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Arbitrarily thin metamaterial structure for perfect absorption and giant magnification

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Abstract: In our common understanding, for strong absorption or amplification in a slab structure, the desire of reducing the slab thickness seems contradictory to the condition of small loss or gain. In this paper, this common understanding is challenged. It is shown that an arbitrarily thin metamaterial layer can perfectly absorb or giantly amplify an incident plane wave at a critical angle when the real parts of the permittivity and permeability of the metamaterial are zero while the absolute imaginary parts can be arbitrarily small. The metamaterial layer needs a totally reflective substrate for perfect absorption, while this is not required for giant magnification. Detailed analysis for the existence of the critical angle and physical explanation for these abnormal phenomena are given.

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References and links

1. Introduction

When a wave propagates in a lossy material, it is absorbed gradually. To reduce the interface reflection, absorbers usually possess tapered micro-structures [1]. Recently, some thin flat metamaterial absorbers have been demonstrated [2–5], which can easily fulfill impedance match and large loss. In these cases, strong absorption in a short distance requires large loss, but perfect absorption (100%) cannot be archived. As another type of absorbers based on resonance, the well-known Salisbury screen can perfectly absorb a normally incident plane wave theoretically [1], but the spacer between the resistive sheet and the perfect electric conductor (PEC) substrate cannot be very thin (typically of the order of the wavelength in the spacer). Here we want to raise an interesting fundamental question, namely, can perfect absorption be realized with an arbitrarily thin structure whose material loss can be very small? If the answer is positive, an absorber can physically be made arbitrarily thin, and one can greatly improve the performance of, e.g., detectors since their dependence on material loss can be greatly relaxed. Similarly, another interesting question can be raised, that is, whether giant amplification can be achieved with an arbitrarily thin structure of low gain. Usually, large gain is expected for strong amplification, but the gain cannot be very large in practice. A special material of zero permittivity \( \varepsilon \) or permeability \( \mu \) provides us some special properties [6–10]. Several interesting applications have been reported, such as directive radiation and spatial filtering [6–8], squeezing electromagnetic energy [9] and nonlinear optics [10]. Here we will show that special materials with zero real(\( \varepsilon \)) and real(\( \mu \)) (hereinafter referred to as ZRMs) can find amazing applications in absorption and magnification. With the assumption of a time harmonic factor \( \exp(-i\omega t) \), positive imag(\( \varepsilon \)) and imag(\( \mu \)) represent loss, and negative ones represent gain. A lossy or active ZRM layer can perfectly absorb or strongly amplify an incident plane wave, while amazingly the thickness of the ZRM layer, as well as \( \text{imag}(\varepsilon) \) and \( \text{imag}(\mu) \), can be arbitrarily small. Metamaterials hold the potential to realize ZRMs. By careful tuning the electric and magnetic resonant units that form a metamaterial [11], both real(\( \varepsilon \)) and real(\( \mu \)) may vanish at some frequency. At some Dirac point with a zero Bloch wave vector, a specially designed photonic crystal may be homogenized as a metamaterial with both effective \( \varepsilon \) and \( \mu \) approaching zero [12]. As a special metamaterial composed of small atoms, a gas of electromagnetic induced transparency (EIT) may possess negative \( \varepsilon \) and \( \mu \) [13]. By choosing appropriately some EIT parameters, it is also possible to make real(\( \varepsilon \)) and real(\( \mu \)) vanish.

2. Perfect absorption and giant magnification

Figure 1(a) illustrates the investigated structure. A slab is sandwiched between two semi-infinite layers, and the top layer, the slab, and the bottom layer are denoted as layers 0, 1 and 2, respectively. The permittivity and permeability of layer \( n \) are denoted by \( \varepsilon_n \) and \( \mu_n \), respectively. The magnetic field is along the \( z \) axis (TM polarization). When a plane wave impinging in the downward direction on the slab (the incident angle is \( \theta \) and the corresponding transverse wave vector is \( k_s \)), the magnetic and electric fields in layer \( n \) can be expressed as

\[
H_{n,\pm}(\mathbf{r}) = (H_{n}^{x}e^{ik_nz} + H_{n}^{-x}e^{-ik_nz})e^{ik_sy},
\]

\[
E_{n,\pm}(\mathbf{r}) = -(k_n/\omega\varepsilon_n)(H_{n}^{y}e^{ik_nz} + H_{n}^{-y}e^{-ik_nz})e^{ik_sy},
\]

