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Dou, Suguang; Jensen, Jakob Søndergaard

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Optimization in Nonlinear Structural Dynamics with Reduced Order Models

Suguang Dou and Jakob Søndergaard Jensen
Department of Mechanical Engineering, Section for Solid Mechanics
Technical University of Denmark, 2800 Kgs. Lyngby, Denmark

Introduction

Why account for nonlinear vibration?

- Coupled Nonlinear Dynamics/Aeroelasticity of very Flexible Aircraft
- Vibration-based MicroElectroMechanical Systems (MEMS)
- Long, Light and Flexible Blade of Wind Turbine
- High Speed Compliant Actuator
- Squeal of the Brake System

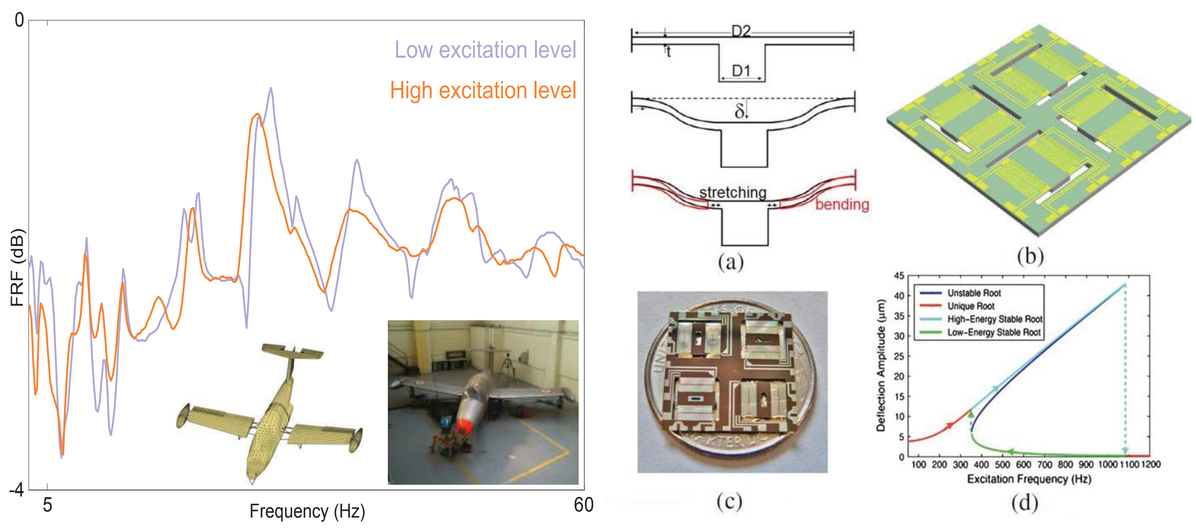

Figure 1: Nonlinear vibration in airplane and MEMS. Left: dynamic testing of an airplane presented by Gaëtan Kerschen [1]; Right: ultra-wide bandwidth piezoelectric energy harvesting device developed by Arman Hajati and Sang-Gook Kim [2].

What is the problem in optimization?

- Today's design procedures are often based on linear finite element (FE) models.
- Nonlinear structural dynamics is analyzed after a full optimization procedure.
- High computation costs of nonlinear structural dynamics are prohibitive for iterative optimization.

The goal of this PhD project

Focus on developing reduced-order models to facilitate efficient analysis and optimization.

- Eliminate the time dimension to compute the steady-state vibration efficiently.
- Reduce the spatial dimension to obtain a model with fewer degree-of-freedom.
- Do sensitivity analysis and design optimization using reduced-order models.

