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Development of Energy and Reserve Pre-dispatch and Re-dispatch Models for Real-time Price Risk and Reliability Assessment

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ABSTRACT

In the future energy framework of European Union and other countries, renewable energy plays an important role tackling the problems of the climate change and security of energy supply. The share of fluctuating and less predictable renewable power production will increase significantly the needs of securing proper balancing between generation and demand. The high penetration of renewable energy sources will also increase the burden of system operator for maintaining system reliabilities. However the current strategy of reliability management developed for conventional power systems and existing electricity market design may not cope with the future challenges the power system faces. The development of smart grid will enable power system scheduling and the electricity market to operate in a shorter time horizon for better integrating renewable energy sources into power systems. This paper presents an electricity market scheme including a multi-period energy and reserve pre-dispatch model and an energy re-dispatch model for real time operation considering their coupling with the day-ahead market, respectively. The multi-period energy and reserve pre-dispatch model is formulated using the multi-period optimal power flow technique, which pre-schedules the generation output for satisfying the expected demand and determines up and down spinning
reserve for each time interval in the operational hour. The *ex-ante* electricity prices and reserve capacity prices are also evaluated correspondingly. During the real time operation, the energy re-dispatch model is used for contingency management and providing balancing services based on the results of the energy and reserve pre-dispatch model. The energy re-dispatch model is formulated as a single-period AC OPF model, which is used to determine generation re-dispatch, load curtailment as well as real-time electricity prices. The modified IEEE-RTS has been analyzed to illustrate the techniques. The proposed market scheme coupled with a contingency analysis methodology has been used to evaluate both real-time electricity price risk and short term reliabilities during the operational hour in the new environment.

**Index Terms**—energy and reserve dispatch, reliability, price risk, optimal power flow

**NOMENCLATURE**

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\( \Delta Q_{fi}^\sigma(t) \) Reactive power change of flexible demand from the reference point in the day-ahead market

\( \bar{P}_d(t) \) Real power of fixed demand in pre-dispatch

\( \bar{Q}_d(t) \) Reactive power of fixed demand in pre-dispatch

\( V_{i_{\text{min}}} \) Lower limit of voltage at bus \( i \)

\( V_{i_{\text{max}}} \) Upper limit of voltage at bus \( i \)

\( S_{i_{\text{max}}} \) Limit of apparent power of line \( l \)

\( V_i = |V_i| e^{j\theta_i} \) Bus voltage at bus \( i \)

\( Y_{ik} = |Y_{ik}| e^{j\delta_{ik}} \) Element of admittance matrix

\( p_{g(i)}^{\text{up}} - p_{g(i)}^{\text{down}} \) Up and down ramp limit of generator

\( p_{g(i)}^{\text{min}} - p_{g(i)}^{\text{max}} \) Minimum and maximum real power output of generator

\( q_{g(i)}^{\text{min}}, q_{g(i)}^{\text{max}} \) Minimum and maximum reactive power output of generator

\( CC_{si} \) Interruption cost of customer sector

\( \Delta P_{g(i)}(t) \) Generator real power re-dispatched from the reference point in the day-ahead scheduling

\( \Delta Q_{g(i)}(t) \) Generator reactive power re-dispatched from the reference point in the day-ahead scheduling

\( CP_{si}(t) \) Real power curtailment of customer sector

\( CQ_{si}(t) \) Reactive power curtailment of customer sector

\( P_d(t) \) Real power of demand in re-dispatch

\( Q_d(t) \) Reactive power of demand in re-dispatch

1. **Introduction**

The future electricity consumption will be based on a low carbon generation mix, hence substantially increasing the contribution from renewable and more efficient energy sources. In the European Union’s (EU) future energy plan [1], renewable energy will reach a 20% share in total energy consumption by 2020. In Denmark, the wind power penetration will reach 50% of the electricity consumption by 2025 [2]. The high penetration of fluctuating and less predictable renewable energy sources (RES) entails a need for future power system scheduling that is able to adapt rapidly to the fluctuation in the power system. Continually securing proper balancing between generation and demand and maintaining system reliabilities are the major objectives faced by the system operator. Therefore it is critically important to dispatch energy and reserve resources with fast responses for real time balancing both in normal state and contingency states.

