The influence of the group delay of digital filters on acoustic decay measurements

Sobreira-Seoane, Manuel A.; Cabo, David Pérez; Jacobsen, Finn

Published in:
Applied Acoustics

Link to article, DOI:
[10.1016/j.apacoust.2012.03.003](https://doi.org/10.1016/j.apacoust.2012.03.003)

Publication date:
2012

Document Version
Early version, also known as pre-print

[Link back to DTU Orbit](#)

Citation (APA):
Sobreira-Seoane, M. A., Cabo, D. P., & Jacobsen, F. (2012). The influence of the group delay of digital filters on acoustic decay measurements. *Applied Acoustics*, 73(9), 877-883.
<https://doi.org/10.1016/j.apacoust.2012.03.003>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

The influence of the group delay of digital filters on acoustic decay measurements

Manuel A. Sobreira-Seoane^{a,*}, David Pérez Cabo^a, Finn Jacobsen^b

^aSignal Theory and Communications Department, ETSIT Building, University of Vigo, E36310, Spain

^bAcoustic Technology, Department of Electrical Engineering, Technical University of Denmark, Denmark

ARTICLE INFO

Article history:

Received 18 January 2012

Accepted 6 March 2012

Available online 9 May 2012

Keywords:

Reverberation time

Digital filters

Group delay

Error

Acoustic decay

ABSTRACT

In this paper the error due to the phase response of digital filters on acoustic decay measurements is analyzed. There are two main sources of errors when an acoustic decay is filtered: the error due to the bandwidth of the filters related to their magnitude response, and the error due to their phase response. In this investigation the two components are separated and the phase error analyzed in terms of the group delay of the filters. Linear phase FIR filters and minimum phase IIR filters fulfilling the class 1 requirements of the IEC 61260 standard have been designed, and their errors compared. This makes it possible to explain the behavior of the phase error and develop recommendations for the use of each filtering technique. The paper is focused on the filtering techniques covered by current versions of the standards for measurement of acoustic decays and in the evaluation of the acoustic decay for narrow filters at low frequencies and low reverberation times ($BT < 16$).

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Several authors have examined the influence of filters on measurement of acoustic decays. The standard ISO 3382, parts 1 and 2 [1,2] presented several equations for calculating the standard deviation of measurement of the reverberation time in rooms, based on papers published by Davy in 1979 and 1980 [3,4]. In 1988 Davy proposed some empirical corrections to these expressions for low frequency measurements [5]. The corrections were needed because of discrepancies observed in real measurements compared with the analytical expressions. Davy assumed a smooth magnitude response of the filters and thus his expressions were based on the behavior of decays of sound in reverberant spaces without taking any influence of the filters into account. One of the reasons for the differences between Davy's estimation and real measurements could be due to the large errors that narrow filters may introduce in the evaluation of acoustic decays.

In 1987 Jacobsen studied the influence of detectors and filter bandwidths on measurements of acoustic decays [6] and concluded that the condition $BT_{60} > 16$, where B is the bandwidth and T_{60} is the reverberation time, ensures acceptable conditions in reverberation time estimations from the average slope of the acoustic decay. An even stronger condition was found when the evaluation of the initial part of the decay is needed: $BT_{60} > 64$ must be satisfied. Later Jacobsen and Rindel proposed the use of time-reversed decays in order to improve the estimations of the

reverberation time [7]. This technique makes it possible to relax the condition to $BT_{60} > 4$. Restrictions on reverberation time and loss factor measurements were presented, comparing the traditional direct filtering technique with the time-reversed one.

In Kob and Vörlander's investigation [8], the influence of the filters was studied with computer simulations. They detected how the error in the estimation of the reverberation time changes depending on the position of the resonances of the system under test with respect to the center frequency of the filter. This investigation led the present authors to suspect that the influence of the phase of the filters should be taken into account.

More recently, in 2008, Huszty et al. presented a description of the effects of filtering on the estimation of room acoustic parameters; however, the causes of the errors were not identified [9]. There is no report in the literature on the influence of the phase of the filters on the time-envelope of acoustic decays.

In the present work, the terms “phase distortion”, “effects of the non-linear phase” and “error due to the group delay” refer to the same concept. The group delay of the filter has been chosen as a measure of the phase distortion because it gives a direct view of the relative phase shifts of the signal within the filter band and information on temporal distortion of the acoustic decays.