Method

Time-reduced models

For the time-reduced model, we consider only problems with time-harmonic excitation. These are of major relevance in machinery with rotating parts. The FE model becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}\cos\Omega t$$

in which $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ is the discretized displacement, velocity and acceleration vector, respectively. The matrices $\mathbf{M}$ and $\mathbf{C}$ represent mass and damping, and $\mathbf{g}(u)$ is a vector with the nonlinear forces, and $\mathbf{f}\cos(\Omega t)$ is the time harmonic load.
A semi-analytical method called Incremental Harmonic Balance (IHB) method is used to solve the equation of motion [3, 4]. For the incremental harmonic balance method, the governing equation becomes

$$\left(\omega^2\bar{\mathbf{M}} + \omega\bar{\mathbf{C}} + \bar{\mathbf{K}}_T(\bar{\mathbf{u}})\right)\Delta\bar{\mathbf{u}} = \bar{\mathbf{f}} - \left(\omega^2\bar{\mathbf{M}}\bar{\mathbf{u}} + \omega\bar{\mathbf{C}}\bar{\mathbf{u}} + \bar{\mathbf{g}}(\bar{\mathbf{u}})\right)$$

in which $\bar{\mathbf{u}}$ is a vector containing all coefficients of harmonics in Fourier series of $\mathbf{u}$. And $\bar{\mathbf{M}}$, $\bar{\mathbf{C}}$ and $\bar{\mathbf{K}}_T$ are augmented matrices of mass, damping and tangent stiffness, respectively. And $\bar{\mathbf{g}}$ and $\bar{\mathbf{f}}$ are augmented vectors of elastic force and external force, respectively.

Space-reduced models (future work)

In case of transient loads, e.g. encountered in machinery start-up or with impact loads such as blasts, we need to consider the time response of the structure. The finite element model in this case becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}(t)$$

in which $\mathbf{f}(t)$ is the specific time-dependent load. Instead of computing the transient response for the full FE model, an analysis model based on nonlinear modal reduction will the applied. The space-reduced model becomes

$$\mathbf{M}_r\ddot{\mathbf{q}} + \mathbf{C}_r\dot{\mathbf{q}} + \mathbf{g}_r(\mathbf{q}) = \mathbf{f}_r(t)$$

in which $\mathbf{u}$ has been reduced to a set of generalized coordinates $\mathbf{q}$. And $\mathbf{M}_r$ and $\mathbf{C}_r$ are reduced matrices for mass and damping, respectively. And $\mathbf{g}_r$ and $\mathbf{f}_r$ are reduced vectors for elastic force and external force, respectively. The use of nonlinear modal reduction can potentially reduce computational costs by orders of magnitude.

Design optimization using reduced-order models

A general optimization problem concerning vibration is to minimize the amplitude of vibration. Based on the time-reduced models, this problem can be expressed as

$$\begin{aligned}
&\min_{\rho_e}\ c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T\mathbf{L}\bar{\mathbf{u}} \\
&\text{s.t.}:\ \omega^2\bar{\mathbf{M}}\bar{\mathbf{u}} + \omega\bar{\mathbf{C}}\bar{\mathbf{u}} + \bar{\mathbf{g}} = \bar{\mathbf{f}}, \\
&\sum_{e=1}^{N_e}\rho_e V_e - V^* \le 0, V_e = L_e A_e, \\
&0 < \rho_{min} \le \rho_e \le 1, V^* = \alpha V_0, \\
&A_e = \rho_e A_{max}, V_0 = L_e A_{max}.
\end{aligned}$$

where ρ_e are design variables, N denotes the total number of design variables, the symbol α defines the volume fraction, V_0 is the volume of the admissible design domain, and V^* is the given available volume of solid material.

Examples

Design of nonlinear beam dynamics

The structure is a doubly clamped beam with periodic load applied at the midspan. The design variable is the width $w(x)$. The objective function will be given in each example. The nonlinearity in the model arises from the midplane stretching. The axial strain ε_0 and the curvature κ are defined as

$$\varepsilon_0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \kappa = \frac{\partial^2 w}{\partial x^2}$$