\[
E_{n,\pm}(\mathbf{r}) = (k_n/\omega\varepsilon_n)(H_{n}^{x}e^{ik_nz} - H_{n}^{-x}e^{-ik_nz})e^{ik_sy},
\]

where \( k_{n,\pm} = (k_n^2 - k_s^2)^{1/2} \), \( k_n \) is the wave number in layer \( n \), and \( H_{n}^{\pm} \) is the magnetic field amplitude of a down- or up-going plane wave component in layer \( n \) as shown in Fig. 1. According to the electromagnetic boundary conditions, one can obtain \( H_n^\pm \) and the corresponding field distribution in layer \( n \). Due to the symmetry of the structure, it is sufficient to study the electromagnetic response for \( k_s \geq 0 \).

First we assume that layer 0 is of free space, layer 1 (ZRM) is lossy with \( \varepsilon_1 = i\varepsilon_1, \mu_1 = i\mu_1 \), \( \varepsilon_0, \varepsilon_2 \geq 0 \) and \( \mu_0, \mu_2 \geq 0 \), and layer 2 is a PEC. The reflection coefficient of the slab is

\[
\frac{1145051 - $15.00 USD  Received 30 Mar 2011; revised 4 May 2011; accepted 17 May 2011; published 23 May 2011  (C) 2011 OSA 6 June 2011 / Vol. 19, No. 12 / OPTICS EXPRESS  11115
\[ R_{\text{loss}} = \frac{H_0^*}{H_0^0} = \frac{1}{\tanh(\gamma d_1) - \frac{1}{\tanh(\gamma d_1) + \frac{1}{\gamma / k_0}}}. \] (2)

where \( \gamma = (\varepsilon_{1r} / \mu_{1r} / k_0^2 + k_y^2)^{1/2} \). Since all the variables on the right-hand side of Eq. (2) are real numbers when \( k_y < k_0 \), the reflected wave is either in phase or out of phase with respect to the incident wave. The numerator on the right-hand side of Eq. (2) is denoted by \( f_{\text{loss}} \), and may become zero for some \( k_y < k_0 \) (corresponding to some incident angle) in some situations. The first term in \( f_{\text{loss}} \) decreases as \( k_y \) increases from 0 to \( k_0 \), and is always larger than or equal to 1. The second term in \( f_{\text{loss}} \) increases from a minimal value of \((\mu_{1r} / \varepsilon_{1r} / k_y)^{1/2}\) to infinity as \( k_y \) increases from 0 to \( k_0 \). Thus, when \( \mu_{1r} / \varepsilon_{1r} / k_y \leq 1 \), there always exists some value of \( k_y < k_0 \) making \( f_{\text{loss}} \) zero, no matter how small \( d_1 \) becomes. Then \( R_{\text{loss}} \) also becomes zero. This means that there exists an incident angle at which the incident plane wave is completely (100%) absorbed by the lossy slab since the absorptivity is equal to \( 1 - |R_{\text{loss}}|^2 \). This incident angle is referred to as the critical angle (denoted by \( \theta_c \)) hereafter. When \( \mu_{1r} / \varepsilon_{1r} / k_y > 1 \), the existence of \( \theta_c \) depends on the value of \( d_1 \). If \( d_1 \) is too large compared with the wavelength in free space \( (\lambda_0) \), \( \theta_c \) does not exist, because the first term in \( f_{\text{loss}} \) is smaller than \( 1/\tanh[(\varepsilon_{1r} / \mu_{1r} / k_y)^{1/2}(k_0 d_1)] \), which approaches 1 when \( d_1 \) is very large, whereas the second term is always larger than 1. When \( \varepsilon_{1r} \) and \( \mu_{1r} \) are given with \( \mu_{1r} / \varepsilon_{1r} / k_y > 1 \), the threshold of \( d_1 \) allowing the existence of \( \theta_c \) is determined by the following equation

\[ \tanh(\varepsilon_{1r} / \mu_{1r} / k_0 d_1) - \frac{1}{\tanh(\varepsilon_{1r} / \mu_{1r} / k_y)} = 0. \] (3)

