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Distance measurements by speckle correlation of objective speckle patterns, structured by the illumination

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Objective speckles produced by two beams overlapping and interfering on a rough object surface contain information about the angle of incidence of the two beams, and how well they overlap. We obtain the autocovariance function for such a speckle pattern, and demonstrate how the information carried by the objective speckles can be used to probe the distance between the object and the observation plane. From a distance of 75 mm to a distance of 150 mm, and using an angle of 0.3 deg between the two incident beams, we can measure the actual distance with an uncertainty of better than $\pm 0.1\%$ of the full range. As long as the beams overlap at the object surface, the proposed method can measure distance with an uncertainty inversely proportional to the spot size at the object. © 2012 Optical Society of America
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1. Introduction

A coherent laser beam illuminates an object. The object surface is rough on scales similar to or larger than the optical wavelength. The field, which is scattered off from the object surface, is propagated to an observation plane, where accordingly, speckles [1] appear. If the object moves, as a result the speckles move and/or boil in the observation plane. The dynamics of speckles have found several applications within fields such as speckle photography, speckle interferometry, and spatial filtering velocimetry. Typical applications of speckle photography, where speckle pattern intensities are recorded only, are measurements of in-plane displacement/velocity [2], angular displacement/velocity [3], and in-plane object deformation [4] due to, e.g., strain, torque [5], and temperature [6]. In speckle interferometry, interferograms are recorded [7] as they are applied to obtain two- and three-dimensional deformation fields

[8] of a deforming object. As the electronic cameras have been developed and reached the consumer market, this field has been dominated by the electronic speckle pattern interferometry (ESPI) [9,10]. The dynamics of the speckle patterns obtained within all these applications are typically processed by speckle correlation [11] or spatial filtering velocimetry [12]. Instantaneous distance measurements have been carried out by probing the speckle velocity at two different distances to a moving object. When the object is illuminated with, e.g., a diverging beam [13] the speckle velocity depends on the beam curvature at the object and the distances between the object and the observation planes, thus facilitating the measurement of both velocity and distance.

With two mutually coherent beams incident on the object with different angles of incidence and coinciding at the object surface, a laser Doppler velocimeter is implemented in the image plane of the object for measuring in-plane displacement or velocity [1,14]. If the imaging lens is removed from the above system, the speckle pattern will still provide information of in-plane motion [15]. The speckle statistics

produced by $2n + 1$ for $n \in \mathbb{N}$ number of beams, all interfering on a rough surface and observed as objective speckles in an observation plane, have been described in [15]. In the observation plane, cross-correlation reveals a correlation between the speckle patterns produced by each of the two beams individually. The averaged wavevectors of these two speckle patterns will originate from the same position on the object surface, and will have directions defined as the directions of the diffraction orders diffracted from a specific grating component of the resulting field illuminating the object surface. The duplets of this objective speckle pattern can be found within the entire half plane.

In this paper, we will introduce the normalized autocovariance function of speckle patterns produced by two coherent beams, overlapping at the object surface. The speckle statistics provide a tool to determine, e.g., the distance between an object and the observation plane. This will be demonstrated in the experimental section. In case the two waves illuminate the object at two distinct spots, still the distance between the object and the observation plane can be retrieved from the objective speckles. Both cases are

similar Gaussian field distributions. The angle between the wave vectors ($\mathbf{k}_i$) of the two beams is 2θ , and at incidence at the object, the beam diameter and radius of curvature of their phase front is w and R , respectively. The center distance between the two Gaussian fields at the surface is Λ . We assume that the rough surface has a rms surface roughness that is larger than the optical wavelength, and that the diameters of the beams illuminating the object are larger than any lateral scales of the surface roughness. In this case the scattered fields will propagate through free space a distance z to the observation plane where the intensity distributions $I(\mathbf{p}_i)$ will generate fully developed speckles. $I(\mathbf{p}_i)$ denotes the optical intensity in the observation plane at position $\mathbf{p}_i = (p_{x,i}, p_{y,i})$. The positions $\mathbf{p}_1$ and $\mathbf{p}_2$ are two-dimensional vector coordinates for two points of interest in the observation plane.

A. Autocovariance Function

The spatial statistics of the speckle pattern will here be described with the normalized spatial autocovariance function of ensembles of intensity distributions obtained in the observation plane [16]:

$$R_{cn}(\mathbf{p}_1, \mathbf{p}_2) = \frac{\langle I(\mathbf{p}_1)I(\mathbf{p}_2) \rangle - \langle I(\mathbf{p}_1) \rangle \langle I(\mathbf{p}_2) \rangle}{\left\{ \left[\langle I(\mathbf{p}_1)^2 \rangle - \langle I(\mathbf{p}_1) \rangle^2 \right] \left[\langle I(\mathbf{p}_2)^2 \rangle - \langle I(\mathbf{p}_2) \rangle^2 \right] \right\}^{1/2}}, \quad (1)$$

apparent from the model, although we will focus the experiments on the case where the two beams overlap.

