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Published in:
Physical Review B Condensed Matter

Link to article, DOI:
10.1103/PhysRevB.89.041201

Publication date:
2014

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):

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Acoustic gain in piezoelectric semiconductors at $\varepsilon$-near-zero response

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(Received 13 November 2013; revised manuscript received 10 January 2014; published 27 January 2014)

We demonstrate strong acoustic gain in electric-field biased piezoelectric semiconductors at frequencies near the plasmon frequency in the terahertz range. When the electron drift velocity produced by an external electric field is higher than the speed of sound, Cherenkov radiation of phonons generates amplification of sound. It is demonstrated that this effect is particularly effective at $\varepsilon$-near-zero response, leading to giant levels of acoustic gain. Operating at conditions with strong acoustic amplification, we predict unprecedented enhancement of the scattered sound field radiated from an electrically controlled piezoelectric slab waveguide. This extreme sound field enhancement in an active piezo material shows potential for acoustic sensing and loss compensation in metamaterials and nonlinear devices.

DOI: 10.1103/PhysRevB.89.041201 PACS number(s): 77.10.-t, 73.20.Mf, 81.05.Xj, 85.50.—n

We distinguish between a low-frequency regime where acoustic gain unambiguously is explained by the emission of sound due to Cherenkov radiation and a high-frequency one where $\varepsilon$ approaches zero. In ENZ materials, light propagates with almost no phase advance due to the extended sizes of the wavelength [14,15]. This has been achieved by metamaterials and has resulted in prominent applications such as supercoupling and directive emission of light and sensing, to name a few [16]. In the context of PZ acoustic amplification at ENZ response, we show that gain can be many orders of magnitude larger compared to amplification caused by Cherenkov emission. In addition, we design an optomechanical device giving rise to enhanced acoustic radiation triggered by electrical switching with the cycle of half a wave round-trip.

Consider the constitutive relations for a piezoelectric material,

$$T = \varepsilon E - \varepsilon E,$$

$$D = \varepsilon E + P + eS,$$

where $T$, $S$, $D$, $E$, $P$, $c$, $e$, and $\varepsilon$ are the stress, strain, electric displacement, electric field, spontaneous polarization, stiffness, piezoelectric constant, and permittivity, respectively. In cubic (zinc blende) structures the spontaneous polarization is zero, but in hexagonal (wurtzite) structures the spontaneous polarization is nonzero and is usually higher than the piezoelectric contribution to the electric displacement. In reality, the above equations are tensor equations for the crystal;
however, discarding field variations in space except along one coordinate, \( z \), the above scalar system suffices for the analysis.

The constitutive relations above are for isentropic conditions such that Onsager relations apply. This approximation will only be used in the constitutive relations, and losses are accounted for by using a finite and frequency-dependent complex carrier mobility. The elastic equation in one dimension is

\[
\frac{\partial^2 u}{\partial t^2} = \frac{\partial T}{\partial z} = c \frac{\partial^2 u}{\partial z^2} - \varepsilon \frac{\partial E}{\partial z},
\]

where \( \rho \) and \( \omega \) are the mass density and angular frequency, respectively, and \( S = \frac{\rho}{\rho_0} \), with \( u \) being the material displacement. We consider only electrons as the acoustic response of the much heavier holes can be discarded. The continuity equation reads

\[
\frac{\partial J}{\partial z} = -\frac{\partial \rho_e}{\partial t}, \quad J = q \mu_n n E + q F D_n \frac{\partial n}{\partial z},
\]

where \( J \) and \( \rho_e \) are the free-current density and the space-charge density, respectively, \( F \) denotes the fraction of acoustically generated electrons that are free to move, \( q \) is the elementary charge, \( \mu_n \) is the electron mobility, \( D_n \) is the electron diffusivity, and \( n = n_0 + n_s, \ n_e, \) and \( n_0 \) are the total electron density, the generated acoustic electron density, and the electron density at equilibrium, respectively. Combining the above expressions implies an electromechanical dispersion relation:

\[
\rho \omega^2 = c k^2 - \frac{k^2 \omega^2}{\omega + \mu_n n_s E_0 k^2} - \frac{\varepsilon}{\mu_\text{DC}}.
\]

