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Guiding Simulations and Experiments using Continuation

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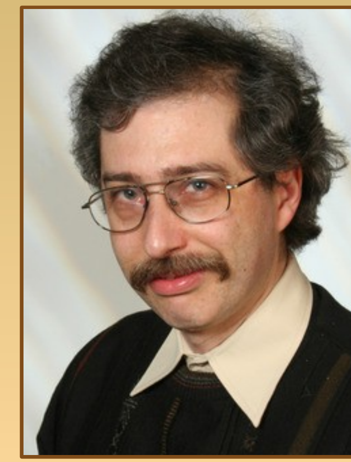

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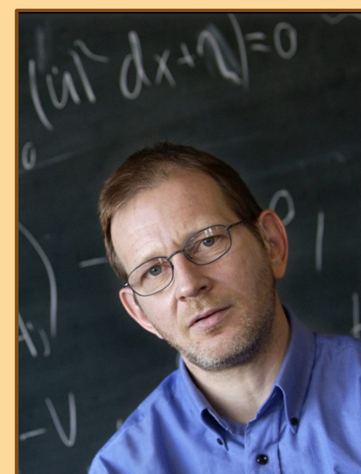

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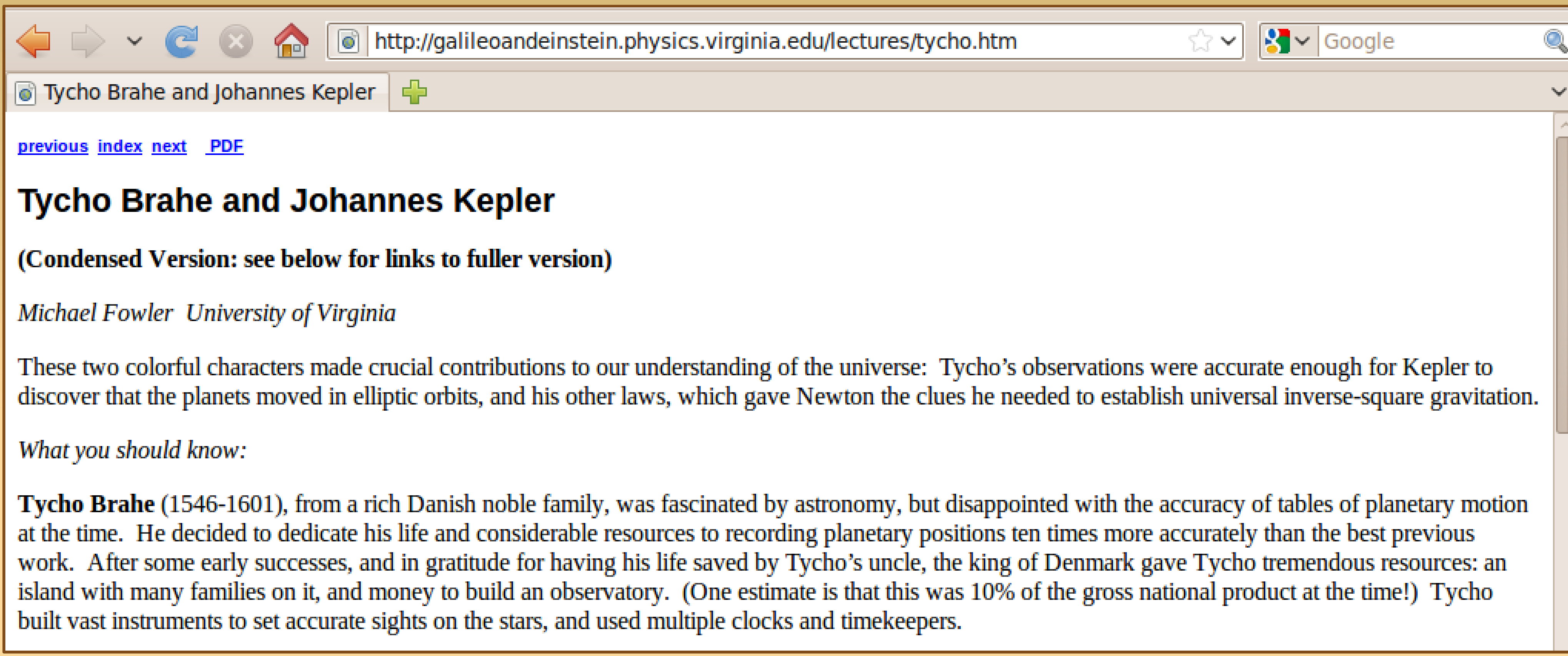
http://galileoandeinstein.physics.virginia.edu/lectures/tycho.htm

Tycho Brahe and Johannes Kepler

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Tycho Brahe and Johannes Kepler

(Condensed Version: see below for links to fuller version)

Michael Fowler University of Virginia

These two colorful characters made crucial contributions to our understanding of the universe: Tycho's observations were accurate enough for Kepler to discover that the planets moved in elliptic orbits, and his other laws, which gave Newton the clues he needed to establish universal inverse-square gravitation.

What you should know:

Tycho Brahe (1546-1601), from a rich Danish noble family, was fascinated by astronomy, but disappointed with the accuracy of tables of planetary motion at the time. He decided to dedicate his life and considerable resources to recording planetary positions ten times more accurately than the best previous work. After some early successes, and in gratitude for having his life saved by Tycho's uncle, the king of Denmark gave Tycho tremendous resources: an island with many families on it, and money to build an observatory. (One estimate is that this was 10% of the gross national product at the time!) Tycho built vast instruments to set accurate sights on the stars, and used multiple clocks and timekeepers.

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http://www.imm.dtu.dk/~ht/

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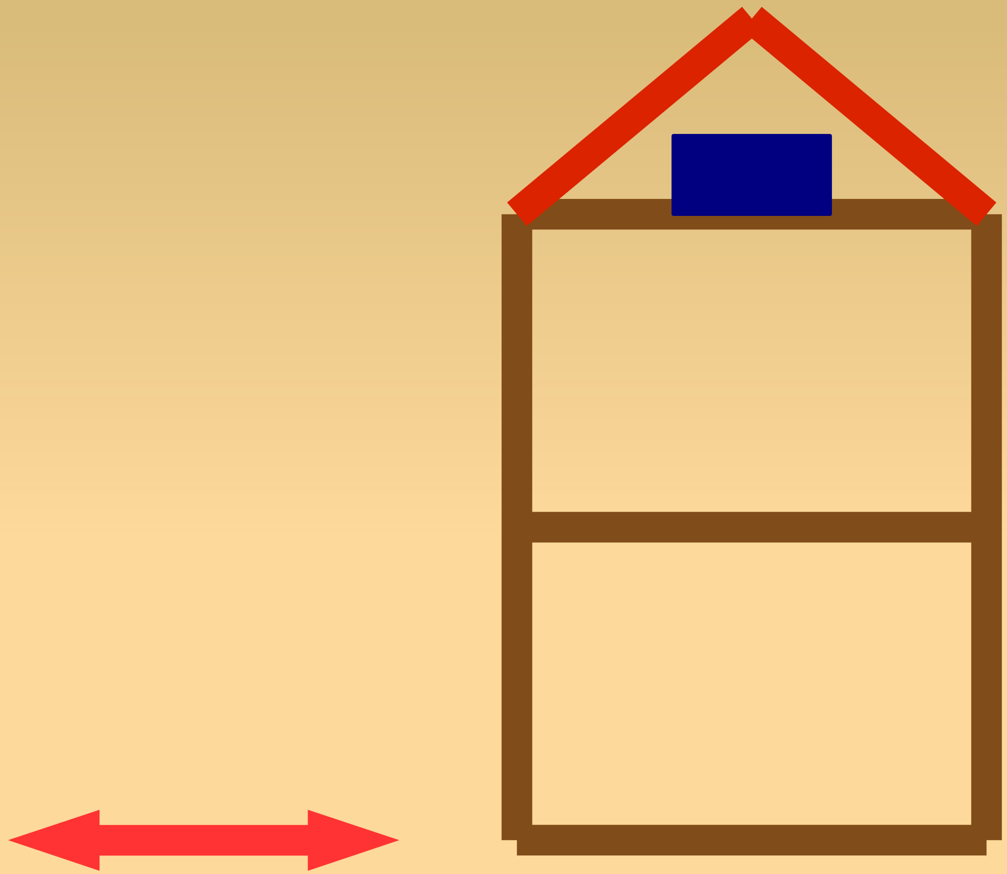

