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An Equivalent Source Method for Modelling the Global Lithospheric Magnetic Field

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Summary
We produce a new model of the global lithospheric magnetic field based on 3-component vector field observations at all latitudes from the CHAMP satellite using an equivalent source technique.

Method
A regularized iteratively reweighted least squares algorithm is applied. Data error covariance matrices are implemented, including both the latitude dependence of data error variances $\sigma^2$ (Fig.1) and covariances $\mathbf{C}$ between the vector field components due to unmodelled sources. The regularization norm $\mathbf{R}$ is defined to be the Euclidean length of the model solution. Our scheme iteratively minimizes:

$$\Theta(m) = \| \mathbf{d} - \mathbf{G}m \|^2 + \lambda \mathbf{R}(m)$$

Huber weighting ensures a robust solution in the presence of non-Gaussian data errors

$$H_{i,i} = \min\left( \frac{1.5}{\sigma_i^2}, 1 \right)$$

An initial unregularized ($\lambda = 0$) model is derived using 10 iterations. The final model is obtained with 5 further iterations using quadratic regularization and $\lambda = 3\times10^{-13}$.

Equivalent Source Method
The equivalent potential field sources $m_i$ (monopoles) are arranged in an icosahedron grid (Fig.2), consisting of $K = 30722$ vertices and midpoints, placed at a depth of 100km below the Earth’s surface. The derived model can be transformed into a spherical harmonic representation by:

$$g_{lm} = \sum_{k=1}^{K} \frac{m_i}{\alpha_{i}^{3/2}} \mathbf{P}_{lm}(\cos \theta_i) \cos (\phi_i)$$

$$h_{lm} = \sum_{k=1}^{K} \frac{m_i}{\alpha_{i}^{3/2}} \mathbf{P}_{lm}(\cos \theta_i) \sin (\phi_i)$$

Fig 2: Right: Icosahedron grid with 483 of vertices (red) and midpoints (green). Left: The corresponding K equivalent potential field sources $m_i$ can be directly transformed into spherical harmonics of order $l$ and degree $m_i$ = Earth mean radius, $(\theta_i, \phi_i) = source location.$

Results and Outlook
The presented model has a power spectrum that compares well to CHAOS-4, MF7 and CM5 (cf. Poster EGU2014-6883) models to degree $n = 100$ (Fig.4). Ongoing investigations concern non-quadratic regularization using maximum entropy. Looking forward, we plan to explore local grid refinement options in order to incorporate aeromagnetic survey data.

Fig 4: Left: Modeled lithospheric radial magnetic field at the Earth’s surface for degree $n = 16$-180. The scale saturates at $200$ nT. Right: Power spectrum for MF7, CM5 and CHAOS-4 models in comparison to model results with different regularization damping values. The chosen model is represented by the red line.