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Investigation of switching frequency variations in self-oscillating class D amplifiers

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ABSTRACT
Class D audio amplifiers have gained significant influence in sound reproduction due to their high efficiency. One of the most commonly used control methods in class D amplifiers is known as self-oscillation. An parameter of key interest in self-oscillating class D amplifiers is the switching frequency, which can be directly related to the performance of the amplifier. This paper will clarify the myth of the switching frequency through investigation of its dependency on modulation index and reference frequency. Validation is done using simulations and an 50 W amplifier providing 0.2 % of distortion. The switching frequency is tracked through accurate spectrum measurements, and very good compliance with simulation results are observed.

1. INTRODUCTION
Amplification of any signal is all about converting energy from one source to a some kind of consumer or load. Pulse Width Modulation (PWM) is a very well known method of carrying out this task, and plays due to its high efficiency an important part in the design of today’s power supplies. However PWM is not limited to the application of power supplies, and can thus also be used in the amplification of audio. Such amplifiers are known as class D amplifiers and have gained significant usage in commercial audio amplifiers. One of the most commonly used control scheme of class D audio amplifiers is known as sliding mode control or self-oscillation. Self-oscillating class D audio amplifiers are characterized by having a open loop bandwidth equal to the switching frequency as oppose to traditional PWM amplifiers, where the loop bandwidth typically will be limited to one tenth of the switching frequency. This increased loop bandwidth provides valuable loop gain at low frequency, which are beneficial with respect to reduction of Total Harmonic Distortion (THD). Many publications exists showing the excellent performance of self oscillating class D audio amplifiers. These are among others [1], [2], [3], [4] and [5]. The control scheme of self oscillating class D audio amplifiers are characterized by having an open loop function shaped to obtain 180° of phase shift at the desired switching frequency [6]. This is typically achieved in one of two ways. The first is by use of extra phase shift in the forward path as seen in [1], while the second uses a hysteresis window in order to obtain the desired phase shift, [6]. This paper will only consider the later known as hysteresis modulators.

One of the most basic class D audio amplifiers is the Astable Integrating Modulator (AIM) topology, which is described in [11]. The AIM amplifier are shown in figure 1, and consists of integrator in the forward path, feedback taken before the output filter using a P regulator, an power stage plus an hysteresis window. Note that in figure 1 is the output omitted as this has no influence on the
switching frequency. The AIM amplifier will be considered throughout this paper. Obviously does many different hysteresis modulators exists. The primary difference of these modulator is however the feedback(s), and the way it is implemented. As feedback are used to reduce THD within the audio band, and all self-oscillating amplifiers need to fully the same requirements in order to oscillate, will the AIM amplifier provided an reasonably basis for carrying out the desired investigation. The paper is further more limited to voltage mode controlled class D audio amplifier.

![Diagram](image)

Fig. 2: Power stage output voltage, carrier, reference signal and hysteresis window.

Hysteresis modulators as the AIM amplifier are known to change the switching frequency with modulation index. This is among others shown in [6], [7] and [9], where it is found that

$$f_{Sw}(M) = \frac{V_S}{4} \frac{1 - M^2}{\tau V_{Hyst}}$$  \hspace{1cm} (1)

In (1) is $V_S$ the power supply voltage, $M = \frac{V_{Out}}{V_S}$ the modulation index, $V_{Hyst}$ the height of the hysteresis window and $\tau$ the integration time constant. It is evident from (1), that the switching frequency will go towards zero as $M$ approaches 1. This is the reason why the modulation index is normally limited to 0.8 in hysteresis controlled self oscillating class D amplifiers. Limiting the modulation index to 0.8 are done in order to keep the switching frequency outside the audio band. Note that the $\tau$ of (1) is equal to $\tau_{int} K_{FB}$ of figure 1.

(1) can be extended to include the loop propagation delay, $t_d$. An example of this is found in [6], yielding

$$f_{Sw}(M) = \frac{V_S}{4} \frac{1 - M^2}{\tau V_{Hyst} + \frac{1}{2} t_d V_S (1 + M^2)}$$  \hspace{1cm} (2)

Mikkel Høyerby has also derived an expression including the loop propagation delay, but using the duty cycle $D$ instead of the modulation index. This is found in [8], and also shows that the switching frequency travels towards zero in a parabolic fashion as the duty cycle is increased i.e. as the modulation index is increased.

(1) and (2) are plotted in figure 3 using an idle switching frequency of 300 kHz, $\tau = 99.58 \mu s$, $V_{Hyst} = 500 mV$ and $t_d = 7 ns$. It is seen, that the loop propagation delay has no influence on the switching frequency. Further more it is evident that the switching frequency falls for increasing modulation index as expected.