The integrated market design for the procurement of energy and reserve has been studied in several previous researches [3]-[5], which can produce accurate price signals for both energy and reserve. An AC optimal power flow (OPF) model has been developed for simultaneously procuring and pricing energy and reserve in [3]. Reference [4] proposed a joint energy and reserve market model including demand side reserve offers. A simultaneous energy and reserve market considering transmission network flow limits and security constraints has been proposed in [5].
However the current design of electricity markets and reserve management strategy may not be sufficient to manage the future challenges the power system faces. Firstly time resolution for markets e.g. one hour used in the Nordic power system [6] may be too rough an approximation of the dynamics in the power system with high penetration of RES, which cannot precisely reflect actual status of power system operation. Secondly the developed integrated energy and reserve markets are forward markets. The correlations of these forward markets with the real-time energy market and corresponding impacts on system and customers’ reliabilities in the operational hour have not been comprehensively studied.

The reliability evaluation techniques of conventional power systems have been well developed [7-9]. In the deregulated environment, customers are more concerned about their individual reliabilities than the system reliabilities [10]. Customers’ reliabilities and price risk are also correlated. The methods for determining nodal reliabilities and nodal electricity price risk for restructured power systems have been developed in [10] and [11]. However these developed methods are more focused on the long-term reliability and price risk evaluation applied in power system planning. In the operational phase, they may be too rough an approximation of intra-hour fluctuations and contingencies. Moreover the correlation between forward markets such as the day-ahead market and the real-time energy market should be incorporated in the reliability and price risk assessment methods of restructured power systems.

Smart grids will be the backbone of the future electricity network, which will facilitate the integration of the high penetration of RES. The development and expansion of information and communication technology (ICT) in smart grid will enable the electricity market to operate in a shorter time horizon and provide valuable information to the system operator taking into account the existing status of the power system operation.

In this paper, a market scheme has been proposed, which includes a day-ahead scheduling model, an energy and reserve pre-dispatch model, and an energy re-dispatch model, respectively. In the day-ahead scheduling model, the status of generating units and the generation scheduling are determined for the operational hour of the next day, which will be used as the reference point in the energy and reserve pre-dispatch model and energy re-dispatch model. The multi-period energy and reserve pre-dispatch model is firstly used for scheduling the generation output for satisfying the expected demand and determining up and down spinning reserve for each time interval in the operational hour. The *ex-ante* electricity prices and reserve capacity prices are also determined correspondingly. In a further step and during the real time operation of the power system, the energy re-dispatch model is used for handling operational condition changes and possible contingencies, which can determine generation re-
dispatch, load curtailment as well as real-time electricity prices. The proposed market scheme coupled with a contingency analysis methodology has been used to evaluate both real-time electricity price risk and short term reliabilities in the new environment. The modified IEEE Reliability Test System (RTS) [12] has been analyzed to illustrate the validity and benefits of the proposed approach.

2. Market Models

An overview of the proposed market structure is illustrated in Fig.1, which includes a day-ahead scheduling model, an energy and reserve pre-dispatch model, and an energy re-dispatch model. In the day-ahead scheduling model, the status of generating units, the generation scheduling and the hourly spot price are determined for the operational hour of the next day. For example, in the Nordic electricity market [6], the Nord Pool day-ahead (spot) market settles at around 13:00 (not later than 14:00) and hourly spot price for the next day will be published. However the hourly time resolution for the day-ahead scheduling and market is a rough approximation of the power system operation, which cannot handle the fluctuations especially from the high penetration of RES and possible contingencies during the operational hour. Therefore the energy and reserve pre-dispatch model and energy re-dispatch model are developed for handling intra-hour fluctuations and contingencies, which will maintain the reliable system operation during the real time. The generation scheduling and status of units determined by the day-ahead (spot) market will be used as the reference point in the energy and reserve pre-dispatch model and energy re-dispatch model.