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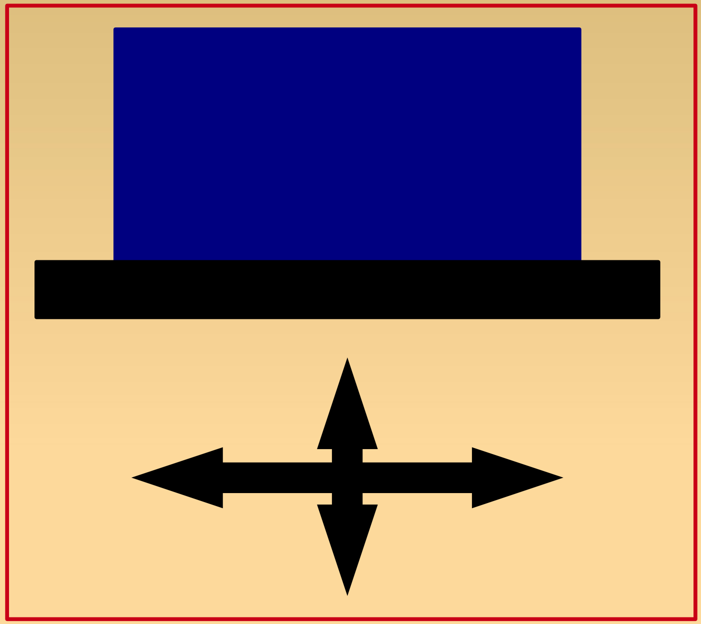

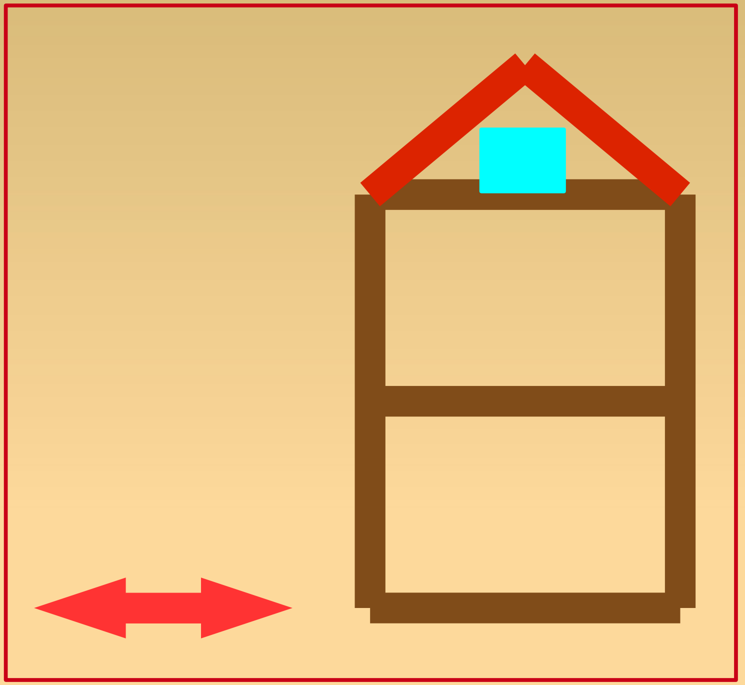

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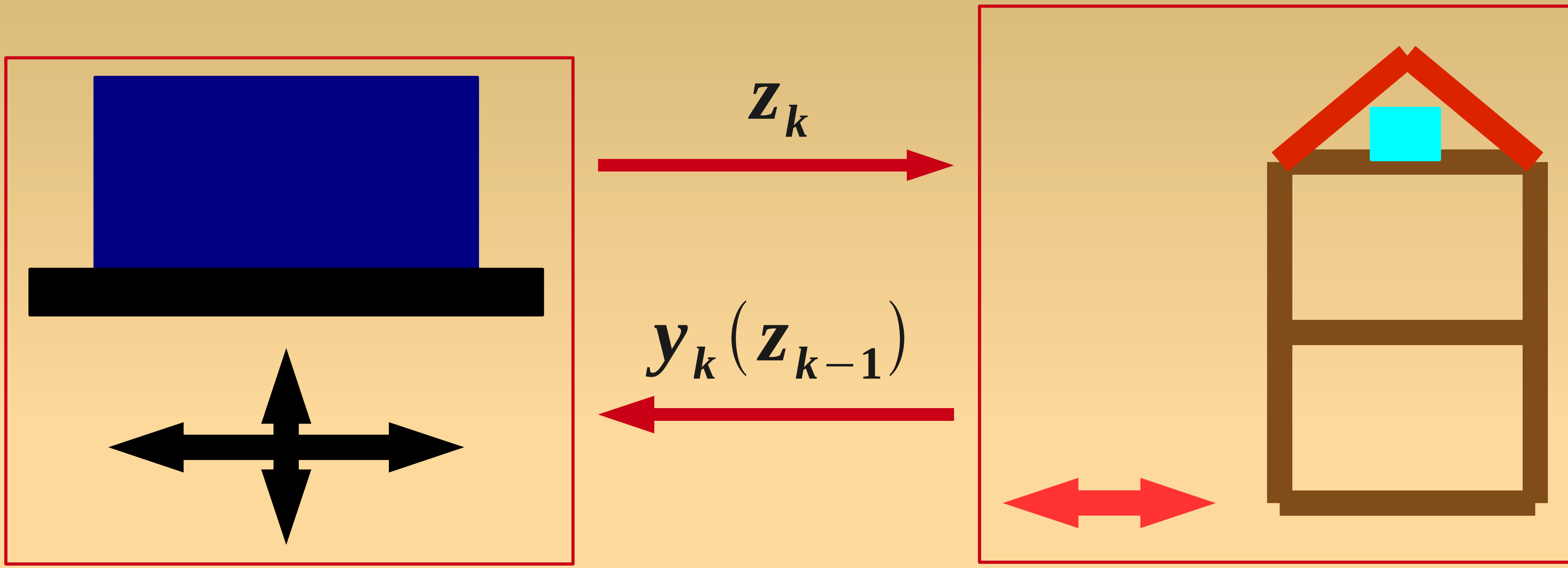