The paper takes the model proposed by Kob and Vörlander as a reference, and identifies the influence of the non-linear phase of the filters on estimation of the reverberation time of resonant systems. The use of the term “resonant system” has been chosen because it will be shown how the error due to the phase distortion is strongly dependent on (1) the position within the filter band of the resonances of the system under test for a single resonance inside the band, and (2) the number of modes in the band. Another reason

* Corresponding author.

E-mail addresses: msobre@gts.uvigo.es (M.A. Sobreira-Seoane), dperez@gts.uvigo.es (D.P. Cabo), fja@elektro.dtu.dk (F. Jacobsen).

to choose this term is because if the errors in the estimation of room acoustic parameters are important, the problem may be even more serious in structural dynamics and in the determination of loss factors.

2. The simulation model

The model defined by Kob and Vörlander [8] has been taken as the starting point. The acoustic decay in a resonant system can be modeled as the superposition of several decaying resonant modes [10]. This effect can be modeled as a sum of decaying cosines,

$$h_{model} \sum_{i=1}^n A_i \cos(2\pi f_i t + \phi_i) \exp\left(\frac{-3 \ln 10}{T_i} t\right), \quad (1)$$

where ϕ models the phase of the modes, A_i models the amplitude of the modes at the measurement point at $t=0$, and $\exp\left(\frac{-3 \ln 10}{T_i} t\right)$ models an exponential decay with an attenuation of 60 dB when $t = T_i$ (therefore T_i is the reverberation time associated with a given normal mode with resonance frequency f_i). Averaging over ϕ has a similar effect as averaging several decays measured at different points of the system under test.

The signal model h_{model} is used as input to a filter with impulse response $h[n]$. The aim of this paper is to separate the errors produced when the acoustic decay is filtered by a one-third octave bandpass filter. Three kinds of filters have been designed: (1) an Infinite Impulse Response (IIR) Butterworth 8th order filter fulfilling the class 1 specifications of IEC 61260 [11]; (2) a linear phase Finite Impulse Response (FIR) filter matching the magnitude re-

sponse of the previous filter; and (3) an all-pass filter having a group delay with the exact shape as the IIR filter, so that the relative phase shifts within the band are the same. Table 1 shows the values of the reverberation times of the designed filters. It can be seen how the reverberation time of the all-pass filter tends to zero. As it has constant magnitude response for all frequencies, its time response should be a delta function. Fig. 1 shows the magnitude and group delay of the designed filters.

If the frequency response of a given filter is written as a function of its magnitude and phase responses,

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j\angle H(e^{j\omega})}, \quad (2)$$

the group delay is defined from the phase function as [12],

$$\tau_g(\omega) = \text{grad}(H(e^{j\omega})) = -\frac{d}{d\omega} \angle H(e^{j\omega}). \quad (3)$$

The reverberation time is estimated at the input and at the output of the filters by backwards integration of the squared impulse response [2,13]. After the integration, the reverberation time is evaluated by linear regression with the aim of finding the line which best fits the integrated decay. The line fitting is performed using a limited dynamic range of the acoustic decay, and the value of the reverberation time is obtained by extrapolation to a decay of –60 dB. The estimation of the reverberation time performed over the first 10 dB of the decay is called the Early Decay Time (EDT). The 60 dB decay times calculated by a line fit to the portion of the decay curve in the range from –5 dB to $-(R+5)$ dB are notated as T_R . Values of $R = 10, 20, 30$ and 60 dB are used throughout this paper. The influence of the filters on the estimation of the reverberation time can be described in terms of the relative difference between the reverberation time of the filtered acoustic decay and the reference reverberation time measured at the input of the filter,

$$\epsilon_f = \frac{T_{output} - T_{input}}{T_{input}} \quad (4)$$

where T_{input} and T_{output} defines the reference reverberation time estimated before and after the filter. Both T_{input} and T_{output} are estimated using different dynamic ranges of the decay.

Fig. 1. (a) Magnitude response of the IIR and FIR filters (identical), centered at 63 Hz; dashed line, magnitude response of the all-pass filter; (b) group delay of the IIR filter; (c) group delay of all pass (dashed line) and FIR filter.