2. Theory

Figure 1 shows an object with a rough surface illuminated by two mutually coherent beams, having

Fig. 1. The figure illustrates the principle configuration of the coordinate systems, describing the fields. Further, the two beams illuminating the object are illustrated. The r -coordinate system is located in the object plane, while the $\mathbf{p}$ -coordinate system is located in the observation plane.

where the angular brackets denote the ensemble average. In this paper, we assume quasi monochromatic conditions, stationary statistical conditions, and a stationary object. Further, as mentioned above, the complex optical scalar field in the observation plane is assumed to obey circular symmetric complex Gaussian statistics [1] with zero mean. Therefore all higher-order moments of the field can be reduced to products of second-order field moments. Consequently, Eq. (1) can be written as follows:

$$R_{cn}(\mathbf{p}_1, \mathbf{p}_2) = \frac{|\Gamma(\mathbf{p}_1, \mathbf{p}_2)|^2}{\Gamma(\mathbf{p}_1, \mathbf{p}_1)\Gamma(\mathbf{p}_2, \mathbf{p}_2)}, \quad (2)$$

where

$$\Gamma(\mathbf{p}_1, \mathbf{p}_2) = \langle U(\mathbf{p}_1)U^*(\mathbf{p}_2) \rangle, \quad (3)$$

is the mutual intensity of the optical field and the asterisk denotes the complex conjugate.

The optical scalar field $U(\mathbf{p})$ in the observation plane can be expressed via the Green's function, $G(\mathbf{r}, \mathbf{p})$, as a function of the field, $U_0(\mathbf{r})$, scattered off the target surface (S):

$$U(\mathbf{p}) = \int_S d^2\mathbf{r} U_0(\mathbf{r}) G(\mathbf{r}, \mathbf{p}), \quad (4)$$

where

$$G(\mathbf{r}, \mathbf{p}) = -\frac{ik}{2\pi z} \exp\left(-\frac{ik}{2z}(\mathbf{r}^2 - 2\mathbf{r} \cdot \mathbf{p} + \mathbf{p}^2)\right), \quad (5)$$

is the paraxial version of Green's function [17] for free-space propagation. In the object plane $\mathbf{r}$ denotes the two-dimensional vector coordinate of position. The length of the optical wavevectors is denoted $k = 2\pi/\lambda$, where λ is the optical wavelength.

The reflected optical field at the object plane is modeled as follows:

$$U_0(\mathbf{r}) = U_i(\mathbf{r})\Psi(\mathbf{r}), \quad (6)$$

where $U_i(\mathbf{r})$ is the optical field, incident on the object plane, and $\Psi(\mathbf{r})$ is the stochastic complex reflection coefficient that describes the surface. The modulus of $\Psi(\mathbf{r})$ is assumed to be constant, and set equal to unity. Following the analysis, described in [16], the correlation function for the complex reflection coefficient can be found, assuming the surface giving rise to fully developed speckles:

$$\langle \Psi(\mathbf{r}_1)\Psi^*(\mathbf{r}_2) \rangle = \text{const.} \times \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (7)$$

i.e., the complex reflection coefficient in one position is uncorrelated with the reflection coefficient in any other position on the object, except with the reflection coefficient in the position itself. Therefore, the mutual coherence function for the reflected field follows as

$$\Gamma_0(\mathbf{r}_1, \mathbf{r}_2) = \langle U_0(\mathbf{r}_1)U_0^*(\mathbf{r}_2) \rangle \propto U_i(\mathbf{r}_1)U_i^*(\mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_1), \quad (8)$$

The optical field incident on the object consists of two Gaussian beams propagating in the xz -plane defined by $y = 0$. The angle between the two beams is 2θ , while the center distance between the two beams at incidence with the object plane is Λ . Therefore, θ and Λ can be introduced to the r -plane via the unity vector $\mathbf{e}_x$. We find