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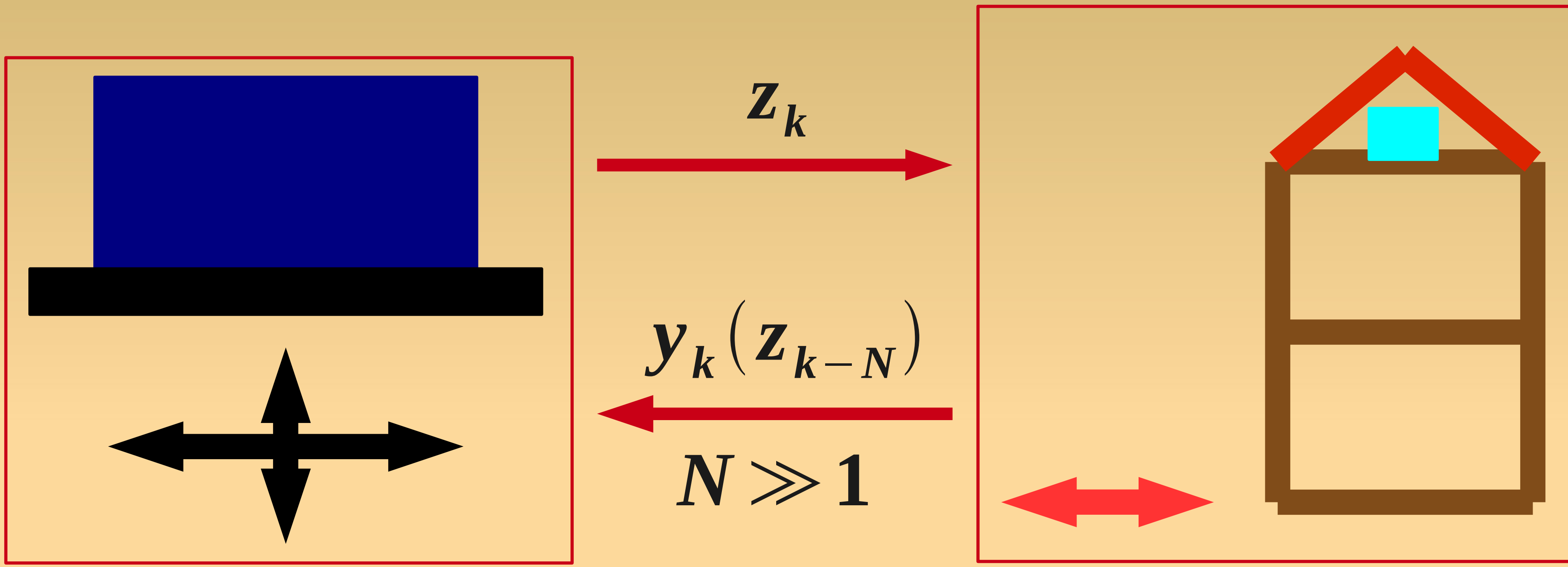

Guiding Simulations and Experiments using Continuation

Sources of delay in dynamic sub-structuring:

- **Computation time of time step of model**
- **Delayed actuation of physical experiment**

This delay is usually sufficiently large to destabilise the combined system.

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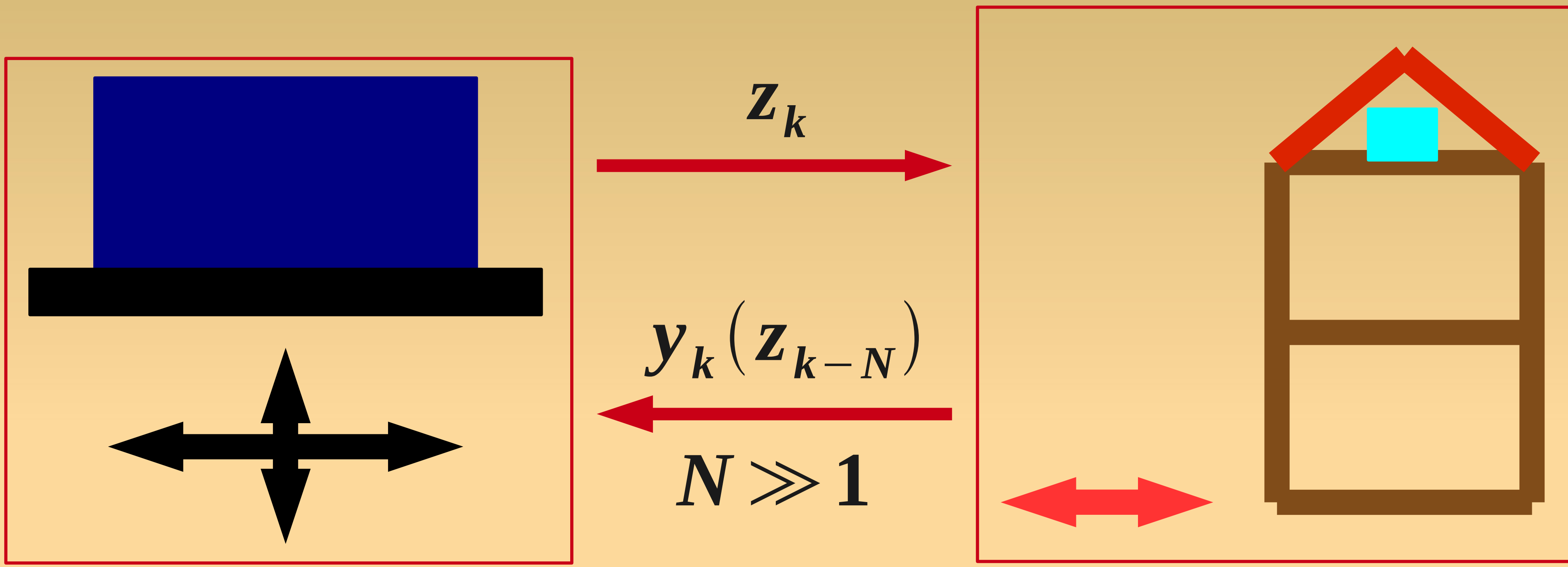