When an acoustic decay is filtered there are several mixed effects that can lead to errors in the estimation of the reverberation time: (1) the time response of the filters will influence the decay time observed after the filter, and its influence will be noticeable mainly when measurements on systems with short reverberation times are carried out; (2) the magnitude response of the filter, i.e., the effect of limiting the bandwidth of the signal, will modify the slope of the decay, and it will be seen how the distortion depends on the position of the resonance frequency within the filter band; and (3) the non-linear phase of the filters will introduce

Fig. 2. Influence of the filter on a single resonance decay with $T_i = 0.5$ s. A one-third octave bandwidth filter centered at 63 Hz has been used. Filter specifications: IIR, 8th order butterworth, class 1 according to IEC 61260. Dashed line: non-filtered decay. Solid lines: decays with different f_i in the filter band.

different time shifts for each frequency component so that the temporal envelope of the signal will change. The first and second effects have already been described in the literature [2,6]. In this paper, the error due to the non-linear phase of the filters is evaluated and separated from the other effects. The use of the group delay of the filters instead of the phase itself helps to identify the source of error and also helps in the discussion of the nature of the error. It is well described in the classic literature of signal processing that the group delay is a good measure of the linearity of the phase of a filter, and how a non-constant group delay will lead to the distortion of the temporal envelope of a narrowband signal [12]. This effect is of great interest for understanding the changes and the distortions of the slope of the acoustic decay after passing it through a filter. The 8th order butterworth IIR filter has a non-linear phase as may be deduced from the shape of its group delay shown in Fig. 1. The FIR filter has the same magnitude response with linear phase (i.e., a constant group delay); therefore it will have the same error as the IIR due to the magnitude response, but no contribution due to phase distortion. The error due to the phase distortion can be evaluated from the all-pass filter as it has a constant magnitude response.

3. The energy decay of a single mode resonant system

Let us start by evaluating the influence of the filters when only one resonance is present in the filter band. We have chosen the

Fig. 3. Relative error as a function of the position of the frequency of the decaying mode relative to the center frequency of the filter [%] ($T_{input} = 0.5$ s); solid line, FIR filtered; dash-dotted line, IIR filtered; dashed line, all-pass filtered; and dotted line, the sum of the errors of the all-pass and FIR filter. (a) EDT; (b) T_{10} ; (c) T_{20} ; (d) T_{30} ; (e) T_{60} .

one-third octave band filter centered at 63 Hz to show the effects. Fig. 2 shows the backwards integrated decay at the input (dashed line) and at the output of the IIR filter [13]. It is clear that the slope of the decay changes for different resonance frequencies, so the error in the evaluation of the reverberation time will not only depend on the filter itself, but also on the position of the resonance frequencies of the system under test.

Fig. 3 shows the error in the estimation of the EDT, T_{10} , T_{20} , T_{30} and T_{60} for the different filters. The error is calculated according to Eq. (4), and the x-axis shows the tuning of f_i relative to the center frequency of the filter, $(f/f_0 - 1) \times 100\%$. In these figures two sources of error may be identified: the effect of the bandwidth of the filter and the effect of the group delay:

Table 2

Mean error $\bar{\epsilon}_f$ and standard deviation in the estimation of the reverberation time and initial decay in the 63 Hz band. $T_{input} = 0.5$ s.

	$\bar{\epsilon}_{f-EDT}$ (%)	$\bar{\epsilon}_{f-T_{10}}$ (%)	$\bar{\epsilon}_{f-T_{20}}$ (%)	$\bar{\epsilon}_{f-T_{30}}$ (%)	$\bar{\epsilon}_{f-T_{60}}$ (%)
IIR	29.4	4.2	2.8	1.9	0.8
FIR	15.7	2.6	1.5	1	0.5
All-pass	15.9	0.9	0.5	0.4	0.2
	σ_{e-EDT} (%)	$\sigma_{e-T_{10}}$ (%)	$\sigma_{e-T_{20}}$ (%)	$\sigma_{e-T_{30}}$ (%)	$\sigma_{e-T_{60}}$ (%)
IIR	20.8	19.4	12.8	9.3	4.3
FIR	9.4	9.7	6.4	4.6	2.2
All-pass	16	11.4	7.3	5	2.3

- FIR filter: there is no contribution to the error of the group delay as it is constant. The increase of the error close to the band limits of the filter has already been detected and described in [8].
- All-pass filter: this filter only makes a contribution due to the group delay. It can be observed how this contribution is of the same order as the previous one: the two lines overlap and the error follows the shape of the group delay of the filter: minimum in the center of the band.
- IIR filter: this filter has both contributions. The dotted line shown in Fig. 3 represents the sum of the errors due to the bandwidth and the phase distortion. The error due to the IIR filters trends to the sum of the two contributions.