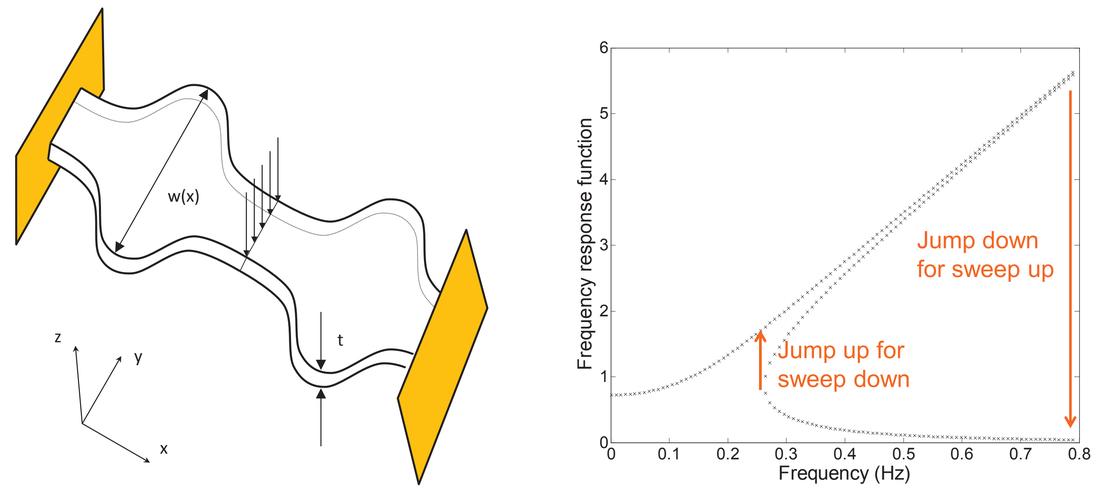

Figure 2: Schematic of the model and a typical frequency-amplitude curve.

Minimize the resonant peak

$$\min_{\rho_e}\ c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T\mathbf{L}\bar{\mathbf{u}} = a_{i1}^2 + b_{i1}^2$$

in which a_{i1} and b_{i1} are the coefficients of the fundamental harmonic $a_{i1}\cos(\omega t) + b_{i1}\sin(\omega t)$ for the deflection at the midspan.

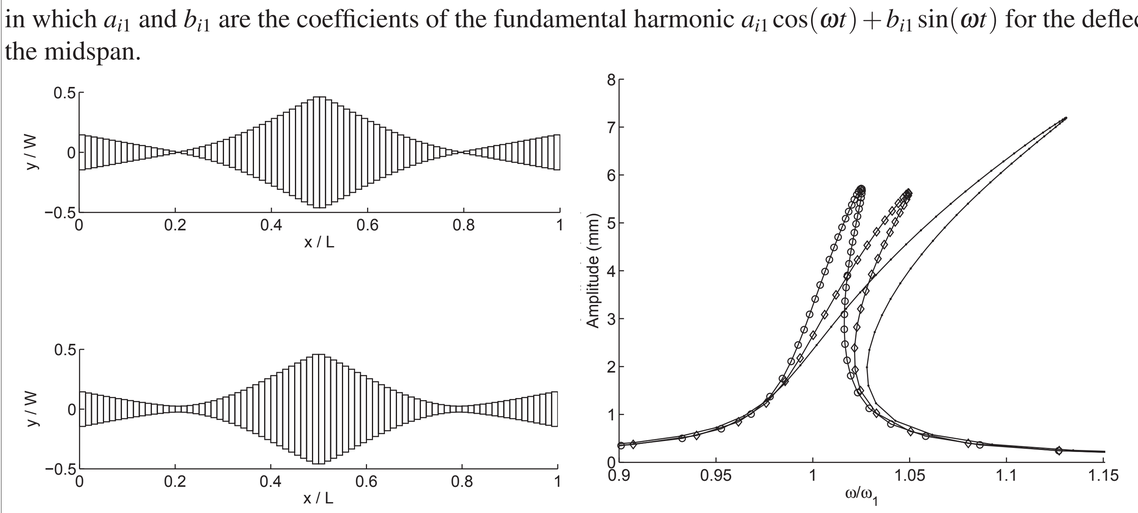

Figure 3: Optimized width for minimizing the resonant peak using linear FE model(left top) and using noninear FE model (left bottom), and nonlinear frequency-amplitude curves: circles denote optimized width using linear FE model, diamonds denote optimized width using nonlinear FE model and dots denote uniform width.