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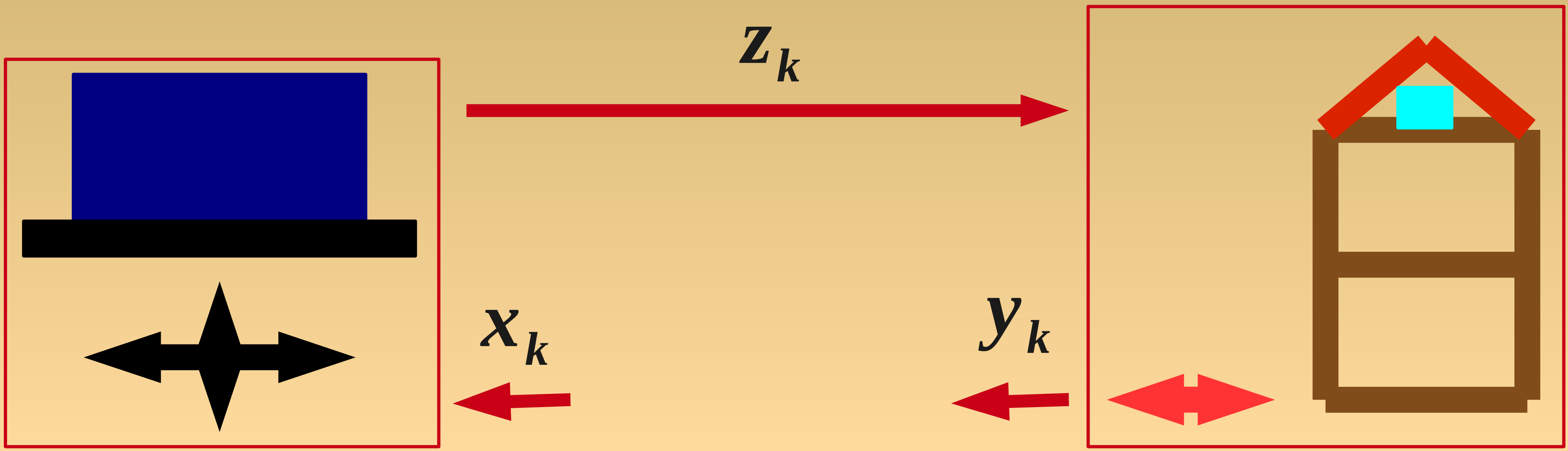

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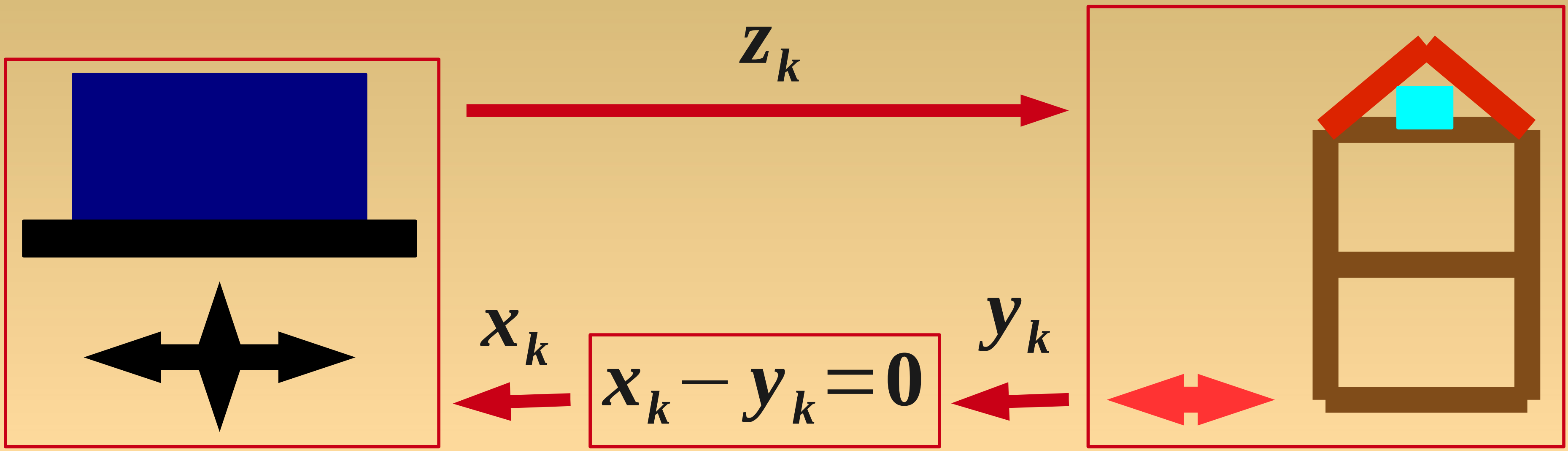

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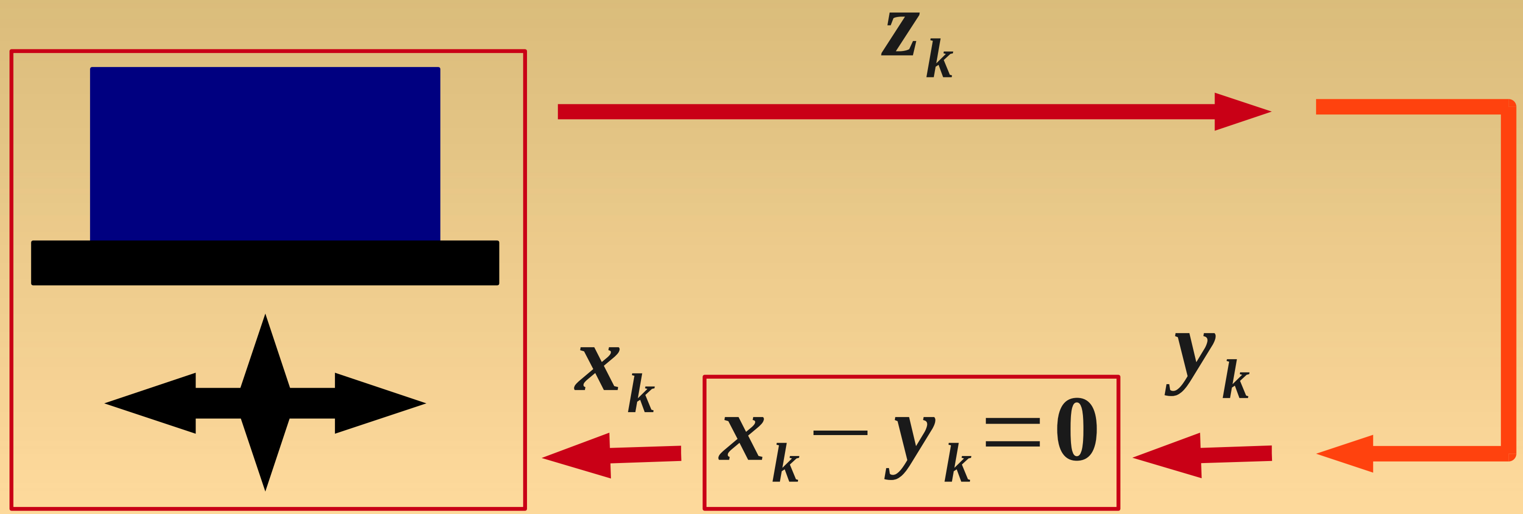
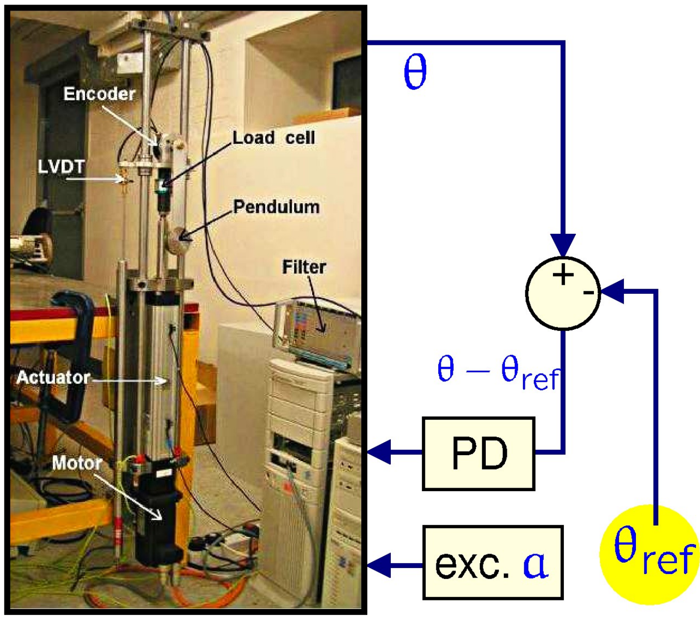