The time resolution of the energy and reserve pre-dispatch model and energy re-dispatch model can be 5-min, which more accurately reflects status of power system operation. The update interval of 5 minutes enables the distributed energy resources
(DERs) with fast responses and fast generating units for more actively participating in system operation with the development of smart grid. Therefore the 5-min interval has been used in a new EU real time market demonstrated in the Ecogrid EU project [13] [14]. This is also in line with the smart grid project - Olympic Peninsula [15] in US, where a 5-minute interval has documented successes. However the 5-min interval requires the market participants reach their scheduled dispatch target in a short-term horizon, which may preclude a significant portion of generating units with relatively slow responses. Moreover some important operational definitions such as the spinning reserve normally utilizing 10-min lead time would need to be re-defined. Fortunately the fundamental concept and the infrastructure of two developed models may work equally well with longer intervals, e.g. 10 minutes, though better responses can be limited by the interval length. This allows the models to be adapted to include most generating units for providing spinning reserve with 10-min lead time. The 5-min interval can be progressively deployed in some developed countries such as Denmark, where conventional centralized power plants have been gradually replaced by more flexible DERs. With the future development of power systems, the time interval can be further shortened, e.g. 1-min, which may achieve better response for system operation. However it is not very realistic in current situations.

The energy and reserve pre-dispatch model is a multi-period AC OPF model, which will optimally pre-dispatch the generation output and determine the \( \text{ex-ante} \) electricity prices for each time interval of the operational hour based on the day-ahead scheduling. The energy and reserve pre-dispatch model is also used to procure spinning reserve capacity and determine reserve capacity prices.

The energy re-dispatch model is used for contingency management and providing balancing services during the real time system operation according to results of the energy and reserve pre-dispatch model. The energy re-dispatch model is a single-period AC OPF model, which is used to determine generation re-dispatch, load curtailment as well as real-time electricity prices. The following paragraphs provide the mathematical formulations of two optimization models.

### A. Energy and Reserve Pre-dispatch Model

The objective function of the model is to minimize the total system social cost in the studied periods, which equals to the total energy and up and down spinning reserve cost minus the total benefit of flexible demand:

\[
\begin{align*}
\text{Minimize} & \sum_{m} \sum_{g \in M_{g}} \left( \sum_{t} \left( GC_{g} \left( P_{g}^{e} + \Delta P_{g}^{e} (t) \right) + RCU_{g} \left( RU_{g} (t) \right) + RC_{g,d}^{d} \left( RD_{g} (t) \right) \right) \right) \\
& - \sum_{g \in M_{g}} B_{g} \left( P_{g}^{d} + \Delta P_{g}^{d} (t) \right) 
\end{align*}
\]

The decision variables in the objective function include the real power output deviation of unit \( g \) at time interval \( t \) from the day-ahead scheduling.
ahead scheduling \( (\Delta P^d_{gi}(t)) \), up and down spinning reserve capacity of unit \( g \) at time interval \( t \) \( (RU^d_{gi}(t)) \) and \( (RD^d_{gi}(t)) \) respectively, and change of flexible demand \( i \) at time interval \( t \) from the reference point in the day-ahead market \( (\Delta P^f_{gi}(t)) \).