As a demonstration, Figs. 2(a) and 2(b) show the reflectivity \(|R_{\text{loss}}|^2\) of the slab for various \( k_y \). When \( \mu_{1r} / \varepsilon_{1r} / k_y \leq 1 \), there are always deep dips representing perfect absorption on the reflectivity curves [see curves 1–4 in Fig. 2(a) and curves 1 and 2 in Fig. 2(b)]. When \( \mu_{1r} / \varepsilon_{1r} / k_y > 1 \) and \( d_1 \) is large enough, such dips disappear [see curve 5 in Fig. 2(a) and curve 3 in Fig. 2(b); note that perfect absorption is still possible for some appropriate thickness \( d_1 \) when \( \mu_{1r} / \varepsilon_{1r} / k_y > 1 \)]. Comparing Figs. 2(a) and 2(b), one sees that when \( \mu_{1r} / \varepsilon_{1r} = \text{match} \) (i.e., matched impedance), if \( \varepsilon_{1r} \) and \( \mu_{1r} \) are large, \( \theta_c \) is near to zero (i.e., perfect absorption at nearly normal incidence) and strong absorption over a wide range of incidence angle can be achieved. If \( \varepsilon_{1r} \) and \( \mu_{1r} \) are small (i.e., small loss), perfect absorption near normal incidence can still be achieved when \( \varepsilon_{1r} / \mu_{1r} \) is much smaller than \( \mu_{1r} / \varepsilon_{1r} \) (see curve 4 of Fig. 2(a)). Note that contrary to all the absorbers reported previously, the values of \( d_1 \), \( \varepsilon_{1r} \) and \( \mu_{1r} \) can be very small (arbitrarily small) in the situation of perfect absorption.

![Fig. 1. (a) Configuration for a slab of ZRM sandwiched between two semi-infinite layers. (b) Wave decomposition of the reflected or transmitted field.](image)

The value of \( \theta_c \) depends on \( d_1 \), \( \varepsilon_{1r} \) and \( \mu_{1r} \) as illustrated well in Fig. 2(a). The second term in \( f_{\text{loss}} \) is approximately inversely proportional to \( \varepsilon_{1r} / \mu_{1r} \). As \( \varepsilon_{1r} / \mu_{1r} \) decreases gradually (with fixed \( \mu_{1r} / \varepsilon_{1r} / k_y \) and \( d_1 \)), \( \theta_c \) should approach zero so that the first term in \( f_{\text{loss}} \) can cancel the second term. If \( \varepsilon_{1r} / \mu_{1r} \) decreases further and becomes smaller than some value which satisfies Eq. (3) with the other parameters given, \( \theta_c \) will not exist. Similarly, as \( \mu_{1r} / \varepsilon_{1r} \) increases, the second term in \( f_{\text{loss}} \) also increases, and \( \theta_c \) should approach zero in order to make \( f_{\text{loss}} \) zero. And if \( \mu_{1r} / \varepsilon_{1r} \) increases further, \( \theta_c \) will not exist. When \( d_1 \) is gradually reduced with fixed \( \varepsilon_{1r} / \mu_{1r} \) and \( \mu_{1r} / \varepsilon_{1r} \), \( \theta_c \) becomes larger. Note that the first term in \( f_{\text{loss}} \) is very large when \( d_1 \) is very small. To make the second term in \( f_{\text{loss}} \) also large, \( k_y \) needs to approach \( k_0 \) (i.e., \( \theta_c \) approaches 90 degrees). On the other hand, \( \theta_c \) approaches zero as \( d_1 \) increases. \( \theta_c \) will not exist any more when \( d_1 \) increases further and becomes larger than some value satisfying Eq. (3).

The above perfect absorption can be understood by coherent cancelling. As shown in Fig. 1(b), the reflected wave in Fig. 1(a) can be considered as a composition of infinite plane wave
The reflection coefficient of the slab is

\[ R_{\text{loss}} = r_{0,1} + t_{0,1} e^{-2i\gamma_1 t + r_{0,1} e^{-2i\gamma_1 d_1} + t_{1,0} e^{-2i\gamma_1 d_1} + ...}, \]  