Vertically excited pendulum

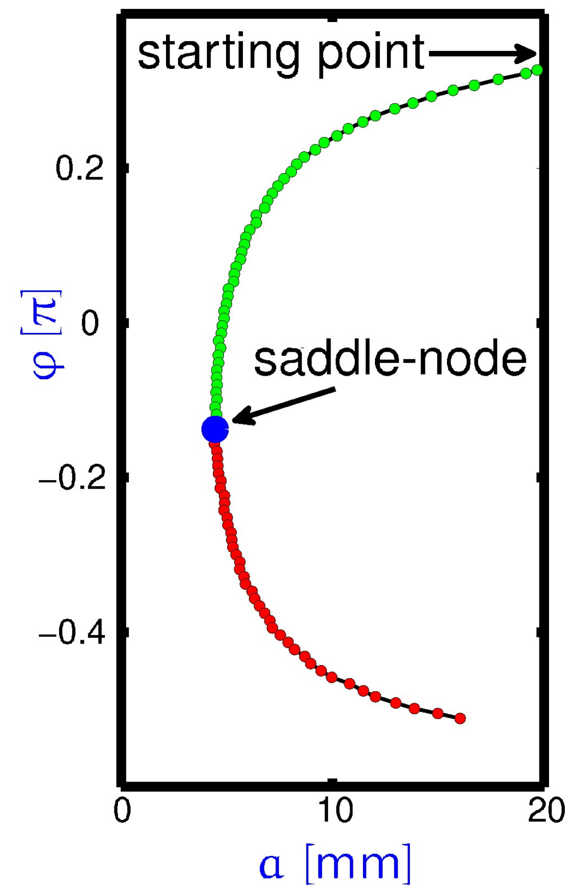
Set-up

feedback control for angle θ

reference signal θ_{ref}



Continuation of rotations



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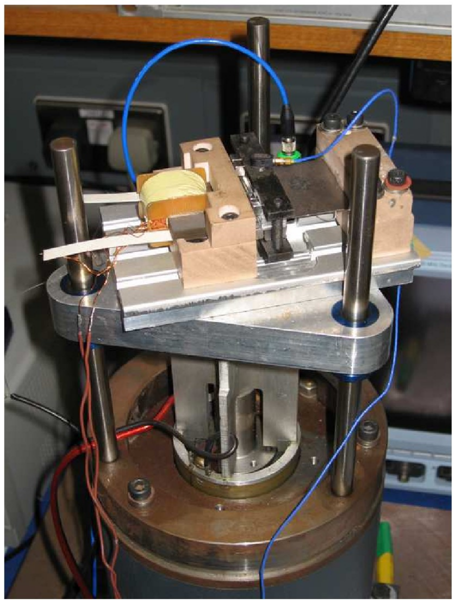
Sieber and Krauskopf, *Nonlinear Dynamics*, **2008**

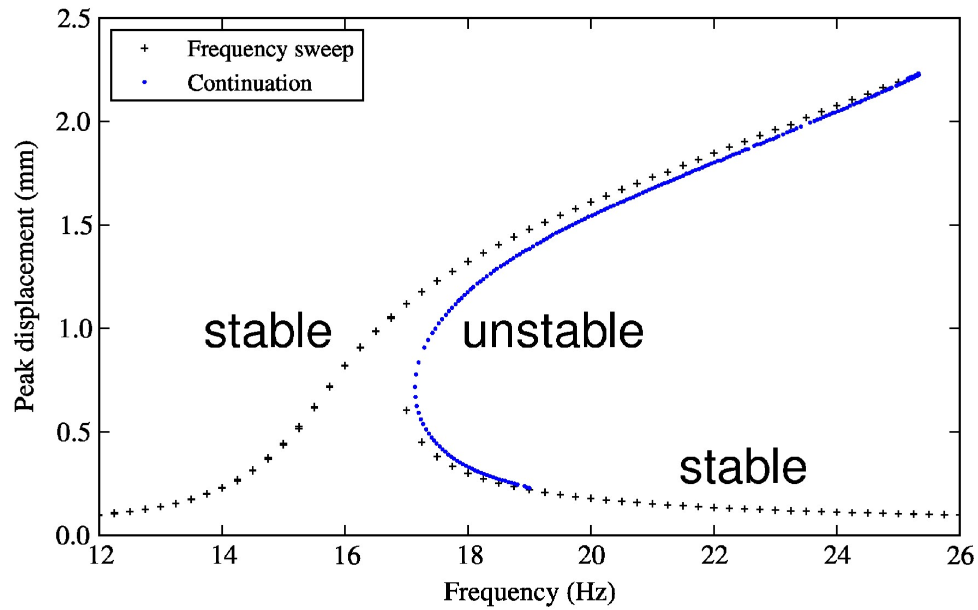
Sieber, Krauskopf, Wagg, Neild and Gonzalez-Buelga, *JCND*, **2011**

Continuation in an energy harvester

continuation experiment by David Barton and Stephen Burrow



setup



nonlinear frequency response

[Barton & Burrow, ASME 2009 Proceedings]