In the flexible demand’s response model, a linear function describing the electricity consumption and the electricity price is considered, which have been used in most technical literatures [16] [17] [18] and a new Ecogrid EU real time market [13] [14]. The “intercept” parameter \( \beta_{gi} \) in S/MWh and the “slope” parameter \( \alpha_{gi} \) in S/MW\(^2\)h are used to determine the flexible demand’s response linear function [17] [18]. Therefore the flexible demand’s benefit can be represented as the quadratic function:

\[
B_i \left( P^0_{gi} + \Delta P^f_{gi}(t) \right) = \frac{1}{2} \alpha_{gi} \cdot \left( P^0_{gi} + \Delta P^f_{gi}(t) \right)^2 + \beta_{gi} \cdot \left( P^0_{gi} + \Delta P^f_{gi}(t) \right)
\]  (2)

The objective (1) is subject to network and market constraints, which are illustrated as follows:

Power flow constraints:

\[
\sum_{i \in N_{N_g} \forall k \in N_{gi}} \left( P_{gi}^0 + \Delta P^f_{gi}(t) \right) - \left( P^0_{gi} + \Delta P^f_{gi}(t) \right) - P_{gi}(t) = \quad \forall i \in N, \; \forall t \in T
\]  (3)

\[
\sum_{i \in I \forall j \in I_{gi}} V^0_{gi}(t) V^0_{pl}(t) \left[ \cos\left( \theta^0_{gi}(t) - \theta^0_{pl}(t) - \delta^0_{gi}(t) \right) \right]
\]

\[
\sum_{i \in N_{N_g} \forall k \in N_{gi}} \left( Q_{gi}^0 + \Delta Q^f_{gi}(t) \right) - \left( Q^0_{gi} + \Delta Q^f_{gi}(t) \right) - Q_{gi}(t) = \quad \forall i \in N, \; \forall t \in T
\]  (4)

\[
\sum_{i \in I \forall j \in I_{gi}} V^0_{gi}(t) V^0_{ql}(t) \left[ \sin\left( \theta^0_{gi}(t) - \theta^0_{ql}(t) - \delta^0_{gi}(t) \right) \right]
\]

Spinning Reserve Requirement:

\[
\sum_{i \in N_{N_g} \forall k \in N_{gi}} RU^d_{gi}(t) \geq RU^{req}(t) \quad \forall t \in T
\]  (5)

\[
\sum_{i \in N_{N_g} \forall k \in N_{gi}} RD^d_{gi}(t) \geq RD^{req}(t) \quad \forall t \in T
\]  (6)

Equations (5) and (6) specify the total amount up and down spinning reserve required by the system, respectively.

Generating unit limits:

\[
\left( P^0_{gi} + \Delta P^f_{gi}(t) \right) + RU^d_{gi}(t) \leq P^{max}_{gi} \quad \forall g \in NG, \; \forall t \in T
\]  (7)

\[
\left( P^0_{gi} + \Delta P^f_{gi}(t) \right) - RD^d_{gi}(t) \geq P^{min}_{gi} \quad \forall g \in NG, \; \forall t \in T
\]  (8)

\[
P^{down}_{gi} \leq \Delta P^f_{gi}(t) \leq P^{up}_{gi} \quad \forall g \in NG, \; \forall t \in T
\]  (9)

\[
Q^{min}_{gi} \leq Q^0_{gi} + \Delta Q^f_{gi}(t) \leq Q^{max}_{gi} \quad \forall g \in NG, \; \forall t \in T
\]  (10)
Notice that $P_{g_t}^D$ and $Q_{g_t}^D$ represent the real and reactive power outputs of unit $g$ for the operational hour obtained from the day-ahead scheduling. Equation (9) is an inter-temporal constraint, which specifies the ramping limits of generating units from the time interval $t-1$ to the time interval $t$.

