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Publication date: 2014

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Gyrokinetic Linearized Landau Collision Operator

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Abstract

Here the full gyrokinetic electrostatic linearized Landau collision operator, entering the ∆F formulation of gyrokinetic theory, is calculated including the equilibrium operator C[F_M, F_M]. Energy exchange between plasma species is described by C[F_M, F_M].

Furthermore, C[F_M, F_M] describes drag and diffusion of the magnetic field aligned component of the vorticity associated with the E × B drift.

Contrary to conventional wisdom it is shown that C[F_M, F_M] ≠ 0 for like-particle collisions and has the same order of magnitude as the gyrokinetic test- and field-particle operators. C[F_M, F_M] must be included in gyrokinetic ∆F collision operators.

Particles Collide

Gyrokinetic theory decouples the fast time-scale associated with the fast cyclotron motion of charged particles in strongly magnetized plasmas from the low-frequency dynamics of gyro-centers. The gyrokinetic theory describes the time-evolution of gyro-centers.

A gyro-center is a mathematical construction. Gyro-centers do not carry charge nor do they collide. Therefore, caution must be exercised when describing particle dynamics in the gyro-center picture.

For instance in the gyrokinetic Gauss’s Law the charge distribution must be deduced from the distribution of gyro-centers. Therefore, the gyro-center charge contribution is accompanied by polarization density and inverse gyro-angle averages, which in combination give the low-frequency particle charge distribution. The polarization density originates from the Maxwellian (constant density) part of the gyrokinetic distribution function.

Similarly, the Landau collision operator (and collisions in general) describes collisions between particles, not gyro-centers. Therefore, collision operators in gyrokinetic theory are expected to have terms similar to those in the gyrokinetic Gauss’s Law. A polarization-density-like contribution is expected in gyrokinetic collision operators.

Gyrokinetic Maxwellian

The Landau collision operator C[F_M, F_M] = 0 when \( f_{k,M}(x,v) \) are Maxwellian: \( f_{k,M}(x,v) = n_{M}(2\pi m_{M}k_{B}T_{M})^{-3/2}exp\{-\frac{\epsilon_0\sqrt{\nu_{d,O}}}{T_{M}}\} \), with the same temperature. However, when evaluating the collision operator with two gyrokinetic Maxwellians: \( F_{M} = N(2\pi m_{M}k_{B}T_{M})^{-3/2}exp\{-\frac{\epsilon_0\sqrt{\nu_{d,O}}}{T_{M}}\} \) having the same temperature the Landau collision operator does not vanish:

\[ C[F_{M}, F_{M}] \neq 0 \]

\( f_{k,M} \) and \( F_{M} \) have the same functional form but do not agree to lowest order in e:

\[ f_{k,M}(z) = f_{k,M}(z)1 - \epsilon_0\phi(z)/T_{M} + O(\epsilon^2) \]

Test-particle-like operator

The gyro-angle average of \( C_{F}^{\parallel} \) becomes:

\[ \langle C_{F}^{\parallel} \rangle = \int d^{3}k \epsilon^{\parallel} \epsilon^{\perp} \sum_{n=0}^{\infty} F_{M,a,n}(x,v) \left[ \frac{m_{n}}{m_{a}} (2\pi m_{a}k_{B}T_{a})^{-3/2}exp\{-\frac{\epsilon_0\sqrt{\nu_{d,O}}}{T_{a}}\} \right] \]

\[ \times \left[ \frac{m_{n}}{m_{a}} (2\pi m_{a}k_{B}T_{a})^{-3/2}exp\{-\frac{\epsilon_0\sqrt{\nu_{d,O}}}{T_{a}}\} \right] \]

\[ \left[ 1 - \epsilon_0\phi(x)/T_{a} + O(\epsilon^2) \right] \]

where \( \epsilon_0 \) and \( g_{a,n} \) are time-independent, and can be precomputed and re-used for all time-steps in numerical simulations.

References


Like-particle collisions

Utilizing \( C[f_{k,M}, f_{k,M}] = 0 \) allows us to evaluate the equilibrium operator:

\[ C[F_{M}, F_{M}] = C[F_{M}, f_{k,M}] + C[F_{M}, f_{k,M}] \]

using the gyrokinetic test- and field-particle operators\([4, 5, 2]\). \( C[F_{M}, f_{k,M}] \) has the same order of magnitude as the standard test- and field-particle operators. Must be retained in order to describe ion-ion collisions correct.