The main drawback of (1) and (2) is that they relay on small signal models linearized around the modulation index. This is unfortunate as the carrier signal for large modulation indexes no longer is linear. Looking at figure 2(a) and 2(b) is it evident that the carrier can be considered to be linear for small modulation index. However at high modulation indexes are the carrier degraded, and takes the form of an exponential function. This is caused by the well-known response of an step function to an first order system, which will be shown in section 2. Small
signal models are thus not necessarily the best way to investigate variations in the switching frequency. The following section will illuminate the problems of deriving a switching frequency model of hysteresis controlled self-oscillating class D audio amplifiers.

![Fig. 5: Proposed large signal model.](image)

### Fig. 5: Proposed large signal model.

### 2. SWITCHING FREQUENCY MODEL

The AIM topology has already been introduced in section 1. Using a passive integrator can the AIM amplifier be realized as presented in figure 4, where the output filter has been omitted. Note that the comparator can be modeled to include the power stage if necessary. However in order to investigate the switching nature of the AIM amplifier, feedback from the comparator can easily be assumed.

An simple model of the AIM amplifier can be done by replacing the comparator with a voltage source generating pulses according to the PWM methodology. Further more is the comparator assumed to have a infinite input resistance. One thus obtain an model as seen in figure 5. Notice that \( R_{H1} \) and \( R_{H2} \) are omitted for simplicity as these just determines the hight of the hysteresis window. The AIM amplifier will switch each time the voltage across the capacitor becomes equal to the hight of the hysteresis window. Note that for this derivation will the loop propagation delay be omitted. The following equation can now be obtained

\[
V_{Hystr}(s) = \frac{R_1}{R_1+R_2} v_{Ref}(s) + \frac{R_1}{R_1+R_2} v_{Comp}(s) \tag{3}
\]

The clear benefit of figure 5 is, that the AIM amplifier can be considered as two first order systems. This is illustrated in (3), where each switching point is determined by the superposition of the reference and comparator response to the first order systems with time constants \( \tau = \frac{R_1 R_2 C}{R_1+R_2} \).

Converting (3) to the time domain requires two well-known operations. These are the responses of a sine wave and a step function to an first order filter. The response of a step is a exponential function, while the response of an sine wave also will be a sine wave, [10]. It is assumed, that the cut off frequency of the filter related to \( v_{Ref} \) is well above the reference frequency, \( f_{Ref} \). This allows for the phase and amplitude change of the sine wave be neglected. Grouping constants one obtain an expression of the form

\[
k = \sin(2\pi f_{Ref} t) + e^{-\frac{t}{\tau}} \tag{4}
\]

In 4 is \( k \) an arbitrary constant. Note that the sin wave and exponential function might be weighted with respect to each by including yet another constant. However for simplicity is this neglected.

(4) might look like a fairly simple equation. The authors have however not be able to find any general solution to
this equation. Obtaining an switching frequency model of hysteresis controlled self-oscillating class D audio has thus not be possible, and the attention will now be turn to results obtained through simulations.

3. SIMULINK SIMULATIONS

Simulations have been preformed using the Simulink model of figure 6. Further more an FFT-function has been written in Matlab allowing for obtaining the switching node spectrums, [14]. These spectrum will be used to track the switching frequency. All simulated spectrums as presented in this paper are plotted using an Hanning window. The simulation model of figure 6 are made according to figure 1, and is thus an AIM amplifier. Notice that an output filter has been added in order to complete the simulations model. The plots of figure 2 are obtained using the simulation model of figure 6. All simulations are performed with an idle switching frequency of 300 kHz, an output filter cut off frequency of 49.5 kHz, $t_{int} = 12.45\mu s$, $V_{hyst} = 500mV$, $V_S = 30V$, $t_d = 7ns$, $K_{Fb} = \frac{1}{8}$ and $K_{hyst} = \frac{V_{hyst}}{V_S}$.

3.1. Spectrum simulations

The switching node output spectrums obtained by simulations are shown in figure 7. Notice that the spectrum of a self-oscillating class D audio amplifier deviates significantly from the one of fixed frequency amplifiers. Thus are the sidebands no longer of equal magnitudes. This gives an indication of the complex mathematics, which are need to describe such spectrums. Remember that double fourier series are used to calculate the spectrum of fixed frequency amplifiers, [15]. In these derivations becomes the sidebands Bessel functions.

3.2. Switching frequency

Starting out with an relatively small modulation index of 0.1 is the spectrum of figure 7(a) obtained. Here can the switching frequency and its harmonics clearly be identified. At modulation index 0.1 is the switching frequency identified to be 293 kHz, which is a reduction of 7 kHz comparing with the idle switching frequency of 300 kHz. Increasing the modulation index to 0.3 causes an drop in switching frequency of 18 kHz. Finally is the modulation index increased to 0.6 resulting in the spectrum of Figure 7(c). This shows the general problem of self-oscillating class D audio amplifiers, namely that at large modulation index can the switching frequency not be identified. The switching frequency simply drowns and only an spectral distribution of peaks are observed. Is can thus be questioned whether talking about a switching frequency makes sense for large modulation indexes.