Spinning reserve limits

$$RU_{g_t}(t) \leq RU_{g_t}^{\text{max}} \quad \forall g \in G, \forall t \in T \tag{11}$$

$$RD_{g_t}(t) \leq RD_{g_t}^{\text{max}} \quad \forall g \in G, \forall t \in T \tag{12}$$

Line flow constraints:

$$S_{l_t}^i(t) \leq S_{l_t}^{\text{max}} \quad \forall l \in L, \forall t \in T \tag{13}$$

Voltage limits:

$$V_{i_t}^{\text{min}} \leq |V_{i_t}^p(t)| \leq V_{i_t}^{\text{max}} \quad \forall i \in N, \forall t \in T \tag{14}$$

The general form of the OPF formulations (1), (3) – (14) can be rewritten as:

**Objective function:**

$$\text{Min } f(X) \text{ for } X \tag{15}$$

where $X$ represents vector of decision variables such as $\Delta P_{g_t}^D(t)$, $RU_{g_t}(t)$, $RD_{g_t}(t)$ and $\Delta P_{g_t}^Q(t)$.

**Subject to:**

Equality constraints (column vector):

$$G(X, \eta) = 0 \tag{16}$$

Inequality constraints (column vector):

$$H(X, \eta) \leq 0 \tag{17}$$

where $\eta$ represents vector of state variables, such as $V_{i_t}^p(t)$ and $\theta_{i_t}^p(t)$.

According to Karush-Kuhn-Tucker (KKT) condition, the optimal solution of the OPF problem (15) – (17) must satisfy [19]:

$$\frac{\partial f}{\partial X} + \lambda \frac{\partial G}{\partial X} + \bar{\lambda} \frac{\partial \bar{H}}{\partial X} = 0 \tag{18}$$

$$G(X, \eta) = 0 \tag{19}$$

$$\bar{H}(X, \eta) = 0 \tag{20}$$

where $\bar{H}$ represents column vector that have active inequalities among $H$, $\lambda$ and $\bar{\lambda}$ are Lagrangian multipliers associated with $G$ and $\bar{H}$, respectively.
The Lagrangian function of the OPF problem can be defined as the sum of system social cost plus a number constraint charges in the studied periods [19]:

\[ \hat{L}(X, \lambda, \rho) = f(X) + \lambda G(X, \eta) + \rho H(X, \eta) \]  

The sequential quadratic programming (SQP) algorithm [20] is used to obtain the optimal solutions, which combines the Newton method with the quadratic programming, has proved to be a highly efficient method for solving the nonlinear programming problem. The SQP has the capabilities of directly handling inequality constraints. After solving the problem, the generation scheduling and up and down spinning reserve can be determined for each time interval.

The ex-ante electricity prices of real power at bus \( i \) for the time interval \( t \) can also be obtained, which are represented as:

\[ \hat{\rho}_p(t) = \frac{\partial \hat{L}}{\partial \dot{P}_d}(t) \]  

\( \hat{\rho}_p(t) \) is the Lagrangian multiplier associated with the power flow constraint (3), which represents the marginal energy cost at bus \( i \) for the time interval \( t \).

The prices of up and down spinning reserve capacity are evaluated as:

\[ \hat{\rho}_U(t) = \frac{\partial \hat{L}}{\partial \dot{RU}^{req}}(t) \]  

\[ \hat{\rho}_D(t) = \frac{\partial \hat{L}}{\partial \dot{DU}^{req}}(t) \]  

where \( \hat{\rho}_U(t) \) and \( \hat{\rho}_D(t) \) are prices for up and down spinning reserve capacity for the time interval \( t \), respectively.

B. Energy Re-dispatch Model

The obtained results from the energy and reserve pre-dispatch model are used as the basic inputs for the energy re-dispatch model, which will re-dispatch generation within reserve capacity limits for maintaining system balance during the real time operation.

The objective is to minimize the total system cost after considering network and market constraints in real time operation. The system cost includes the generation re-dispatched cost and customers’ interruption cost of different customer sectors. For contingency state \( j \) at time interval \( t \), the objective function is:
Minimize $\Delta P_g^i(t), CP_{gi}^i(t)$

$$C^i(t) = \sum_{g \in N_g} \sum_{j \in N_j} GC_{gi}(P_{gi}^0 + \Delta P_{gi}^0(t)) + \sum_{j \in N_j} \sum_{s \in N_s} CC_{si}(CP_{si}^i(t))$$

(25)

The decision variables in the objective function include real power re-dispatched of unit $g$ from the reference point in the day-ahead scheduling ($\Delta P_{gi}^0(t)$) and real power curtailment of customer sector $s$ at bus $i$ ($CP_{si}^i(t)$) for contingency state $j$.