$$\begin{aligned} U_i(\mathbf{r}) = & \left(\frac{2P}{\pi w^2}\right)^{1/2} \exp\left(-\left(\mathbf{r} - \frac{\Lambda}{2}\mathbf{e}_x\right)^2\left(\frac{1}{w^2} + \frac{ik}{2R}\right)\right. \\ & \left.+ ik\left(\mathbf{r} - \frac{\Lambda}{2}\mathbf{e}_x\right) \cdot \theta\mathbf{e}_x + i\varphi_1\right) \\ & + \left(\frac{2P}{\pi w^2}\right)^{1/2} \exp\left(-\left(\mathbf{r} + \frac{\Lambda}{2}\mathbf{e}_x\right)^2\left(\frac{1}{w^2} + \frac{ik}{2R}\right)\right. \\ & \left.- ik\left(\mathbf{r} + \frac{\Lambda}{2}\mathbf{e}_x\right) \cdot \theta\mathbf{e}_x + i\varphi_2\right), \end{aligned} \quad (10)$$

where P is the optical power of the individual beams. The other beam parameters, w and R , denote the e^{-2} intensity radius and the radius of curvature, respectively—here assumed to be identical for the two beams in the object plane. The sign of R is positive for an illuminating beam diverging as z increases. The individual phases of the beams are denoted φ_i . We will without loss of generality ignore the difference in phases between the beams and tacitly assume that they are zero. In practice they will be arbitrary, and a finite difference will merely shift the phase of the fringe pattern to be observed in the intensity of the combined resulting fields in the object plane. All beams are assumed to have the same state of polarization, and any depolarization effects at reflection are ignored. The normalized mutual coherence function only depends on the difference $\Delta\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1$.

Substituting Eqs. (5) and (10) into Eq. (9), performing the resulting Gaussian integrations, and simplifying yields

$$R_{cn}(\Delta\mathbf{p}) = R_{c0} \left(\begin{aligned} & \exp\left(-\frac{\Lambda^2}{2w^2} - \frac{1}{\rho^2}\left((\Delta p_x - (2\theta - \frac{\Lambda}{R})z)^2 + \Delta p_y^2\right)\right) \\ & + \exp\left(-\frac{\Lambda^2}{2w^2} - \frac{1}{\rho^2}\left((\Delta p_x + (2\theta - \frac{\Lambda}{R})z)^2 + \Delta p_y^2\right)\right) \\ & + 2 \exp\left(-\frac{1}{\rho^2}(\Delta p_x^2 + \Delta p_y^2)\right) \cos\left(\frac{k}{2z}\Lambda\Delta p_x\right) \end{aligned} \right)^2, \quad (11)$$

where an unimportant constant value associated with the intensity reflection coefficient for the object has been discarded.

Inserting Eqs. (4) and (8) into Eq. (3), the mutual coherence function of the optical field obtained at the observation plane becomes

$$\Gamma(\mathbf{p}_1, \mathbf{p}_2) = \int_S d^2\mathbf{r} U_i(\mathbf{r})U_i^*(\mathbf{r})G(\mathbf{r}, \mathbf{p}_1)G^*(\mathbf{r}, \mathbf{p}_2). \quad (9)$$

where R_{c0} is independent of $\Delta\mathbf{p}$:

$$R_{c0} = \left(2 + 2 \exp\left(-\frac{\Lambda^2}{2w^2} - \frac{z^2}{\rho^2}\left(2\theta - \frac{\Lambda}{R}\right)^2\right)\right)^{-2}. \quad (12)$$

Generally, three peaks are present in the auto-covariance function, a self-correlation peak located in the origin of the covariance plane, and two distance-correlation peaks, located symmetrically

around the self-correlation peak. The self-correlation peak is modulated by the cosine function, having a spatial frequency of $\Lambda/(2\lambda z)$. The radius of each of the three peaks is given by the mean speckle size ($\rho = 2^{3/2}z/(kw)$).

The normalized spatial power spectral density of the speckle pattern can be determined as the inverse Fourier transform of the normalized autocovariance function. By applying the inverse Fourier transform to Eq. (11), we find $\mathcal{G}_{\text{nAC}}$, the power spectral density function, excluding the DC pedestal and normalized by the square of the average intensity of the speckle pattern.

$$\mathcal{G}_{\text{nAC}}(\mathbf{f}) = \int_{-\infty}^{\infty} R_{\text{cn}}(\Delta\mathbf{p}) \exp(2\pi i \Delta\mathbf{p} \cdot \mathbf{f}) d\Delta\mathbf{p}. \quad (13)$$