(4)

where \( r_{0,1} = (1 - \gamma E_{1,x}^r k_{0,x})(1 + \gamma E_{1,x}^r k_{0,x}), \) \( r_{1,0} = -r_{0,1}, \) \( t_{0,1} = 2e_{1,r} E_{1,x}^r k_{0,x} + \gamma, \) \( t_{1,0} = 2\gamma E_{1,x}^r k_{0,x} + \gamma \). The first term on the right-hand side of Eq. (4) represents component \( p_0 \) in Fig. 1(b), and the second term represents the composition of the other components, \( p_n \) \( (n = 1, 2, ...) \). From Eq. (2), one sees that \( \gamma E_{1,x}^r k_{0,x} \) must be larger than 1 to make \( f_{\text{loss}} \) zero since \( \tanh(\gamma d_1) < 1 \). Thus, \( r_{0,1} \) is negative and \( r_{1,0} \) is positive. Then, all components \( p_1, p_2, ... \) are in phase, and they are out of phase with component \( p_0 \). At critical angle \( \theta_c \), the two groups cancel each other. This leads to the disappearing of the reflected wave in layer 0, and the incident plane wave is perfectly absorbed by the lossy slab. From Eq. (1), one sees that inside the slab, \( E_{1,i}(r) \) is out of phase with \( H_{1,i}(r) \), and time-averaged energy stream density \( P_i(r) \) has a zero component along the \( y \) axis. This indicates that when the incident plane wave enters the slab, it will just be normally multi-reflected by surface \( S_{0,1} \) and \( S_{1,2} \) and repeatedly absorbed. During this process, there is no phase introduced, leading to a result that the exponents in the square on the right-hand side of Eq. (4) just possess negative real variables (instead of complex variables). There is no transverse shift along the \( y \) axis among components \( p_0, p_1, ... \) in Fig. 1(b). As a numerical example, Figs. 2(c)-2(e) show the electric and magnetic fields and time-averaged energy stream density (represented by arrows) around the slab when \( d_1 = 0.4\lambda_0, e_{1,x}'' = 0.3, \mu_{1,y}'' = 0.1, \) and \( \theta_c = 19.8 \) degrees. To show clearly the distributions of the field and time-averaged energy stream density inside the slab, a relatively large value of \( d_1 \) is chosen for Figs. 2(c)-2(e). The distributions inside a thinner slab are similar. These distributions clearly show that there is no wave reflected by the slab. When the position approaches surface \( S_{1,2} \), \( E_{1,i}(r) \) has to tend to zero as required by the boundary condition at PEC surface \( S_{1,2} \), and so is \( P_i(r) \), whereas \( E_{1,i}(r) \) and \( H_{1,i}(r) \) have no such tendency. For perfect absorption, a PEC as the substrate of the slab is necessary. If it is removed, the plane wave components refracted into the substrate (after being multi-reflected by surfaces \( S_{0,1} \) and \( S_{1,2} \)) cannot cancel each other, and the incident plane wave can partially transmit through the slab as a total effect.

![Reflectivity and field distributions of a slab with a PEC substrate](image)

Next we investigate an opposite case, namely, \( e_1 \) and \( \mu_1 \) are only of negative imaginary parts (i.e., \( e_1 = -i\epsilon_{1,y}''\epsilon_0 \) and \( \mu_1 = -i\mu_{1,y}''\mu_0 \)). Recently, there have been some efforts in active metamaterials [14,15]. Then, an incident wave is magnified by the active metamaterial slab. The reflection coefficient of the slab is

\[ R_{\text{gain}} = 1/[\tanh(\gamma d_1) + \gamma E_{1,x}^r k_{0,x}]1/[\tanh(\gamma d_1) - \gamma E_{1,x}^r k_{0,x}], \]  

which is reciprocal to Eq. (2) for \( R_{\text{loss}} \). When \( R_{\text{loss}} \) is zero, \( R_{\text{gain}} \) is infinite. Then, there exists a
critical angle \( \theta_c \) at which the plane wave impinging on the slab is infinitely magnified. The relation between \( \theta_c \) and the values of \( d_1, \varepsilon_{1,r}^{''}, \mu_{1,r}^{''} \) is similar to that for the previous lossy slab. Especially, when \( \mu_{1,r}^{''} / \varepsilon_{1,r}^{''} \leq 1, \theta_c \) always exists even if the thickness and gain of the slab are arbitrarily small. Curve 1 in Fig. 3(a) shows a numerical example. The infinite magnification can be understood as follows. The condition of \( R_{\text{gain}} = \infty \) determines the dispersion equation of the slab waveguide. Now, special waveguide modes can exist for \( k_y < k_0 \).

The energy stream inside the slab is normally reflected back and forth by surfaces \( S_1 \) and \( S_2 \) (instead of propagating along the slab). Some electromagnetic energy runs away from the slab, but it can be compensated by the energy generated by the gain. When an incident wave excites a waveguide mode, the total reflected wave will be infinite. The infinite magnification is from the time-harmonic solution. In practice, it may take an infinitely long time to obtain this effect. However, one can still obtain giant magnification after long enough time.