This equation is a fourth-order complex polynomial in \( k = \omega + i \alpha \), where \( \alpha \) is the damping term. Now, since \( |\alpha| \ll \omega \), we may safely, for small fields \( E_0 \), replace \( k \) by \( \frac{\omega}{c} \) in the denominator of the second term on the right-hand side. The resulting dispersion equation is a second-order polynomial in \( k \) whose roots we denote \( k_1 \) and \( k_2 \) henceforth. The dispersion relation is supplemented by a Drude permittivity frequency response for semiconductors,

\[
\varepsilon = \varepsilon_\infty \left( 1 - \frac{\omega_p^2}{\omega^2 + \tau^{-2}} \right),
\]

where \( \omega_p \) is the plasma frequency and the complex mobility \( \mu_n \) is

\[
\mu_n = \mu_{\text{DC}} \frac{\tau^{-1}}{\tau^{-1} + i \omega}.
\]

where \( \mu_{\text{DC}} \) is the dc mobility and \( \tau \) is the carrier collision time. We note that either the plasma frequency or the collision time (or both) are in the terahertz (THz) range for many semiconductors so that the permittivity approaches zero and changes sign only in the range of THz frequencies. A strong mechanical response within this spectral range is predicted since vanishing permittivities lead to a tremendously high stress, \( T \sim 1/\varepsilon \), as derived from Eq. (1), and is responsible for obtaining enhanced gain or absorption, which we will see in the following. The above dispersion relation is solved for the case of zinc-blende InSb using the following parameters: \( \varepsilon = -0.07 \) C/m\(^2\), \( c = 4.7 \times 10^{10} \) Pa, \( m_{\text{eff}} = 0.014 \) (in units of the free-electron mass), \( \rho = 5770 \) kg/m\(^3\), \( n_0 = 2 \times 10^{22} \) m\(^{-3}\), \( \mu_{\text{DC}} = 7.7 \) m\(^2\) V\(^{-1}\) s\(^{-1}\), \( \varepsilon_\infty = 15.7 \), \( \tau = m_{\text{eff}}/\mu_{\text{DC}}/q \), and \( F = 1 \), corresponding to a plasma frequency \( f = 2.71 \) THz. It is evident from the \( k_1 \) wave-number-plots in Figs. 1(a) and 1(b) for a frequency range far below the plasma frequency \( \omega_P \) that an abrupt transition from absorption to gain occurs near the position where the Cherenkov condition is fulfilled, i.e., where the drift speed \( v_d = \mu E_0 \) surpasses the speed of sound \( v_s \). The small deviation in the transition frequency away from the Cherenkov condition stems from the appearance of the small diffusion term in Eq. (4). Gain (absorption) requires the real and imaginary terms of the wave vector to have the same (opposite) sign [refer to Fig. 1(b)]. In the case of InSb and the parameters above, the transition between absorption and gain takes place when the dc electric field equals approximately \(-430 \) V/m. It is also evident from Fig. 1(a) that the strength of the gain or absorption is weak since the damping term of \( k_1 \) is at most \( 0.005 \) m\(^{-1}\), with increasing gain toward increasing electric field strength and low frequencies. In Figs. 1(c) and 1(d), we also plot the wave-number component \( k_2 \). Since the real and imaginary components are always of opposite sign irrespective of the dc electric field value, only absorption is possible for this mode, and \( k_2 \) excitations are always damped during propagation. Further, it can be seen that the absorption strength is rather weak for \( k_2 \) modes, a result that is similar to earlier absorption results [11].