Using the model of figure 6 has a surface plot been produced, which maps variations in switching frequency as function of modulation index and reference frequency. The plot can be seen in figure 9. Note that the plot only considers reference frequencies between 14 kHz and 20 kHz, while the modulation index is limited to the interval 0.1-0.5. The switching frequency is tracked by identifying the highest peak in the spectrum above the reference frequency and below twice the first harmonic of the idle switching frequency (600 kHz in this case). An clear and important conclusion of figure 9 is that the switching frequency is independent of the reference frequency. This comply very well with the theory as presented in section 1.

In order to investigate the switching frequency dependency on modulation index even closer, is the plot of figure 8 produced. In this case is the reference kept fixed at 10 kHz, while modulation index is limited to the interval 0.1-0.5. Comparing is made with (2), and it is seen, that the simulated switching frequency actually falls a bit slower than predicted by the linearized model. As shown in section 2 must is be caused by the carrier, which no longer is linear, but an exponential function. It can thus be concluded, that (2) is to be considered as an worst case approximation of the switching frequency dependency on modulation index.
4. VERIFICATION BY PROTOTYPING

Experimental measurements are performed on a 50 W self-oscillating class D audio amplifier with a rated Total Harmonic Distortion of 0.2%. The prototype can be seen in figure 10, while table 4 collects the key parameters of the prototype. THD over power are obtained using an AP2 from Audio Precision, and the result are shown in figure 11. Notice that all simulations and calculations as presented earlier in this paper are performed with respect to table 4.

4.1. Spectrum measurements

All spectrum measurements are performed with an EMI Test Receiver (ESI7 from Rohde & Schwarz). Again is the switch node selected as the measuring point. Figure 12 and 13 is an selection of the measured spectrums. As observed in section 3 is the sidebands not of equal magnitude which is the case in fixed frequency class D
Switching frequency (idle) | 300 kHz
---|---
Supply | ±30V
Output power | 50W
Load | 4 Ω
Gain | 8
Time delay | 7 ns
Output filter cut off | 49.5kHz

| Table 1: Key parameters of prototype. |

4.2. Switching frequency measurements

Figure 12 shows the spectrum of modulation index 0.1, 0.3 and 0.6. Remembering that the idle switching frequency is 300 kHz one observe an drop in switching frequency of 2.6 kHz at modulation index 0.1. As expected does the reduction in switching frequency continue yielding a drop of 18.4 kHz at modulation index 0.3. Notice that the simulation predicted an drop of 18 kHz with modulation index 0.3. This complies very well with the measurement. At modulation index 0.6 is the problem of identifying the switching frequency again observed.

Investigation of the switching frequency dependency on reference frequency are done by use of figure 13. The measurements are all performed using a modulation index of 0.2. For the three measurements are switching frequencies of 291.2 kHz, 290.8 kHz and 290.4 kHz found. These variation are so small, that they can not be caused by the change of reference frequency. This is as accepted from the simulations.

Measured variations in switching frequency are mapped and shown in figure 14. All measurements are performed with a reference of 10 kHz and an modulation limited to the range 0.1-0.5. Again is the switching frequency identified as the highest peak found in the spectrum between the reference frequency and the first harmonic of the switching frequency.

Figure 14 compares measurements with (2) and the simulation results of figure 8. It is seen, that the simulation results and measurements following each other nicely for modulation index less than 0.3. At modulation index 0.3 is an jump in the measured switching frequency observed. This jump is caused by the problems of tracking the switching frequency as the modulation index is increased. Finally is it observed, that the measurements does not decay as rapidly as predicted by (2). The measurements thus emphasizes the conclusion of section 3, where (2) are characterized as a worst case approximation of the switching frequency dependency on modulation index.

5. CONCLUSION

It has been shown that the switching frequency of self-oscillating class D audio amplifiers is independent of the reference frequency. Further more it concluded, that the spectrum of self-oscillating class D audio amplifier deviates significantly from the one of fixed frequency amplifier. This is seen by the sidebands, which is not of equal magnitudes.
Comparing with the results obtained by linearized models is it to be concluded, that such models provide an worst case approximation of the switching frequency dependency on modulation index. However these models should note be used to describe the switching frequency a high modulation indexes. This is due to the fact, that the switching frequency simply is not defined for such modulation indexes. For high modulation indexes does the switching node spectrum consists of spectral distributed peaks, where no switching frequency are to be identified.

The results of this paper has been verify through Simulink simulations and prototyping an 0.2 % THD class D audio amplifier.

Future work includes investigating whether the audio band is disturbed at high modulation indexes. Further more would it be desirable to have a model of self-oscillating class D audio amplifiers spectrum in order to improve EMI design.

6. REFERENCES


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Fig. 14: Comparison of switching frequency obtained by (2) and measuring. The measurement are performed with 10 kHz reference signal.


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