We find

$$\mathcal{G}_{\text{nAC}}(\mathbf{f}) = \frac{\pi\rho^2}{2} R_0 \left(\begin{aligned} & 2 \exp\left(-\frac{\pi^2\rho^2}{2}(f_x^2 + f_y^2)\right) \left[1 + \exp\left(-\frac{\Lambda^2}{w^2}\right) \cos\left(2\pi\left(\frac{\Lambda}{R} - 2\theta\right)zf_x\right) \right] \\ & + \exp\left(-\frac{\pi^2\rho^2}{2}\left(\left(f_x + \frac{k\Lambda}{2\pi z}\right)^2 + f_y^2\right)\right) \\ & + \exp\left(-\frac{\pi^2\rho^2}{2}\left(\left(f_x - \frac{k\Lambda}{2\pi z}\right)^2 + f_y^2\right)\right) \\ & + 2 \exp\left(-\frac{\Lambda^2}{w^2} - \frac{2z^2}{\rho^2}\left(\frac{\Lambda}{R} - 2\theta\right)^2\right) \exp\left(-\frac{\pi^2\rho^2}{2}(f_x^2 + f_y^2)\right) \\ & + 4 \exp\left(-\frac{\Lambda^2}{2w^2} - \frac{z^2}{2\rho^2}\left(\frac{\Lambda}{R} - 2\theta\right)^2\right) \\ & \times \left[\exp\left(-\frac{\pi^2\rho^2}{2}\left(\left(f_x + \frac{k\Lambda}{4\pi z}\right)^2 + f_y^2\right)\right) \cos\left(\pi z\left(\frac{\Lambda}{R} - 2\theta\right)\left(f_x + \frac{k\Lambda}{4\pi z}\right)\right) \right. \\ & \left. + \exp\left(-\frac{\pi^2\rho^2}{2}\left(\left(f_x - \frac{k\Lambda}{4\pi z}\right)^2 + f_y^2\right)\right) \cos\left(\pi z\left(\frac{\Lambda}{R} - 2\theta\right)\left(f_x - \frac{k\Lambda}{4\pi z}\right)\right) \right] \end{aligned} \right), \quad (14)$$

where $\mathbf{f} = (f_x, f_y)$ is the spatial frequency coordinate in the power spectrum of the intensity distribution in the observation plane. Again, the expression for the power spectral density function consists of a self-correlation peak and two symmetrically displaced distance-correlation peaks, including modulating terms addressing all three peaks. The nature of these expressions can be described conveniently for the following three regimes:

In the case of $\Lambda \ll w$, the two beams will illuminate two almost concentric sections of the object surface; thus the total scattered field can be decomposed into two subfields, which have wavefronts distorted by the same surface structures. The total field propagates into the entire half plane. At the observation plane, the autocovariance function of the speckle pattern predicts a strong correlation at $\Delta p_x = \pm 2\theta z$. In other words, the resulting speckle pattern, observed

from any direction and distance, contains two mostly identical intensity structures that appear to “propagate” from the same point on the object surface towards the observation plane along directions separated by an angle of 2θ . In this case decorrelation has no effect on the height of the distance-correlation peaks in Eq. (11) as $\Lambda/w \approx 0$. The period of the cosine function approaches infinity; thus the cosine function approaches the value of unity.

We find the following expressions from Eqs. (11) and (14), respectively:

$$R_{\text{cn}}(\Delta\mathbf{p}) = R_0 \left(\begin{aligned} & \exp\left(-\frac{1}{\rho^2}\left((\Delta p_x - 2\theta z)^2 + \Delta p_y^2\right)\right) \\ & + \exp\left(-\frac{1}{\rho^2}\left((\Delta p_x + 2\theta z)^2 + \Delta p_y^2\right)\right) \\ & + 2 \exp\left(-\frac{1}{\rho^2}(\Delta p_x^2 + \Delta p_y^2)\right) \end{aligned} \right)^2, \quad (15)$$

and

$$\mathcal{G}_{\text{nAC}}(\mathbf{f}) = \pi\rho^2 R_0 \exp\left(-\frac{\pi^2\rho^2}{2}(f_x^2 + f_y^2)\right) \times \left(\begin{aligned} & 2 + \exp\left(-\frac{8\theta^2 z^2}{\rho^2}\right) \\ & + \left[\cos(4\pi\theta z f_x) + 4 \exp\left(-\frac{4\theta^2 z^2}{2\rho^2}\right) \cos(2\pi\theta z f_x) \right] \end{aligned} \right). \quad (16)$$

In Fig. 2(a), the autocovariance function in Eq. (15) is plotted in the Δp -plane for $\Lambda = 0$, $\Lambda/w = 0$, $z/R = 0$, $z\theta/\rho = 3.37$, and $(k\Lambda)/z = 0$. The power spectral density function is plotted in Fig. 2(b). In this regime the power spectral density function describes an

Fig. 2. Autocovariance function (a) in Eq. (15) and the corresponding power spectral density function (b) in Eq. (16) are plotted as a function of $\Delta \mathbf{p}$ and $\mathbf{f}$, respectively. The parameters for the plot are $\Lambda = 0$ mm, $\Lambda/w = 0$, $z\theta/\rho = 3.37$, and $(k\Lambda)/z = 0$.

oscillation with a period proportional to the product of θz .