Similar plots of \( k_1 \) and \( k_2 \) are shown in Fig. 2 for frequencies around the plasma frequency. It follows from Eq. (4) that when \( \varepsilon = 0 \) (for a Drude permittivity response this occurs when \( \omega^2 = \omega_p^2 - \tau^2 \), assuming \( \omega_p > \tau \), as is the case for, e.g., intrinsic InSb), the imaginary part of \( k \) can take on arbitrarily
large positive (or negative) values as the electric field is increased. Thus, we can tailor the intrinsic acoustic absorption or gain by controlling the frequency and the dc electric field. Significant changes appear in the magnitude of the imaginary part of the wave numbers \( k_1 \) and \( k_2 \), as seen in Fig. 3. The real part of the wave number displays evidence of sound amplification since the sound traverses the slab in, say, the positive \( z \) direction. The surrounding media are assumed to be acoustically well matched to guarantee a high portion of the incoming sound field enters the slab. In Fig. 4(a) we compute the transmission amplitude of an incoming plane wave impinging on a slab when a constant electric field is applied. The surrounding media are assumed to have an acoustic impedance \( Z_3 = 3 \times 10^7 \) kg/(m²s), and the plasma frequency is \( f = 2.69 \) THz. Clearly, due to the exponential increase in the sound field along the positive \( z \) direction in a case with gain, the longer the slab is, the higher the transmission coefficient becomes. Calculations at different frequency values again show that amplification is strongest slightly below \( \omega_p \). Since the real part of the wave number is considerably larger in magnitude than its imaginary part whenever gain is present, many oscillations in the acoustic field will take place over a slab length where gain is pronounced. The basic requirement is to have a high enough applied electric field \( E_0 \) and to operate at ENZ conditions so that \( |\text{Im}(k_1)| \) is high and damping is positive. Im\((k_1)/\text{Re}(k_1) > 0 \) (refer to Figs. 2 and 3). It is important that the material surrounding the slab of length \( L \) is acoustically well matched to guarantee that a high portion of the incoming sound field enters the slab. In Fig. 4(a) we compute the transmission amplitude of an incoming plane wave impinging on a slab when a constant electric field is applied. The surrounding media are assumed to have an acoustic impedance \( Z_3 = 3 \times 10^7 \) kg/(m²s), and the plasma frequency is \( f = 2.69 \) THz. Clearly, due to the exponential increase in the sound field along the positive \( z \) direction in a case with gain, the longer the slab is, the higher the transmission coefficient becomes. Calculations at different frequency values again show that amplification is strongest slightly below \( \omega_p \). Since the real part of the wave number is considerably larger in magnitude than its imaginary part whenever gain is present, many oscillations in the acoustic field will take place over a slab length where gain is pronounced. The basic requirement is to have a high enough applied electric field \( E_0 \) and to operate at ENZ conditions so that \( |\text{Im}(k_1)| \) is high and damping is positive. 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surrounding media is $f = \frac{\text{Transmission}}{\text{Reflection}}$ ($T$) and without switching of $E_0$. Over time, huge transmissions and reflections are built up when switching is imposed. On the other hand, there is no additional amplification of sound in making a slab round-trip for a constant applied field since the gain experienced in acoustic metamaterials have been explored [21–26]. We utilize this finding by designing a switch-controlled optomechanical device producing larger-than-unity scattering coefficients of the radiated sound field, and we foresee that this technique will find many striking applications for sensing and spectroscopy. The fact that acoustic gain can be much higher than realized in the works by White and Hutson [10–12] makes the present idea less sensitive to crystal noise for sound amplification at ENZ conditions, resulting in a high signal-to-noise ratio. To embed this concept in future and current systems, one needs to consider possible saturation of gain as a result of nonlinear sound interaction and increased material absorption. We note also that acoustic dissipation mechanisms play an increasing role at higher frequencies. There are other methods for obtaining acoustic gain, such as parametric amplification in a magnetostrictive solid [17]. Furthermore, we stress that amplification of sound has also been reported in nonclassical systems, giving rise to phonon lasing and amplification in Stark ladder superlattices [18–20]. The present idea, based on the piezoelectric effect that exists in a large class of semiconductors where the unit cell is inversion asymmetric, does not require direct excitation of photons. An incoming acoustic wave will, under the application of a dc electric field, experience amplification. Acoustic amplification can also be generated in the absence of an incoming acoustic wave by applying a dc and an ac electric field component simultaneously. In recent years several intriguing phenomena in acoustic metamaterials have been explored [21–26]. We envision that the present idea of sound amplification could find use in metamaterial-related applications at much lower frequencies, providing active compensation of losses at resonance and the design of audible gain by structuring PZ materials.

We have demonstrated how sound amplification in PZ materials can be substantially enhanced when operated at ENZ response. We utilized this finding by designing a switch-controlled optomechanical device producing larger-than-unity scattering coefficients of the radiated sound field, and we foresee that this technique will find many striking applications for sensing and spectroscopy. The fact that acoustic gain can be much higher than realized in the works by White and Hutson [10–12] makes the present idea less sensitive to crystal noise for sound amplification at ENZ conditions, resulting in a high signal-to-noise ratio. To embed this concept in future and current systems, one needs to consider possible saturation of gain as a result of nonlinear sound interaction and increased material absorption. We note also that acoustic dissipation mechanisms play an increasing role at higher frequencies. There are other methods for obtaining acoustic gain, such as parametric amplification in a magnetostrictive solid [17]. Furthermore, we stress that amplification of sound has also been reported in nonclassical systems, giving rise to phonon lasing and amplification in Stark ladder superlattices [18–20]. The present idea, based on the piezoelectric effect that exists in a large class of semiconductors where the unit cell is inversion asymmetric, does not require direct excitation of photons. An incoming acoustic wave will, under the application of a dc electric field, experience amplification. Acoustic amplification can also be generated in the absence of an incoming acoustic wave by applying a dc and an ac electric field component simultaneously. In recent years several intriguing phenomena in acoustic metamaterials have been explored [21–26]. We envision that the present idea of sound amplification could find use in metamaterial-related applications at much lower frequencies, providing active compensation of losses at resonance and the design of audible gain by structuring PZ materials.

J.C. gratefully acknowledges financial support from the Danish Council for Independent Research and a Sapere Aude grant (Grant No. 12-134776).