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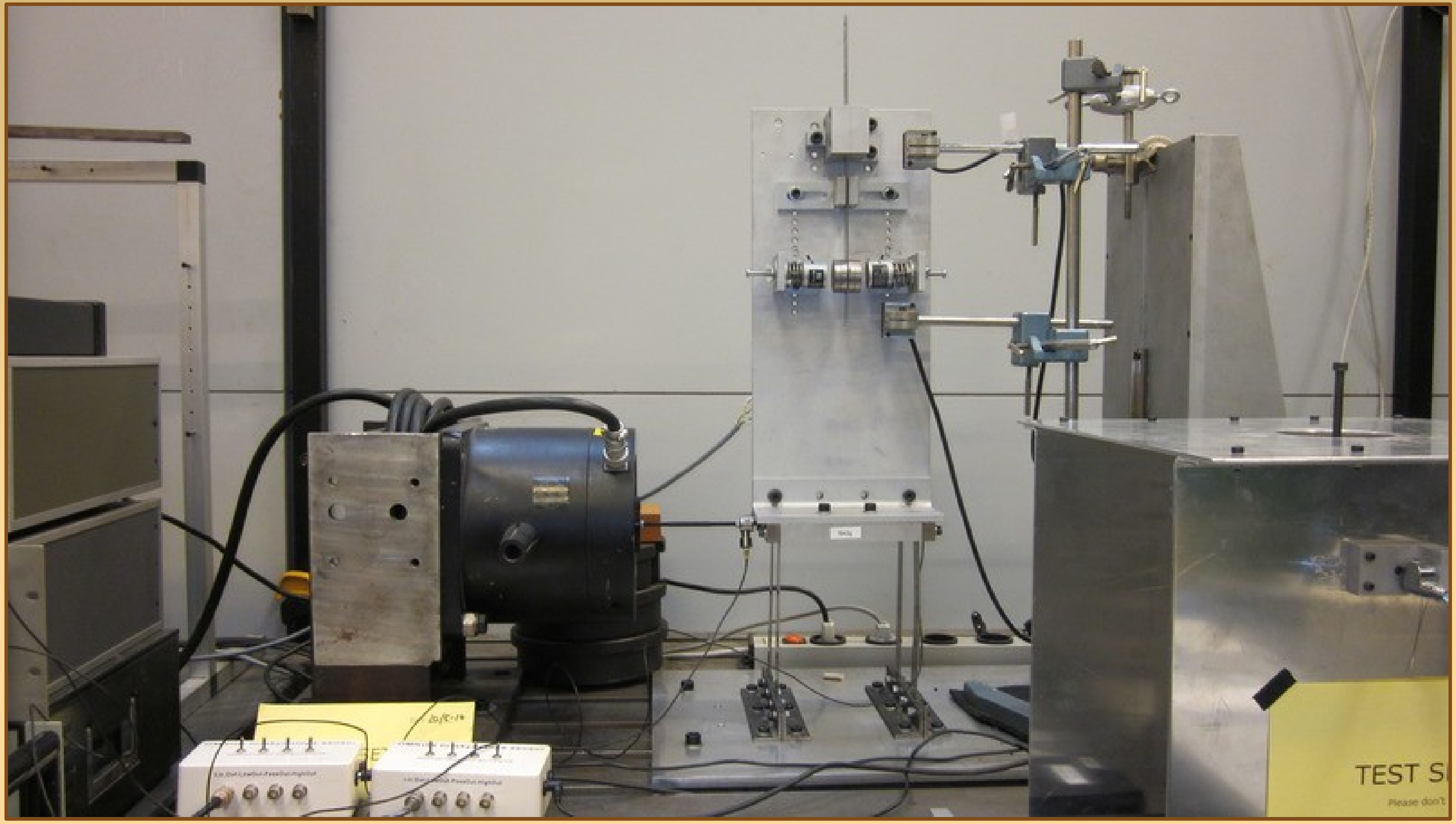
Barton, Mann and Burrow, ***J. Vib. Control*****, accepted manuscript.**

Guiding Simulations and Experiments using Continuation



Test rig with impact oscillator.

Control based Continuation

Continuation rests upon the Implicit Function Theorem. Assume we are given an equation of the form

$$F(u, \mu) = 0, \quad F : X \times R^m \rightarrow Y$$

and we know an initial solution. Then, a continuation algorithm will compute a covering of the solution manifold through the initial solution.

To apply continuation we need to construct a zero problem for our experiment.

Control based Continuation

Fundamental idea: introduce feed back control that uses

$$0 \overset{!}{=} x(t) - z(t)$$

as a control target and apply Newton (like) method to solve the equation

$$x_Q - z_Q = 0,$$

where x_Q and z_Q are discretisations of $x(t)$ and $z(t)$.

Control based Continuation

The control scheme must satisfy a number of conditions.

Notation:

- **Experiments** $Y, \; Z_u$
- **Measurements** $y(\mu, t), \; z(\mu, t, x(t))$
- **Samples** $Y(\mu, N) = \{y_{0,}\ldots, y_{N-1}\}, \; Z_u(\mu, N, x)$

Control based Continuation

1. Consistent: $Y \equiv Z_0$

[Continuous? , Smooth?]

Control based Continuation

1. Consistent: $$Y \equiv Z_0$$

2. Locally stabilising: any equilibrium state *y* of *Y* must become an asymptotically stable equilibrium state of Z_u. In other words, if a controlled experiment Z_u with control target

$$0 \overset{!}{=} y(t) - z(t)$$

is initialized close to an equilibrium state of *Y*, then the state *z* must converge to the state *y* over time.

Control based Continuation

3. Non-invasive: $\|\boldsymbol{u}\| \leq \delta \|\boldsymbol{x}-\boldsymbol{z}\|, \quad \boldsymbol{x} \in \boldsymbol{U}_{\varepsilon}(\boldsymbol{y})$

Remember, the control target is $\boldsymbol{0} \stackrel{!}{=} \boldsymbol{x}(\boldsymbol{t}) - \boldsymbol{z}(\boldsymbol{t})$, so any linear control scheme will typically be consistent and non-invasive.

In many applications a PD controller will be locally stabilising.

In our implementation we use Simulink's PID block, which is non-linear and time-dependent, but locally monotonic around 0.

Control based Continuation

Discretisation.

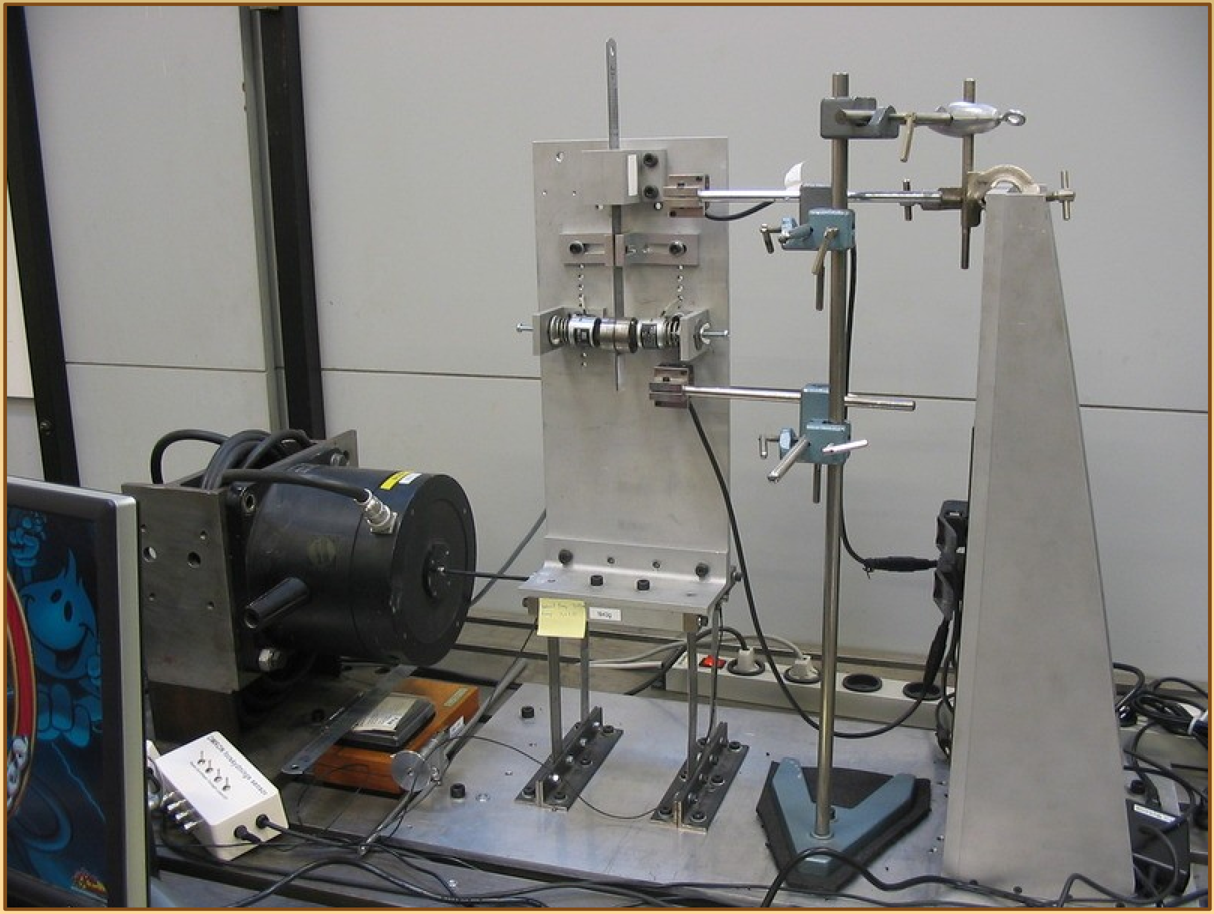
- **Fourier transformation** $\Phi_Q : R^N \rightarrow R^{2Q+1}, \quad \Phi_Q^{-1} : R^{2Q+1} \rightarrow C^{\infty}$
- **Target function** $c \in R^{2Q+1}, \quad x(t) := \Phi_Q^{-1}(c)$
- **Control force** $u(t) := G(x(t)) := PD(x(t) - z(t))$
- **Zero problem** $F(c, \mu) := \Phi_Q(Z_u(\mu, N, G(\Phi_Q^{-1}(c)))) - c$