In the case of $\Lambda \gg w$, the two beams will illuminate two separated sections of the object surface; thus the total scattered field will contain two subfields, which have wavefronts distorted by distinct surface structures. At the observation plane, the autocovariance function of the speckle pattern predicts no correlation on scales larger than the speckle size. Thus, the distance-correlation peaks in Eq. (11) vanish due to decorrelation ($\Lambda/w \gg 1$). However, the cosine function, which modulates the self-correlation peak in the autocovariance function with a period of $\lambda/(2\theta)$, indicates that there is a correlation on scales smaller than the speckle size. Therefore, the individual speckles are modulated with a fringe pattern [15] with a fringe spacing of $\lambda/(2\theta)$. The number of fringes within a speckle is proportional to the ratio of Λ/w , and beyond the mean speckle diameter 2ρ there is no correlation between the phases of the fringes.

We find the following expression from Eqs. (11) and (14):

$$R_{cn}(\Delta \mathbf{p}) = \exp\left(-\frac{2}{\rho^2}(\Delta p_x^2 + \Delta p_y^2)\right) \cos^2\left(\frac{k}{2z}\Lambda \Delta p_x\right), \quad (17)$$

Fig. 3. The autocovariance function (a) in Eq. (17) and the power spectral density function (b) in Eq. (18) are plotted as a function of $\Delta \mathbf{p}$ and $\mathbf{f}$, respectively. The parameters for the plots are $\Lambda/w = 6.2$, $z/R = 0$, $z\theta/\rho = 3.37$, and $(k\Lambda)/z = 8.2 \times 10^5 \text{ m}^{-1}$.

and

$$G_{\text{NAC}}(\mathbf{f}) = \frac{\pi \rho^2}{2} R_0 \left(2 \exp\left(-\frac{\pi^2 \rho^2}{2}(f_x^2 + f_y^2)\right) + \exp\left(-\frac{\pi^2 \rho^2}{2}\left(\left(f_x + \frac{k\Lambda}{2\pi z}\right)^2 + f_y^2\right)\right) + \exp\left(-\frac{\pi^2 \rho^2}{2}\left(\left(f_x - \frac{k\Lambda}{2\pi z}\right)^2 + f_y^2\right)\right) \right). \quad (18)$$

In Fig. 3(a), the autocovariance function in Eq. (17) is plotted in the Δp -plane for $\Lambda/w = 6.2$, $z/R = 0$, $z\theta/\rho = 3.37$, and $(k\Lambda)/z = 8.2 \times 10^5 \text{ m}^{-1}$. The power spectral density function is plotted in Fig. 3(b). Clearly, two frequency peaks, equivalent to the distance-correlation peaks described by the autocovariance function, are present in this regime, and their mutual distance is equal to the spatial frequency $\Delta f_x = 2k\Lambda/(2\pi z)$.

In the case of $\Lambda \approx w$, the two beams will illuminate two partly overlapping sections of the object surface. Accordingly, at the observation plane, the autocovariance function of the speckle pattern predicts a correspondingly weaker correlation at $\Delta p_x = \pm 2\theta z$. In this intermediate regime, the autocovariance function in Eq. (11) and its power spectral density function in Eq. (14) cannot be simplified.

B. Distance Between Object and Observation Plane

Knowing the angle (2θ) between the beams illuminating the object, the product of θz provides the distance (z) between the object and the observation plane, and *vice versa*. In the first regime ($\Lambda \ll w$), this product can be found as the center distance ($\Delta p = 2\theta z$) between the self-correlation peak and one of the two distance-correlation peaks, described by the autocovariance function. The random error $\Delta(\Delta p)$ for measuring the distance Δp between the peaks is proportional to the radius of the correlation peak ρ , as can be found in [18]. Assuming that the two beams are collimated at incidence on the object plane, the relative error of z can be approximated to the following square sum of the relative errors of Δp and θ :

$$z = \frac{\Delta p}{2\theta} \Rightarrow \frac{\Delta z}{z} \simeq \sqrt{\left(\frac{\Delta(\Delta p)}{\Delta p}\right)^2 + \left(\frac{\Delta\theta}{\theta}\right)^2} \propto \frac{\lambda}{\theta w}, \quad (19)$$

where the proportionality factor (rightmost) appears for $\Delta\theta \rightarrow 0$. Therefore, the uncertainty of this method is proportional to $1/w$ within the first regime.