Customer interruption cost ($CC_{si}$) is a function of customer’ load curtailment and the associated interruption characteristics [7] [10]:

$$CC_{si}(CP_{si}^i(t)) = CP_{si}^i(t) \times CDF_i$$

(26)

where $CDF_i$ represents the customer damage function, which is used to evaluate the interruption cost of different customer sectors.

The objective (25) is subject to following network and market constraints:

Power balance constraints:

$$\sum_{g \in N_g} \sum_{j \in N_j} (P_{gi}^0 + \Delta P_{gi}^0(t)) + \sum_{s \in N_s} CP_{si}^i(t) - P_{gi}^i(t) = \forall i \in N$$

(27)

$$\sum_{k \in j} V_{j}^i(t)V_{j}^i(t)\cos(\theta_{k}^j(t) - \theta_{j}^i(t) - \delta_{k}^j(t))$$

$$\sum_{k \in j} V_{j}^i(t)V_{j}^i(t)\sin(\theta_{k}^j(t) - \theta_{j}^i(t) - \delta_{k}^j(t))$$

$$\sum_{k \in j} Q_{j}^i(t)Q_{j}^i(t)\cos(\theta_{k}^j(t) - \theta_{j}^i(t) - \delta_{k}^j(t))$$

$$\sum_{k \in j} Q_{j}^i(t)Q_{j}^i(t)\sin(\theta_{k}^j(t) - \theta_{j}^i(t) - \delta_{k}^j(t))$$

(28)

Generating unit limits:

$$-RD_{gi}^j(t) \leq \Delta P_{gi}^j(t) - \Delta P_{gi}^0(t) \leq RU_{gi}^j(t)$$

(29)

$$P_{gi}^{\text{min}} \leq P_{gi}^0 + \Delta P_{gi}^0(t) \leq P_{gi}^{\text{max}}$$

(30)

$$Q_{gi}^{\text{min}} \leq Q_{gi}^0 + \Delta Q_{gi}^0(t) \leq Q_{gi}^{\text{max}}$$

(31)

Other constraints include line flow constraints, voltage limits, etc.

The SQP algorithm is also used to solve the proposed AC OPF model. After solving the problem, generation re-dispatch and load curtailment can be determined for each contingency state and normal state at time interval $t$. The real time electricity price for state $j$ at bus $i$ for time interval $t$ can also be obtained, which are represented as:

$$\rho_{gi}^j(t) = \frac{\partial L_{gi}^j}{\partial P_{gi}^i(t)}$$

(32)
where $L^{i,j}$ is the Lagrangian function of the OPF problem, which is defined as the total system cost plus a number constraint charges for state $j$ at time interval $t$.

3. RELIABILITY AND PRICE RISK EVALUATION PROCEDURES

A. Reliability and Price Risk Indices

The conventional reliability indexes such as loss of load probability at bus $i$ ($LOLP_i$), and expected energy not supplied at bus $i$ ($EENS_i$) are usually used to assess long-term (steady state) reliabilities of customers at different buses [7]. These indexes have been re-defined to evaluate the operating reliabilities of customers. $LOLP_i(t)$ is defined as the loss of probability at bus $i$ for the time interval $t$, which can be evaluated as:

$$LOLP_i(t) = \sum_{j=1}^{K} p_j(t) \cdot \Omega(\sum_{s \in NL_i} CP_{ij}(t) > 0),$$

where $\Omega(True) = 1, \Omega(False) = 0$. $p_j(t)$ is the probability for state $j$ at time interval $t$, which can be evaluated by equation (35).