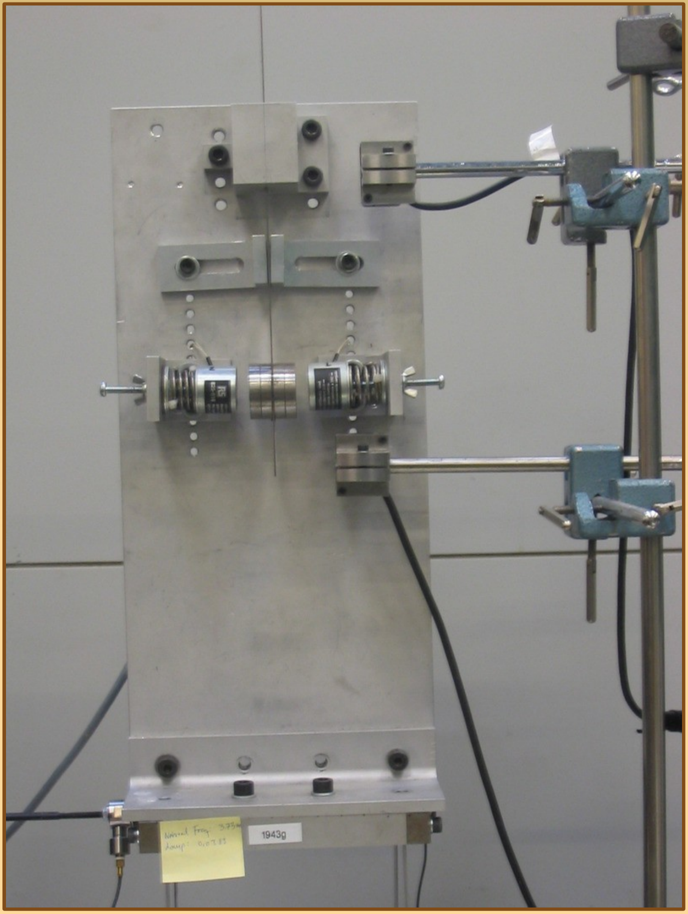
Control based Continuation

Proof of concept.

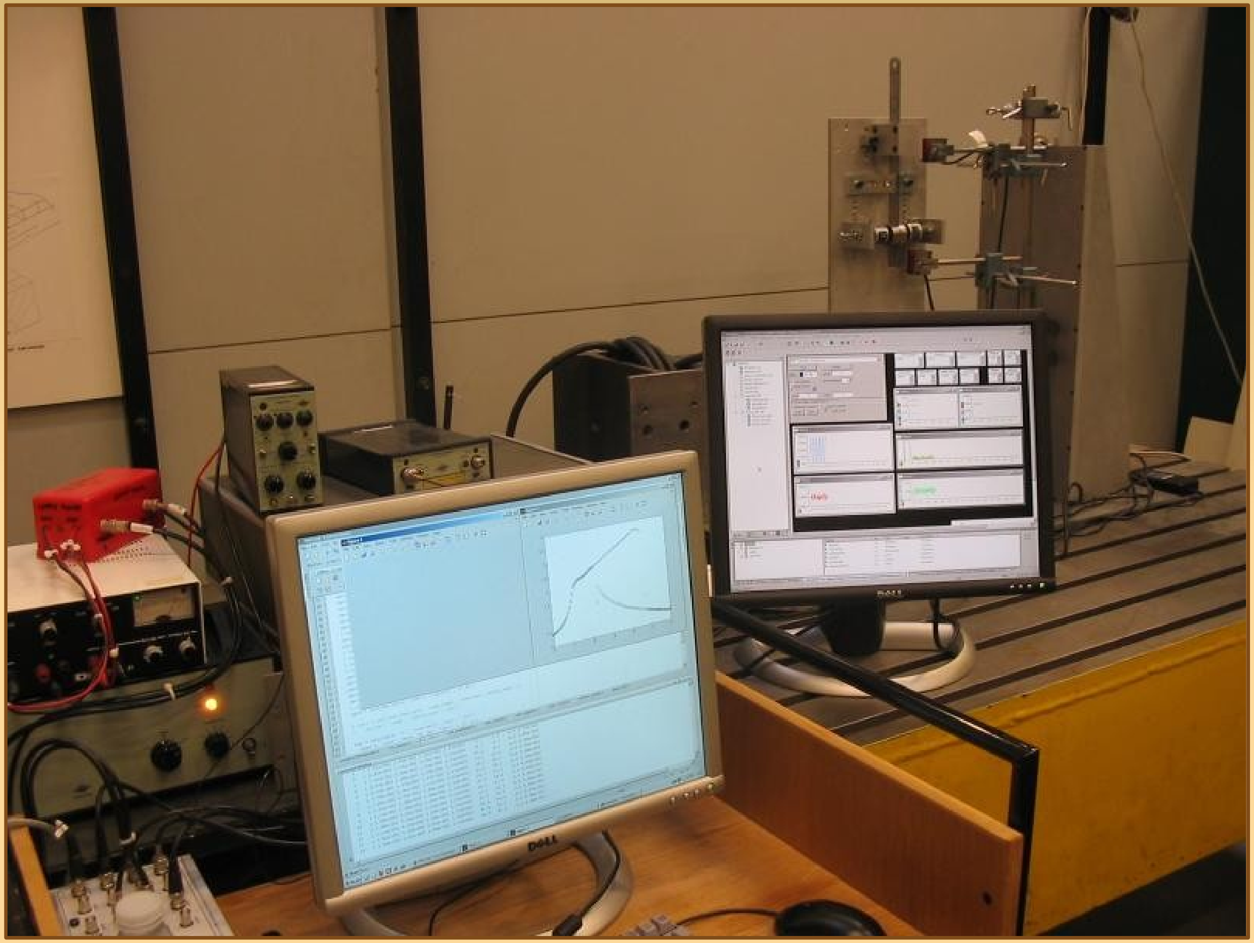
$$\begin{aligned}\|\boldsymbol{u}\| &= \|\boldsymbol{PD}(\boldsymbol{x}-\boldsymbol{z})\| \\ &\le \delta\|\boldsymbol{x}-\boldsymbol{z}\| \\ &\le \delta\kappa\|\Phi_\infty(\boldsymbol{x})-\Phi_\infty(\boldsymbol{z})\| \\ &\le \delta\kappa\left(\|\boldsymbol{F}(\boldsymbol{c},\mu)\|+\boldsymbol{R}_{\boldsymbol{Q}+\boldsymbol{1}}\right)\end{aligned}$$