The error due to the group delay follows the shape of the error due to the bandwidth of the filter. Table 2 shows the values for the averaged error within the band and the standard deviation of the error when the resonance frequency is placed randomly in the band: $B = 14.55$ Hz and $f_i \in [56, 70]$ Hz with a resolution of 0.1 Hz. The standard deviation gives information on the dispersion of the error due to the position of the resonance in the band. Therefore, high values of the standard deviation outliers are expected, i.e., the error in the evaluation of the reverberation time may become large for certain resonant systems.

From these data some conclusions can be derived. Both the error and the standard deviation for IIR filters are close to the sum of the contributions of the influence of the bandwidth of the filter and the phase distortion contribution,

Fig. 4. Error in the estimation of the reverberation time when the acoustic decay with $T_{input} = 0.5$ s is filtered with the time-reversed filter $h[-n]$; solid line, FIR filtered; dashed-dotted line, IIR filtered; dashed line, all-pass filtered. (a) EDT; (b) T_{10} ; (c) T_{20} ; (d) T_{30} ; (e) T_{60} .

Table 3

Mean error $\bar{\epsilon}_f$ and standard deviation in the estimation of the reverberation time and initial decay in the 63 Hz band. Time reversed filters.

	$\bar{\epsilon}_{f-EDT}$ (%)	$\epsilon_{f-T_{10}}$ (%)	$\bar{\epsilon}_{f-T_{20}}$ (%)	$\bar{\epsilon}_{f-T_{30}}$ (%)	$\bar{\epsilon}_{f-T_{60}}$ (%)
IIR	31.2	−0.1	−0.03	−0.01	−0.004
FIR	15.6	3.8	1.5	1.2	−0.6
All-pass	18.2	−13.7	−4.4	−2.1	−0.6
	σ_{e-EDT} (%)	$\sigma_{e-T_{10}}$ (%)	$\sigma_{e-T_{20}}$ (%)	$\sigma_{e-T_{30}}$ (%)	$\sigma_{e-T_{60}}$ (%)
IIR	5.9	0.3	0.08	0.04	0.01
FIR	9.7	9.7	6.4	4.6	1.8
All-pass	9.7	9.7	6.4	4.6	2.2

Fig. 5. Change in the estimated slope of the acoustic decay before (decay on the left) and after the IIR filter.

$$\epsilon_f(IIR) \approx \epsilon_f(FIR) + \epsilon_f(all - pass), \quad (5)$$

$$\sigma_T(IIR) \approx \sigma(FIR) + \sigma(all - pass). \quad (6)$$

It is also clear that the larger dynamic range available to evaluate the slope of the acoustic decay, the smaller the error. We may notice how in the evaluation of the initial part of the decay (EDT), the effect of the group delay may become the dominant factor; although the mean error for both contributions are of the same order, the

standard deviation of the all-pass case (i.e., the contribution to the error of the phase distortion) is almost twice the contribution of the filter bandwidth. The EDT will always be overestimated as the error is always positive within the band (see Fig. 3), whereas overestimations and underestimations of the slope of the acoustic decay are possible as the error takes positive and negative values depending on the position of the resonance frequency.