Maximize the super-harmonic resonance

$$\max_{\rho_e}\ c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T\mathbf{L}\bar{\mathbf{u}} = a_{i3}^2 + b_{i3}^2$$

in which a_{i3} and b_{i3} are the coefficients of the third-order harmonic $a_{i3}\cos(3\omega t) + b_{i3}\sin(3\omega t)$ for the deflection at the midspan.

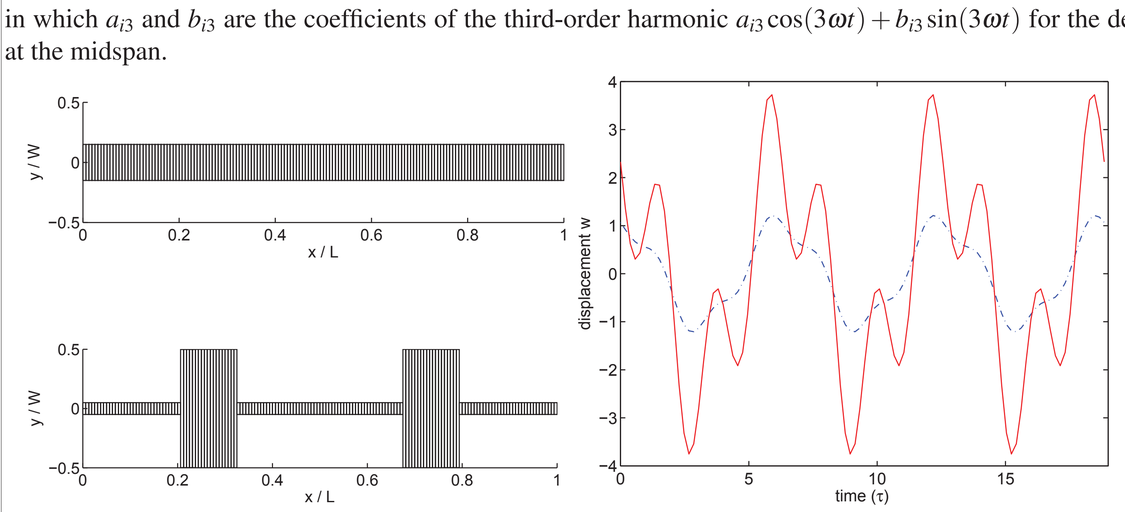

Figure 4: Optimized width for maximizing super-harmonic resonance (Left top: uniform width; Left bottom: optimized width) and the responses before optimization (dashed line) and after optimization (solid line).

Discussion

- Optimized width for minimizing the resonant peak using nonlinear FE model does not have "weak" links.
- Nonlinear structural dynamics is essential for optimizing high-order harmonics in the response.

References

[1] M. Peeters, G. Kerschen, J. Golinval, C. Stephan, and P. Lubrina, "Nonlinear normal modes of real-world structures: Application to a full-scale aircraft," in *Proceedings of the ASME 2011 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, (Washington, USA), August 28-August 31 2011.

[2] A. Hajati and S.-G. Kim, "Ultra-wide bandwidth piezoelectric energy harvesting," *Applied Physics Letters*, vol. 99, p. 083105, aug 2011.

[3] R. Lewandowski, "Non-linear, steady-state vibration of structures by harmonic balance/finite element method," *Computers & Structures*, vol. 44, no. 12, pp. 287 – 296, 1992. Special Issue: WCCM II.

[4] S. H. Chen, Y. K. Cheung, and H. X. Xing, "Nonlinear vibration of plane structures by finite element and incremental harmonic balance method," *Nonlinear Dynamics*, vol. 26, pp. 87–104, 2001.