$EENS_i(T)$ is defined as the expected energy not supplied at bus $i$ during the operational periods $T$, which can be evaluated as:

$$EENS_i(T) = \sum_{t=1}^{T} \left( \sum_{j=1}^{K} p_j(t) \cdot \sum_{s \in NL_i} CP_{ij}(t) \cdot \Delta t \right)$$

where $\Delta t$ is the duration of the interval $t$, which can be 5 minutes or 10 minutes.

Considering a power system with $N_c$ operating components, probability of state $j$ with exactly $b$ failed components can be determined using the following equations:

$$p_j(t) = \prod_{c=1}^{b} ORR_{c}(t) \times \prod_{c=b+1}^{N_c} A_c(t)$$

where $ORR_{c}(t)$ is the outage replacement rate of the component $c$, which is defined as the probability that the component being failed during the time interval $t$. $A_c(t)$ is the probability that the component $c$ being operating during the time interval $t$, which is evaluated as $A_c(t) = 1 - ORR_{c}(t)$.
The real time electricity prices in a contingency state might be quite different from the prices in the normal state. The expected values \( \bar{\rho}_{pi}(t) \) and the standard deviation \( \sigma_{pi}(t) \), at bus \( i \) for time interval \( t \) are used as price risk indices. These indices can be calculated using the following equations:

\[
\bar{\rho}_{pi}(t) = \sum_{j=1}^{SN} p_j(t) \cdot \rho_{pi}^j(t)
\]

\[
\sigma_{pi}(t) = \sqrt{\sum_{j=1}^{SN} \left( \rho_{pi}^j(t) - \bar{\rho}_{pi}(t) \right)^2 \cdot p_j(t)}
\]

where \( SN \) is the number of system states.

**B. Evaluation Procedures**

The basic procedures for evaluating reliabilities and price risk are as follows:

**Step1:** In the day-ahead market, determine the status of generating units and the generation scheduling for the operational hour and forward the obtained results as reference points to the multi-period energy and reserve pre-dispatch model and the energy re-dispatch model.

**Step2:** For the studied operational periods \( T \), build the multi-period energy and reserve pre-dispatch model based on the expected demand and system up and down spinning reserve requirements for each time interval.

**Step3:** Solve the proposed energy and reserve pre-dispatch model and forward the obtained results of the generation output pre-schedule, as well as up and down spinning reserve for each time interval to the energy re-dispatch model.

**Step5:** For each system state, solve the energy re-dispatch model formulated by the single-period AC OPF model for determining generation re-dispatch, load curtailment and real-time electricity prices.

**Step6:** Evaluate the probability of every system state \( j \) for each time interval using equation (35).

**Step7:** Evaluate reliability indices \( LOLP_i(t) \) and \( EENS_i(T) \) using (33) and (34), respectively.

**Step8:** Evaluate real time price risk indices \( \bar{\rho}_{pi}(t) \) and \( \sigma_{pi}(t) \) using (36) and (37), respectively.

**4. System Studies**

The IEEE-RTS 12 [12] as shown in Fig. 2 has been modified to illustrate the proposed models and techniques. The IEEE-RTS has 10 generating buses, 17 load buses, 33 transmission lines, 5 transformers and 32 generating units. The total installed generating capacity for this system is 3405 MW. It is assumed that all generators at bus \( i \) belong to a single generation company \( i \), which submits bidding cost of energy and reserve respectively for participating in the electricity markets. The demand with capacity of
40 MW located at bus 6 is used to provide flexible responses. The studied operational hour has been divided into 12 time intervals. Each time interval has a time resolution of 5 mins.

Fig. 2 IEEE RTS

Fig. 3 illustrates the *ex-ante* electricity prices in the energy and reserve pre-dispatch model at representative buses – Bus 3 and Bus 15, respectively. The *ex-ante* electricity prices at Bus 15 corresponding to different time intervals range from 15.19 $/MWh to 17.35 $/MWh. The *ex-ante* electricity prices for different time intervals at Bus 3 are little higher than those at Bus 15 because of higher transmission losses, which range from 15.93 $/MWh to 18.77 $/MWh.

