical space and thus are independent of the value of secondary attributes. A major advantage of kriging in attribute space is that uncertainty estimates depend also on the attribute values. Thus if the attributes available reflect geological variability, then the uncertainty provided by kriging in attribute space reflects geological similarity.

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REFERENCES

- Almeida, J. S., and A. G. Journel, 1994, Joint simulation of multiple variables with a markov-type coregionalization model: *Mathematical Geology*, **26**, no. 5, 565–588, doi: 10.1007/BF02089242.
- Chiles, J.-P., and P. Delfiner, 1999, *Geostatistics, modeling spatial uncertainty*: John Wiley & Sons, Inc. Wiley series in probability and statistics.
- Cressie, N., 1985, Fitting variogram models by weighted least squares: *Mathematical Geology*, **17**, no. 5, 563–586, doi: 10.1007/BF01032109.
- Diggle, P. J., P. J. Ribeiro Jr., and O. F. Christensen, 2003, *An introduction to model-based geostatistics*: Springer, 43–86. Springer Series in Statistics.
- Doyen, P. M., 1988, Porosity from seismic data — A geostatistical approach: *Geophysics*, **53**, 1263–1275, doi: 10.1190/1.1442404.
- Goovaerts, P., 1997, *Geostatistics for natural resources evaluation*: Oxford University Press. Applied geostatistics series.
- Hampson, D., J. Schuelke, and J. Quirein, 2001, Use of multiattribute transforms to predict log properties from seismic data: *Geophysics*, **66**, 220–236, doi: 10.1190/1.1444899.
- Hansen, T. M., A. G. Journel, A. Tarantola, and K. Mosegaard, 2006, Linear inverse Gaussian theory and geostatistics: *Geophysics*, **71**, no. 6, R101–R111, doi: 10.1190/1.2345195.
- Hansen, T. M., K. Mosegaard, R. Pedersen-Tatalovic, A. Uldall, and N. J. Jacobsen, 2008, Attribute guided well log interpolation — applied to low frequency impedance estimation: *Geophysics*, **73**, no. 6, R83–R95, doi: 10.1190/1.2996302.
- Herrera, V. M., B. Russell, and A. Flores, 2006, Neural networks in reservoir characterization: The Leading Edge, **25**, 402–411, doi: 10.1190/1.2193208.
- Journel, A. G., 1999, Markov models for cross-covariance: *Mathematical Geology*, **31**, no. 8, 955–964, doi: 10.1023/A:1007553013388.
- Journel, A. G., and C. J. Huijbregts, 1978, *Mining geostatistics*: Academic Press.
- Kitanidis, P. K., and R. W. Lane, 1985, Maximum likelihood parameter estimation of hydrologic spatial processes by the gauss-newton method: *Journal of Hydrology (Amsterdam)*, **79**, no. 1–2, 53–71, doi: 10.1016/0022-1694(85)90181-7.
- Mackertich, D. S., and D. R. G. Goulding, 1999, Exploration and appraisal of the South Arne Field, Danish North Sea, in A. J. Fleet, and S. A. R. Boldy, eds., *Petroleum Geology of Northwest Europe, Proceedings of the 5th Conference*: The Geological Society: 959–974.
- Pardo-Igúzquiza, E., and the Pardo-Igúzquiza, 1997, Mlreml: a computer program for the inference of spatial covariance parameters by maximum likelihood and restricted maximum likelihood: *Computers & Geosciences*, **23**, no. 2, 153–162, doi: 10.1016/S0098-3004(97)85438-6.
- Pardo-Igúzquiza, E., 1998, Maximum likelihood estimation of spatial covariance parameters: *Mathematical Geology*, **30**, no. 1, 95–108, doi: 10.1023/A:1021765405952.
- Pebesma, E. J., and C. G. Wesseling, 1998, Gstat: A program for geostatistical modelling, prediction and simulation: *Computers & Geosciences*, **24**, no. 1, 17–31, doi: 10.1016/S0098-3004(97)00082-4.
- Pedersen-Tatalovic, R., A. Uldall, N. L. Jacobsen, T. M. Hansen, and K. Mosegaard, 2008, Event-based low-frequency impedance modeling using well logs and seismic attributes: The Leading Edge, **27**, 592–603, doi: 10.1190/1.2919576.
- Pramanik, A. G., V. Singh, R. Vig, K. Srivastava, and D. N. Tiwary, 2004, Estimation of effective porosity using geostatistics and multiattribute transforms: A case study: *Geophysics*, **69**, 352–372, doi: 10.1190/1.1707054.
- Russell, B., D. Hampson, J. Schuelke, and J. Quirein, 1997, Multiattribute seismic analysis: The Leading Edge, **16**, 1439–1443, doi: 10.1190/1.1437486.
- Russell, B., D. Hampson, T. Todorov, and L. Lines, 2002, Combining geostatistics and multi-attribute transforms: A channel sand case study, Blackfoot oilfield (Alberta): *Journal of Petroleum Geology*, **25**, no. 1, 97–117, doi: 10.1111/j.1747-5457.2002.tb00101.x.
- Samper, F. J., and S. Neuman, 1989a, Neuman, estimation of spatial covariance structures by adjoint state maximum likelihood cross-validation: 1. theory: *Water Resources Research*, **25**, no. 3, 351–362, doi: 10.1029/WR025i003p00351.
- , 1989b, Neuman, estimation of spatial covariance structures by adjoint state maximum likelihood cross-validation: 2. synthetic experiments: *Water Resources Research*, **35**, 363–371.
- , 1989c, Neuman, estimation of spatial covariance structures by adjoint state maximum likelihood cross-validation: 3. application to hydrochemical and isotopic data: *Water Resources Research*, **35**, 373–384.
- Xu, W., T. T. Tran, R. Srivastava, and A. G. Journel, 1992, Integrating seismic data in reservoir modeling: The colocated cokriging alternative: SPE Annual Technical Conference and Exhibition, 833–842.