![Fig. 3. Reflectivity and transmissivity of a slab. In (a), \( d_1 = 0.1\lambda_0, (\varepsilon_1^{''}, \mu_1^{''}) = (-0.3i, -0.3i) \) and layer 2 is a PEC for curve 1 and of free space for curves 2 and 3. In (b), \( d_1 = 0.1\lambda_0 \) for curve 1 and \( d_1 = 0.5\lambda_0 \) for curves 2–4, \((\varepsilon_1^{''}, \mu_1^{''}) = (-0.1i, 0.3i) \), and layer 2 is a PEC for curves 1 and 2 and of free space for curves 3 and 4.](#)

Now, the hybrid cases are analyzed, that is, \( \text{real}(\varepsilon_1) \) and \( \text{real}(\mu_1) \) possess different signs. Only the case of \( \varepsilon_1 = -i\varepsilon_{1,r}^{''} \varepsilon_0 \) and \( \mu_1 = i\mu_{1,r}^{''} \mu_0 \) is investigated here, and the contrary case can be analyzed similarly. The reflection coefficient of the slab then becomes

\[
R_{\text{hybrid}} = \frac{\tan(k_{1,y}d_1) - k_{1,y}/k_{0,y}\varepsilon_{1,r}^{''}}{\tan(k_{1,y}d_1) + k_{1,y}/k_{0,y}\varepsilon_{1,r}^{''}}
\]

(when \( k_y^2 < \varepsilon_{1,r}^{''} \mu_{1,r}^{''} k_0^2 \), \( \varepsilon_{1,r}^{''} \mu_{1,r}^{''} < 1 \)). In Eq. (5), \( \tan(k_{1,y}d_1) \) is a periodic function with its value varying from \( +\infty \) to \( -\infty \). Thus, both the numerator and denominator on the right-hand side of Eq. (5) have a possibility to be zero when \( k_y < k_0 \). When \( d_1 \) is large enough, many critical angles may exist at which the numerator or denominator on the right-hand side of Eq. (5) is zero. At the right-hand side of Eq. (6), the numerator is always larger than zero, and the denominator can be zero at some value of \( k_y \) since \( \beta k_0 \varepsilon_{1,r}^{''} \) increases from zero to infinite in the range of \( \varepsilon_{1,r}^{''} \mu_{1,r}^{''} < 1 \). This indicates that regardless of the values of \( d_1, \varepsilon_{1,r}^{''}, \) and \( \mu_{1,r}^{''} \), there always exists a critical angle \( \theta_c \) at which the incident plane wave can be infinitely magnified when \( \varepsilon_{1,r}^{''} \mu_{1,r}^{''} < 1 \). This is a quite interesting result that although \( \varepsilon_{1,r}^{''} \mu_{1,r}^{''} \) may be very large (representing large loss), small \( \varepsilon_{1,r}^{''} \) (representing low gain) still can lead to infinite magnification, which may bring convenience in practical magnification. Curves 1 and 2 in Fig. 3(b) show the reflectivity for different thickness of the slab when \( \varepsilon_{1,r}^{''} = -0.1 \) and \( \mu_{1,r}^{''} = 0.3 \). When \( d_1 = 0.1\lambda_0 \), there is only one peak on curve 1. When the slab becomes thick, e.g., \( d_1 = 0.5\lambda_0 \), both one dip and one peak appear on curve 2, which indicates that one can obtain both strong absorption and magnification at different special values of the incident angle.

Finally, we give two remarks for the perfect absorption and giant magnification. The first remark is that the PEC substrate can be removed in the case of giant magnification (unlike the case of perfect absorption). When the substrate is also a free space, and \( \varepsilon_1 = -i\varepsilon_{1,r}^{''} \varepsilon_0 \) and \( \mu_1 = -i\mu_{1,r}^{''} \mu_0 \), one has the following reflection and transmission coefficients of the slab.
\[
R_{\text{gain}} = \left[ \frac{\gamma}{k_0, \varepsilon_1, r'' - k_0, \varepsilon_1, r'' / \gamma} \right]/\left[ 2 / \tanh(\gamma d_1) - (\gamma / k_0, \varepsilon_1, r'' + k_0, \varepsilon_1, r'') / \gamma \right],
\]
\[
T_{\text{gain}} = H_2^f / H_0^f = \left[ 4 / (e^{\gamma d_1} - e^{-\gamma d_1}) \right]/\left[ 2 / \tanh(\gamma d_1) - (\gamma / k_0, \varepsilon_1, r'' + k_0, \varepsilon_1, r'') / \gamma \right].
\]