In the second regime $\Lambda \gg w$, the distance z between the object and the observation plane can be determined from the measured distance Δf_x between the origin and one of two similar peaks in the power spectral density function. Defining z_c as the distance from the observation plane to the exact point of intersection of the two laser beams (see Fig. 1), the distance between the illuminating spots on the object can be determined as $\Lambda = 2\theta|z_c - z|$. Accordingly, knowing z_c the distance z can be found as

$$z = \frac{z_c}{\left(1 \pm \frac{\lambda}{2\theta} |\Delta f_x|\right)}, \quad (20)$$

where the plus option applies when $z_c > z$. Because the paraxial version of the Green's function [Eq. (5)] is used to propagate scattered light to the observation plane, we must assume that the Fresnel approximation ($z^3 \gg (k/8)|\mathbf{r} - \mathbf{p}|_{\text{max}}^2$) is always fulfilled [17]. Unfortunately, in this regime the

relation between z and Δf_x is a hyperbolic function, meaning that the sensitivity of the method drops fast for larger z values. The random error $\Delta(\Delta f_x)$ is proportional to the radius of the power spectral density peak as $2/(\pi\rho)$. Assuming that the two beams are collimated at incidence on the object surface, the relative random error on determining z now becomes

$$\frac{\Delta z}{z} \propto \frac{w}{\Lambda}. \quad (21)$$

Therefore, in the second regime, the relative error of the method is proportional to the ratio of w to Λ .

In the third regime, the distance z can be found from the distance-correlation peaks in both the autocovariance function and the power spectral density function. And the precisions will follow from the above considerations, dependent on which function provides the measurement.

3. Experiments

In Fig. 4 the schematic setup for generating the dual-beam illumination of the object is illustrated. An He-Ne laser emits a TEM₀₀ Gaussian beam at an optical wavelength of $\lambda = 633$ nm with a beam divergence of 1 mrad, and an e^{-2} -beam diameter of $2w = 0.81$ mm. The nonpolarizing beamsplitter divides the beam into two parallel propagating beams having approximately equal intensity and diameter. The L3-lens has a focal length of 250 mm, and deflects the beams with the purpose of bringing them to intersection in the back-focal plane of the L3-lens. In order to keep the beams approximately collimated at the point of intersection, the L2-lens provides the beams with the desired phase-front curvature as they enter the L3-lens. The focal length of the L2-lens is 60 mm and will provide a beam radius of approximately $w = 1.7$ mm at the object. The angle between the two beams is measured as the spot separation at a distant screen. Accordingly, the half angle is estimated to $\theta_{\text{meas}} = 0.294$ deg.

The object is a glass-blown glass plate with a thickness of 0.92 mm. The object is mounted on a linear stage, which can translate the object along the optical axis of the setup through a range from $z = 76$ mm

Fig. 4. The schematic for testing the principle is illustrated here. By moving the aperture up or down, either two or any of the two individual beams can illuminate the object.

Fig. 5. The obtained intensity distribution (a) contains two correlated speckle structures, which are produced by two Gaussian beams, overlapping partly on the object. Further, the spatial autocovariance function (b) of the intensity distribution (a) is illustrated as well. The length scales in (a) and (b) are the same.

to $z = 149$ mm with an approximate accuracy of ± 0.01 mm. The intersection of the beams is located at the object surface for $z = 81.4$ mm. The rest of the setup is stationary. The diffuse side (the object) of the glass plate is facing a CCD camera. The CCD camera is located in the observation plane at a distance of z from the rough surface of the object. The manufacture-specified pitch for the pixels in the CCD camera is $\Lambda_{\text{CCD}} = 0.0052$ mm, with an active array of 1280×1024 pixels.

First, the object is illuminated with two mutually coherent beams as shown in Fig. 4. The position of the object is adjusted in order to obtain nearly concentric sections of illumination on its surface; thus we are in the first regime, where $\Lambda \ll w$. In this case, Fig. 5(a) illustrates a resulting speckle pattern, being the sum of two correlated speckle patterns that add on the amplitude basis. The resulting speckle pattern is acquired with the CCD camera, and the corresponding normalized spatial autocovariance function [Fig. 5(b)] of the image is determined via the discrete Fourier transforms and the Wiener-Khinchin theorem. The presence of two similar speckle structures is difficult to observe visually. However, the autocovariance function finds the three correlation peaks as predicted in Eq. (15) and Fig. 2.

The distance between the two distance-correlation peaks in the autocovariance function is measured by applying a peak-finding procedure to the local areas surrounding each of the two peaks. The peak-finding procedure relies on a parabolic fit as suggested by [18]. The measurements are scaled according to the following expression:

$$z = \frac{\Lambda_{\text{CCD}}}{2 \tan \theta_{\text{meas}}} |\Delta p_{\text{CCD}}|, \quad (22)$$

where Δp_{CCD} is the measured peak positions in units of pixels.