4. Time-reversed decay measurements of single resonance systems

Jacobsen and Rindel demonstrated the favorable influence of reversing the decaying signal applied to the bandpass filter. This effect is equivalent to applying the acoustic decay to a time-reversed version of the digital filter, $h[-n]$. If the real filter has an impulse response $h[n]$ and frequency response $H(e^{j\omega})$, the frequency response of the time reversed filter will be $H(e^{-j\omega})$. This means that the time reversed filter will have the same magnitude response and opposite phase, and therefore the group delay of the filter will be

$$\text{grd}(H(e^{-j\omega})) = -\text{grd}(H(e^{j\omega})). \quad (7)$$

As shown in the previous section, the error due to the group delay of the filter tends to follow the shape of the error due to the filter bandwidth, and therefore it may be expected that for the time reversed IIR filter the latter will tend to counterbalance the former. Fig. 4 shows this effect clearly. In the case of filtering with the IIR filter, it can be seen how the error is close to zero for all the evaluation ranges of the acoustic decay. Fig. 4 shows the error obtained when the reverberation time of the system is slightly higher than the reverberation time of the filters (see Table 1). It can be seen in Fig. 4a how the error is non-zero for all the filters. This means that when the acoustic decay is filtered with the time reversed IIR, the contribution to the error of the reverberation time of the filter is the dominant factor in the estimation of the EDT as the phase error counterbalances the bandwidth error. Table 3 shows the

Fig. 6. Average error in the estimation of the reverberation time as a function of the number of modes in the band. The figure shows the error for a decay with $T_i = 0.75$ s, (a) FIR filter, (b) IIR filter, (c) FIR, (d) time reversed IIR; ‘-o-’, EDT; ‘-▽-’, T_{10} ; ‘-.-’, T_{20} ; ‘- - -’, T_{30} ; solid line, T_{60} .

values of the mean error $\bar{\epsilon}_f$ and standard deviation in the estimation of the reverberation time and initial decay in the 63 Hz band.

5. Acoustic decay measurements for higher order systems

In the previous sections the effect of the band filter in the evaluation of systems with one degree of freedom or with only one normal mode within the band has been described. Here the effect on the decays of several modes will be examined. The decay of several modes with different resonance frequencies exhibits an amplitude modulation due to interference effects between the overlapped acoustic decays of modes with close resonance frequencies, and this will be affected by the phase distortion of the filters. Fig. 5 shows the change of the slope of the integrated acoustic decay. Therefore, the expected error is higher than the errors found in the foregoing.

To calculate the error as a function of the mode density, it was assumed that each possible resonance value is random with a uniform distribution in the frequency range of the band. Fig. 6 shows the average error, and Fig. 7 shows the standard deviation after two thousand calculations for each modal density value when a reverberation time of 0.75 s is selected. The results reveal how the error increases with the modal density and tends to become constant for high modal densities. At low frequencies (below 100 Hz) low modal densities are expected, and it is seen the resulting error increases quickly for evaluations of the initial part of the acoustic decay (EDT and T_{10}). The error of IIR filters tends to be twice the error of FIR filters, showing again that the group delay has a similar contribution to the total error as the bandwidth. These results are also valid for EDT evaluations using time-reversed filters (Fig. 6c and d). The results show that the reversed-IIR behaves worse than linear phase FIR filters for the evaluation of the initial part of the decay. This is due to the fact that phase distortions dominates the initial stage of the decay. Therefore, we may conclude that when time reversed IIR filters are used, the group delay counterbalances the bandwidth error even for higher modal densities. Linear phase FIR filters show the best results in the

Fig. 8. Probability density functions of the error in the estimation of the T_{20} after filtering with: '—' time-reversed IIR; '---' FIR; '-.-' IIR filter (dashed dotted line). $BT = 11$.

evaluation of the initial part of the decay, even though the error is still high for low BT products.

Fig. 7 shows the standard deviation of the error depending on the number of modes within the band. It can be observed how even for low errors (time-reversed IIR filter) the standard deviation is high, revealing a strong dependence of the error on the position of the resonances. It can be observed how the mean error is higher and the distribution more spread in the case of IIR filtering; the shift in the mean and the increase of the variance are due to the influence of the phase distortion of the IIR filter. As can easily be found in the literature on statistics, [14], the kurtosis is a good measure of how outlier-prone a distribution is. The skewness of a distribution function measures the asymmetry of the data around its mean. Both parameters give exact information on how far from normal a distribution is: the kurtosis of a normal distribution is 3 and its skewness is 0, as it is symmetric. Fig. 8 shows as example the Probability Density Function of the error in the estimation of the T_{20} with a $BT = 11$ and FIR filters, IIR filter and time-reversed IIR filters. It can be seen how the distribution of the error is not normal: the right tail is longer than the left, indicating that the probability of overestimation is higher than the probability of underestimation in the measurement. The distributions of T_{20} and T_{30} are similar, although the mean expected error is smaller

Fig. 7. Standard deviation of the error as a function of the number of modes inside the band. Values calculated with $T_i = 0.75$ s. '—o—': EDT, '---▽---': T_{10} , '---□---': T_{20} , '---◇---': T_{30} , '---×---': T_{60} .

for the latter. In the example showed, kurtosis values $k > 8$ and skewness $s > 2$ have been calculated for all the cases. In the case of time-reversed IIR filters, the kurtosis takes values over 30, which clearly confirms a high probability of having outliers.