The denominator on the right-hand side of Eqs. (7) and (8) is denoted by \( f(r) \). The first term in \( f(r) \) decreases as \( k_\gamma \) increases from 0 to \( k_\gamma \) and is always larger or equal to 2. The second term in \( f(r) \) is infinite when \( k_\gamma \) approaches \( k_\gamma \) and reaches a minimal value of 2 when \( \gamma k_1, \varepsilon_1, r'' = k_1, \varepsilon_1, r'' / \gamma \). When \( k_1, \varepsilon_1, r'' \leq 1 \), this condition can be fulfilled by some \( k_\gamma \left( < k_\gamma \right) \), and a critical angle \( \theta_c \) exists. In this case, if \( \theta_c \) is a PEC, the incident plane wave at \( \theta_c \) can be infinitely magnified regardless of the values of \( d_1, \varepsilon_1, r'' \) and \( \mu_1, r'' \). If \( \mu_1, r'' / \varepsilon_1, r'' \geq 1 \), this property disappears. As shown in Fig. 3(a), the value of \( \theta_c \) without a PEC substrate is different from that with a PEC substrate. Similarly, giant magnification can still be obtained in a hybrid case when layer 2 is of free space, as shown by the peaks of curves 3 and 4 in Fig. 3(b) as a numerical example. These two curves also indicate that perfect absorption does not occur in this case since transmissivity curve 4 has no dip although there is a dip on reflectivity curve 3. The second remark is that the deviation of \( \text{real}(\varepsilon_1) \) and \( \text{real}(\mu_1) \) from zero may cause the disappearance of perfect absorption and giant magnification. However, as illustrated in Figs. 4(a) and 4(b), if \( \text{real}(\varepsilon_1) \) and \( \text{real}(\mu_1) \) are moderately small compared with \( \text{imag}(\varepsilon_1) \) and \( \text{imag}(\mu_1) \), respectively, strong absorption and magnification effect can still be obtained. As shown in Fig. 4, the influence of the deviation of \( \text{real}(\varepsilon_1) \) from zero on the absorption and magnification is different from that of \( \text{real}(\mu_1) \). In general, when \( \text{imag}(\varepsilon_1) = \text{imag}(\mu_1) \), the influence of the same deviation of \( \text{real}(\varepsilon_1) \) and \( \text{real}(\mu_1) \) from zero (i.e., with impedance match kept) on the absorption and magnification is smaller.

![Fig. 4. Reflectivity of a slab with a PEC substrate when real(\(\varepsilon_1\)) and real(\(\mu_1\)) deviate from zero.](image)

In (a), \( d_1 = 0.1 \lambda_0 \) and \((\varepsilon_1, \mu_1)\) is \((3i, 3i), (0.05 + 3i, 0.05 + 3i), (0.05 + 3i, 3i)\) and \((3i, 0.05 + 3i)\) for curves 1–4, respectively. In (b), \( d_1 = 0.5 \lambda_0 \) and \((\varepsilon_1, \mu_1)\) is \((-0.3i, 0.3i), (0.05-0.3i, 0.05 + 0.3i), (0.05-0.3i, 0.3i)\) and \((-0.3i, 0.05 + 0.3i)\) for curves 1–4, respectively.

3. Conclusion

In summary, we have shown that an incident plane wave can be perfectly absorbed or giantly amplified by a ZRM layer. The existence of a critical angle \( \theta_c \) has been analyzed for various situations. The thickness of the ZRM layer, as well as \( \text{imag}(\varepsilon_1) \) and \( \text{imag}(\mu_1) \), can be arbitrarily small. This challenges our common understanding that strong absorption and magnification seems impossible in a very thin layer of material with finite or small \( \text{imag}(\varepsilon_1) \) and \( \text{imag}(\mu_1) \). It should be noted that in principle the homogeneous ZRM can be arbitrarily thin. In practical realization, the smallest thickness of a ZRM layer is determined by the dimension of the resonant metamaterial units, which is usually one or two order smaller than the wavelength. However, if the ZRM layer can be realized by an EIT gas, the thickness can be reduced even further. Even if the ZRM layer cannot be very thin, it is still rather significant, especially for giant magnification.

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