4. Results

In Fig. 6 the half distance between the distance-correlation peaks of the autocovariance function have been plotted as a function of z . This equals the distance from the self-correlation peak to one of

the two distance-correlation peaks. The data points are scattered tightly around the line of first-order regression, going through $z = 0$. Any deviation from the line of regression is illustrated in Fig. 6 as well. We assume that the systematic deviation can be estimated by fitting the deviation to a regression curve of third order (see Fig. 6) throughout the entire range of z . In this case, the systematic deviation within the entire range causes an erroneous positioning of the object of less than ± 0.076 mm. Further, by assuming that the random errors can be estimated by subtracting the deviation curve from a regression curve of third order, we find a standard deviation on the estimation of z of 0.018 mm (0.036 pixels) within the z range from 75 to 100 mm. The reason for a third-order deviation will be discussed later. For the z ranges from 100 to 125 mm and from 125 to 150 mm, the standard deviations are 0.023 and 0.044 mm, respectively. Obviously, the random errors on the autocovariance measurements increase as the peak values decrease, in a similar way as predicted and proven by [18]. Nevertheless, within a range of 75 mm, the proposed method measures the distance between the object and the observation plane with systematic and random errors of less than $\pm 0.1\%$ and $\pm 0.06\%$ of the full range, respectively.

Fig. 6. The center distance between the self-correlation peak and one of the distance-correlation peaks in the autocovariance function is plotted as a function of distance between object and observation plane.

Figure 7 shows the maximum height of the distance-correlation peaks relative to the height of the self-correlation peaks as a function of z . The data points are scattered around the theoretical curve plotted for $w = 2$ mm, $\theta = 0.294$ deg, and $R \rightarrow \infty$, using Eq. (11) with a modification: Ideally, the relative height of the distance-correlation peaks should peak at a local maximum of 0.25 rather than approximately at 0.19 as we measure for $\Lambda = 0$ (at $z = 81.4$ mm). However, because, the spatial autocovariance function is generated from an image limited in size (1280×1024 pixels), areas corresponding to the shift between the two similar speckles structures will appear at the boundaries of the image with speckles having no match elsewhere in the image. Therefore, we reduce/modify the autocovariance values provided by Eq. (11) according to these areas of uncorrelated speckles and reach a reasonable match between measured and theoretical values. For the same reason, the position of the local maximum of the relative peak level, appearing due to complete beam overlap on the object, can be biased towards smaller z -values, or entirely vanish as in the case above (Fig. 7). The fast and minor oscillations occur because the maximum heights of the distance-correlation peaks are determined directly as a maximum pixel value, and not as the peak value of a parabolic fit.

5. Discussion

In the present experiment, the radius of curvature R of the beam is finite; thus the influence of Λ becomes apparent in Eqs. (11) and (14). Inserting the expression for a finite radius of curvature for a Gaussian TEM₀₀ beam with its waist located at z_w , as $R(z) = (z - z_w)[1 + (b/(z - z_w))^2]$, and the expression for the beam separation at the object, as $\Lambda(z) = 2\theta(z_c - z)$, the peak position in Eq. (11) becomes

$$\Delta p = 2\theta z \left(1 + \frac{(z - z_w)(z - z_c)}{b^2 + (z - z_w)^2} \right). \quad (23)$$

In Fig. 8, the theoretical systematic deviation has been fitted to an experiment carried out for $\theta_{\text{meas}} = 0.678$ deg and $w = 0.8$ mm. The theoretical deviation provides the best fit for the following

Fig. 7. Maximum distance-correlation peak level relative to self-correlation peak level is plotted as a function of distance between object and observation plane.

parameters: $z_c = 55.4$ mm, $z_w = 98.5$ mm, and $b = 0.447$ m ($w = 0.3$ mm). Just as what was the case above, we can subtract the estimated systematic error from our deviation plot and assume that the remaining variation expresses the random errors. We find the corresponding standard deviation on the estimate of z in the order of 0.033 mm, which provides a similar uncertainty of 0.06% relative to the entire z range of 53 mm.