Fig. 3: The *ex-ante* electricity prices for representative buses

The capacity requirement of up spinning reserve is 400 MW, which corresponds to the capacity of the largest generating unit. The up spinning reserve capacity prices for different intervals are shown in Fig. 4 ranging from 2.513 $/MW to 2.529 $/MW.
The capacity requirement of down spinning reserve is 300 MW. The down spinning reserve capacity prices for different intervals are shown in Fig. 5 ranging from 0.953 $/MW to 1.043 $/MW.

Fig. 4: The up spinning reserve capacity prices

Fig. 5: The down spinning reserve capacity prices

Fig. 6: The expected real time electricity prices for representative buses

Fig. 6 illustrates the expected real time electricity prices in the energy re-dispatch model at representative buses – Bus 3 and Bus 15, respectively. The expected real time electricity prices at Bus 3 corresponding to different time intervals are within the range of 17.21 $/MWh to 21.99 $/MWh. The expected real time electricity prices at Bus 15 range from 16.96 $/MWh to 21.63 $/MWh.
Comparing with Fig. 6 with Fig. 3, we can observe that the expected real time electricity prices are higher than the *ex-ante* electricity prices about 8.05% to 18.5% at Bus 3 and 11.6% to 24.7% at Bus 15, respectively because random failures can result in high price volatilities. The high real time electricity prices will encourage customers to more participate in the forward electricity markets therefore reducing fluctuations of power system operation.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Bus 3</th>
<th></th>
<th></th>
<th>Bus 15</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>$LOLP_3(t)$</td>
<td>$\sigma_{p3}(t)$ ($/\text{MWh}$)</td>
<td>$LOLP_{15}(t)$</td>
<td>$\sigma_{p15}(t)$ ($/\text{MWh}$)</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0958%</td>
<td>41.65</td>
<td>0.0696%</td>
<td>41.64</td>
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</tr>
<tr>
<td>2</td>
<td>0.1788%</td>
<td>61.88</td>
<td>0.0699%</td>
<td>61.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.3179%</td>
<td>61.98</td>
<td>0.0702%</td>
<td>61.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
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<td>62.13</td>
<td>0.0705%</td>
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</tr>
<tr>
<td>5</td>
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<tr>
<td>6</td>
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<tr>
<td>7</td>
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<td>62.58</td>
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<tr>
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<td>63.06</td>
<td>0.1009%</td>
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<tr>
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<tr>
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<td>0.1881%</td>
<td>63.55</td>
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<td></td>
</tr>
</tbody>
</table>

Table 1 shows the loss of load probability and the standard deviation of electricity price corresponding to different time intervals at Bus 3 and Bus 15, respectively. The expected energy not supplied at Bus 3 and Bus 15 are 321.75 MWh/yr and 92.44 MWh/yr, respectively. The higher standard deviations are due to the high probability of inadequate generation because of generation failures and line congestions, which result in price spikes.

### 5. Conclusions

The current reliability management strategy has been well developed for long-term reliability and price risk evaluation. However it cannot catch the dynamic behavior of system operation and consider complex correlation among various electricity markets, which becomes more critical for a power system with high fluctuation of RES. Moreover the current electricity market design with low time resolution may not precisely reflect actual status of power system operation. This paper proposes an energy and reserve pre-dispatch model, as well as a real time energy re-dispatch model for real-time price risk and reliability.
assessment. The shorter time resolution of proposed models can provide more accurate knowledge about electricity and reserve prices reflecting status of power system operation. The obtained information of customers’ short-term reliabilities and real time price risk can also be used to assistant system operator and corresponding stakeholders to make optimal decisions for electricity trading and maintaining system security.

6. REFERENCES