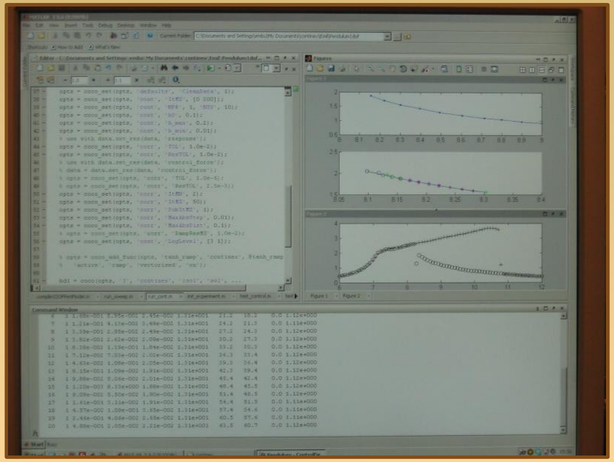
Example: Impact Oscillator

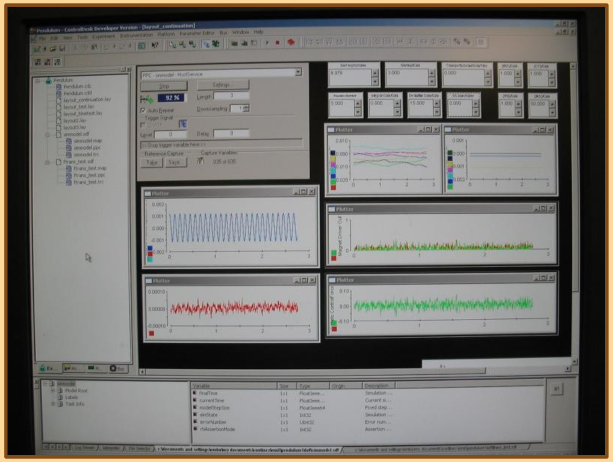




Example: Impact Oscillator



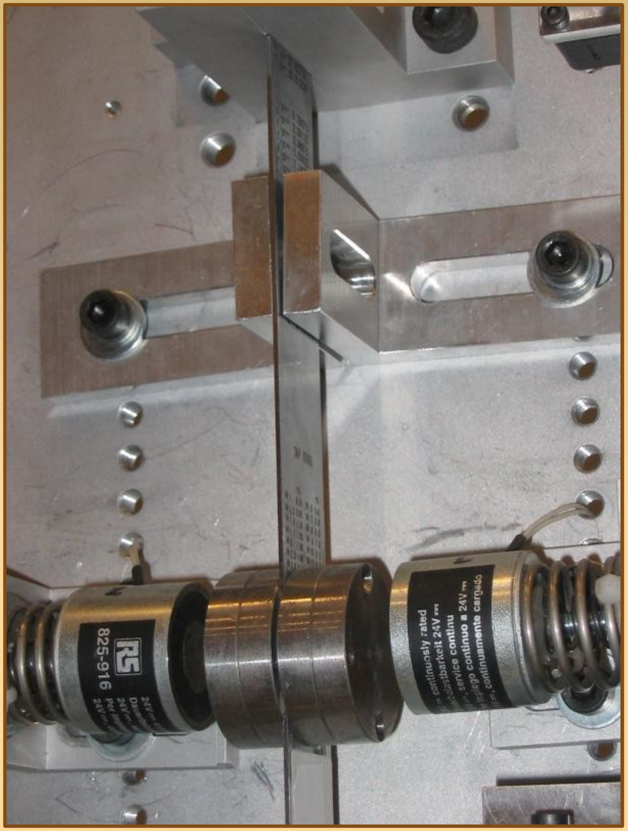


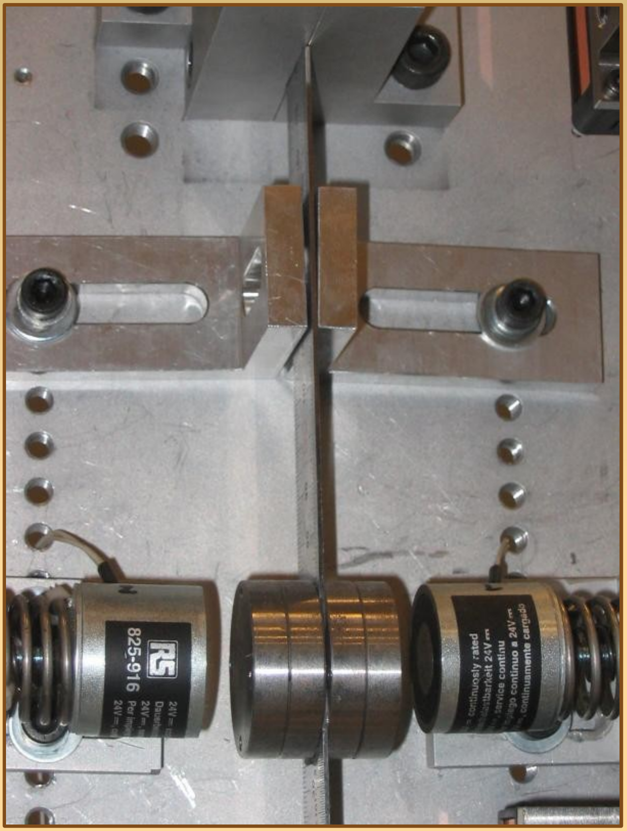


Example: Duffing Oscillator

Simulations by Viktor.

Experimental results.





Current and Future Work

Matlab toolbox for:

- Guided simulations
- Guided experiments
- Equation free methods

Techniques for:

- Measuring stability
- Detecting bifurcations
- Branch-switching etc.

Current and Future Work

Unstable solutions are useful!

Pyragas control: $$\mathbf{0} \overset{!}{=} z(t) - z(t-T)$$

Sieber control: $$\mathbf{0} \overset{!}{=} y(t) - z(t)$$

Very simple (low-tech) auto-adaptive control schemes.

Exploit, don't destroy natural dynamics!