Generally, laser speckles define a fundamental uncertainty [19] in distance measurements by triangulation or in any other technology based on coherent or partially coherent light. In the regime of $\Lambda \ll w$, the technology, described in this paper, facilitates a measurement based on a picture containing a large number of speckles or independent measurements. Therefore, the positions of the corresponding correlation peaks can be determined with an uncertainty that is significantly smaller than the speckle radius: According to the experiment above, the standard deviation on the position of the correlation peaks is less than 0.036 pixels within the z range from 75 to 100 mm. The average speckle radius (ρ) within this range of z is estimated to 0.015 mm, which corresponds to 2.8 pixels on the camera. Therefore, the standard deviation specified for $z \in [75 \text{ mm}; 100 \text{ mm}]$ can be expressed as 0.013ρ . Inserting the parameters for the setup illustrated in Fig. 4 into Eq. (33) in [19], we find the fundamental uncertainty on N^2 independent measurements of Δz given as 0.60 mm/ N . Assuming that the mean spacing (d_{sp}) between two adjacent speckles is between π to 5 times the speckle radius, we find the number of speckles present twice in a speckle pattern as $0.5 \times 1024 \text{ pixels} / (d_{\text{sp}} \times 2.8 \text{ pixels}) \times (1280 - 2|\Delta p_{\text{CCD}}|) \text{ pixels} / (d_{\text{sp}} \times 2.8 \text{ pixels})$, where $2|\Delta p_{\text{CCD}}|$ accounts for the boundary areas on each side of the image where speckles do not have a matching partner. The average displacement $|\Delta p_{\text{CCD}}|$ within the given range of z is 173 pixels. Therefore, we have somewhere in between $N^2 = [2442; 6187]$ independent measurements in the pictures. The corresponding fundamental

Fig. 8. The center distance and ratio of maximum correlation between the self-correlation peak and one of the distance-correlation peaks in the autocovariance function is plotted as a function of distance between object and observation plane. In this setup, the experimental parameters are $\theta_{\text{meas}} = 0.678$ deg, $w = 0.8$ mm.

uncertainty can be found within the range from 0.008 to 0.012 mm, where we measure with an uncertainty of 0.018 mm. The number of speckles present in the image is proportional to the spot size w ; thus as stated in [19], the gain in precision is achieved at the expense of reduced lateral resolution.

The technology is similar to the principles of the well-established technology of triangulation [20]. Specifically, in the regime of $\Lambda \gg w$ where the two spots are separated, the accuracy is similarly defined as being proportional to the spot size (w) on the object. However, in the regime of $\Lambda < w$ where the two spots are overlapping on the object, the experimental accuracy is proportional to the inverse of the spot size (w^{-1}). Therefore, in this regime, the accuracy of this technology does not rely on having a narrowly focused beam on the object. The pattern of the objective speckles is implemented without an imaging lens. Further, the information carried by the speckle pattern can be collected in principle anywhere in the half-plane in which the objective speckles are propagating.

The lateral shift of the peak intensity in the observation, which provides the measure of the product of $z\theta$ in a typical triangulation setup, can be scaled by the magnification of the imaging system in front of the detector array. For that reason triangulation can be applied to a large range of distances. However, to operate the technology proposed here within the regime of $\Lambda < w$ means that the range of operating is limited directly by the size of the camera/detector array. By using an arrangement with two cameras/detector arrays separated by a distance d_{sep} , the working distance can be shifted accordingly to larger values (d_{sep}/θ); however, the range will be the same.

By inserting a second camera/detector arrangement with a different optical distance to the object than the first detector arrangement, both z and θ can be determined. Therefore, absolute distance measurements can be obtained even in case where the angle θ between the beams is not constant. To obtain uncertainties of submicrons for measurements of Δz as achieved by [21], the setup in Fig. 4 must be configured with a larger angle between the two interfering beams; e.g., $\theta = 45^\circ$. The field modulation at the rough object surface can be implemented with a single beam, illuminating the surface through a grating. In case the grating is a surface-relief hologram in a thin polymer film attached to the object surface, the beam can be a free diverging beam from a VCSEL dye because $\Lambda \ll w$. Further, the detector arrangement can be a one- or two-dimensional detector array. Therefore, the technology can provide a simple and compact device as a short range distance transducer, e.g., for a stack of piezo-electric actuators.

6. Conclusion

We have introduced the normalized autocovariance function of speckle patterns produced by two coherent beams, overlapping at the object surface. The speckle statistics provide a tool to determine the distance between an object and the observation

plane. The method has been demonstrated experimentally, and we have achieved a random measurement error not far from the fundamental limit. The technology is applicable within the entire range from having overlapping beams to having separated beams. In this work, the focus has been on the regime where the two beams overlap completely or partly. Working within this regime only puts a practical upper limit to the range of distances that the technology can handle. Therefore, we suggest that the technology is applied to short range applications within the concept of triangulation. The advantage of the technology is that no alignment is required and it can be implemented with a minimum of components.

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