6. Conclusions

Errors in acoustic decay measurements due to the phase distortion introduced by the most common filters have been analyzed. The investigation has focused on short reverberation times and narrow filters ($BT < 16$) in order to get more knowledge of the nature of the errors and develop steps toward the improvement of such measurements. The examples given throughout the paper are based on the response of an 8th order butterworth IIR filter fulfilling the requirements of IEC 61260, a quite common implementation in many acoustic measurement devices. A linear phase FIR filter with the same magnitude response as the IIR and a all-pass filter with the same group delay as the IIR have been implemented. A comparison of the responses of the filters have made it possible to separate and describe the error on the measurement of acoustic decays, leading to the following conclusions.

The magnitude of the error due to the phase distortion tends to be of the same order as the error due to the filter bandwidth. The standards on measurement of reverberation times include some comments on the convenience of using the time-reversed filtering technique to evaluate the acoustic decays for low BT products and relate this convenience to the time response of the filters; the reverberation time of the filters are a limiting factor, i.e., there is no chance of measuring a reverberation time shorter than the filter response. However, this investigation has demonstrated that the error due to the magnitude response of the filters can be counter-balanced by the error due to the phase distortion; hence we may conclude that the use of time-reversed IIR is strongly recommended for acoustic decay measurements with a few modes within a filter band, even for long reverberation times. The evaluation of

the early part of the acoustic decay is still a problem: high errors can be expected because there is a strong dependence of the shape of the initial part of the decay on the phase of the resonant modes involved in the decay and of the phase response of the filter. In this situation, even the time reversed technique shows high errors for low BT products.

The distributions of the errors are clearly non-symmetric, showing high values of skewness and kurtosis, which reveals that outliers can be expected. The outliers are due to the strong dependence of the error on the number of modes and the position of the resonances within the filter band of the system under test.

References

- [1] ISO. 3382-1:2009. Acoustics – Measurement of room acoustic parameters – Part 1: Performance spaces.
- [2] ISO. 3382-2:2008. Acoustics – Measurement of room acoustic parameters – Part 2 reverberation time in ordinary rooms.
- [3] Davy JL. The variance of decay rates in reverberation rooms. *Acustica* 1979;43:12–25.
- [4] Davy JL. The variance of impulse decays. *Acustica* 1980;44:51–6.
- [5] Davy JL. The variance of decay rates at low frequencies. *Applied Acoustics* 1988;23:63–79.
- [6] Jacobsen F. A note on acoustic decay measurements. *J Sound Vib* 1987;115(1):163–70.
- [7] Jacobsen F, Rindel JH. Time reversed decay measurements. *J Sound Vib* 1987;117(1):187–90.
- [8] Kob M, Vörländer M. Band filters and short reverberation times. *Acust United Acta Acust* 2000;86:350–7.
- [9] Torma C, Huszty N, Bukuli, Augusztinovicz F. Effects of filtering of room impulse responses on room acoustic parameters by using different filter structures. In: *Proceedings of Acoustics'08*. Paris; 2008. p. 4497–502.
- [10] Coppens AB, Kinsler LE, Frey AR, Sanders JV. *Fundamentals of acoustics*. 4th ed. John Wiley and Sons; 2000.
- [11] IEC 61260. Electroacoustics – Octave-band and fractional-octave-band filter banks; 1995.
- [12] Oppenheim AV, Schaffer RW. *Discrete-time signal processing*. 3rd ed. Prentice Hall; 2009.
- [13] Schroeder MR. New method of measuring reverberation time. *J Acoust Soc Am* 1965;37:409–12.
- [14] Ruppert D. *Statistics and finance: and introduction*. Springer texts in statistics. Springer; 2004.