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# Topology optimization of optical surfaces

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## Motivation and goals

For many applications the need for specifying and controlling the optical properties of surfaces is of high importance. Furthermore, the advances in nano-technology allow for fabrication of increasingly complex nano-structured surfaces. The problem of designing nano-structured surfaces with specific optical properties can be extremely challenging. A systematical design method is desirable to design surfaces of complex topology with tailored optical properties and to ensure design robustness considering practical dimensional tolerances.

### Goals:

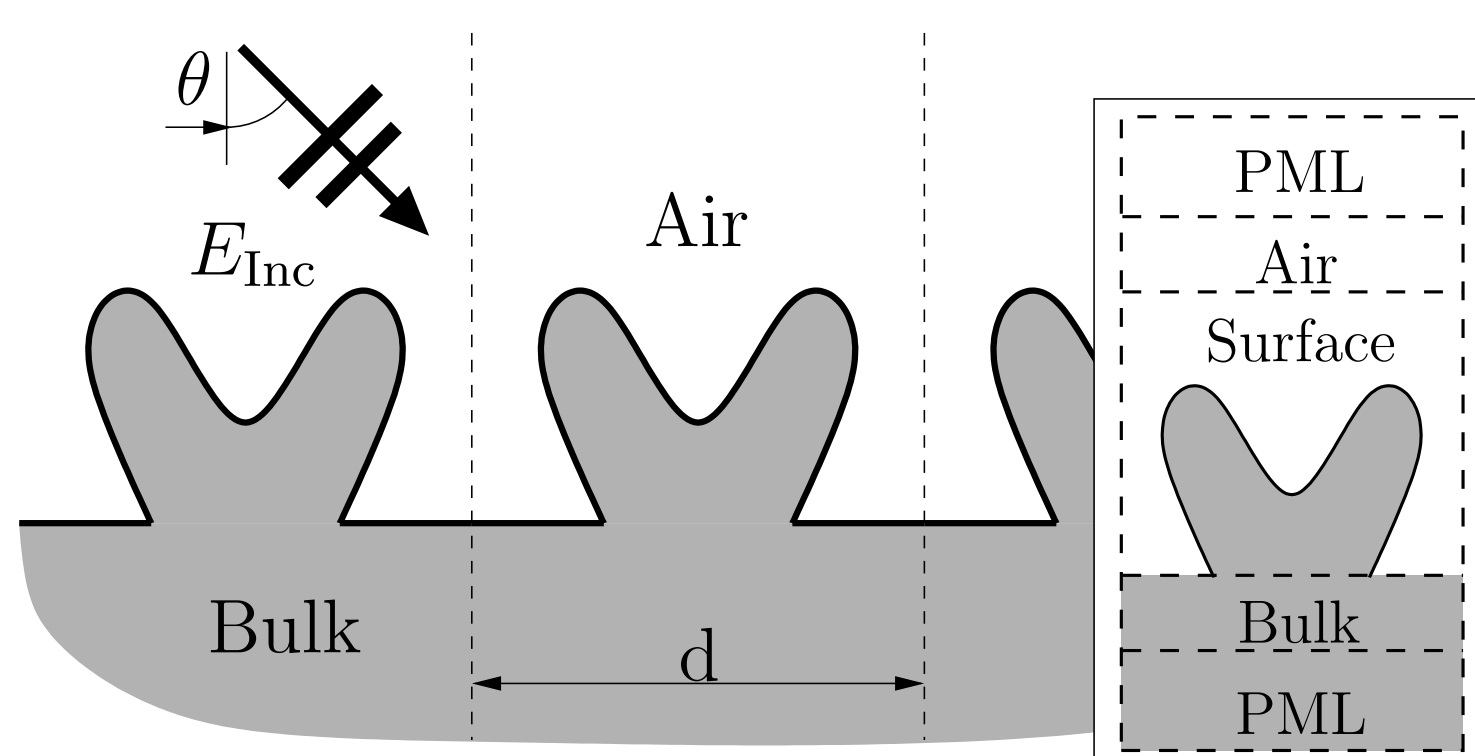
- Development of systematic method for designing optical surfaces
- Consider manufacturing tolerances and minimum length scale
- Verification of method by numerical examples

### Example Applications:

- Advanced high performance gratings
- Structural color surfaces
- Optical filters

## Modelling

The problem is modelled as a periodic cell



The electro-magnetic field is described by the 2D Helmholtz equation

$$\nabla \cdot (A(\mathbf{x}) \nabla u(\mathbf{x})) + \omega^2 B(\mathbf{x}) u(\mathbf{x}) = 0$$

with polarisation dependent parameters

$$A_{\text{TM}} = \frac{1}{\mu_r}, B_{\text{TM}} = \frac{\epsilon_r}{c^2} \quad A_{\text{TE}} = \frac{1}{\epsilon_r}, B_{\text{TE}} = \frac{\mu_r}{c^2}$$

## Parameterization

Material properties are interpolated by elemental material parameters

$$A_e = A_1 + \bar{\rho}_e^p (A_2 - A_1)$$

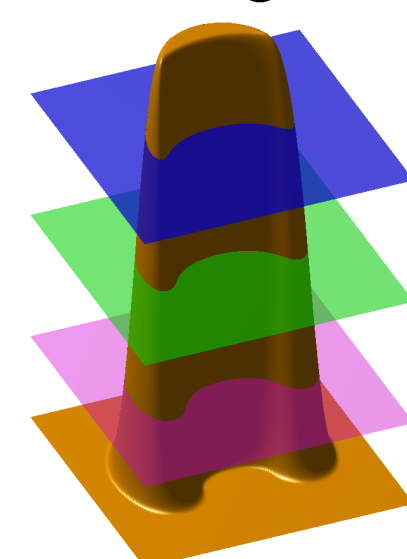
$$B_e = B_1 + \bar{\rho}_e^p (B_2 - B_1)$$

Elemental density:  $0 \leq \bar{\rho}_e(\rho) \leq 1$ , for  $e \in \Omega$

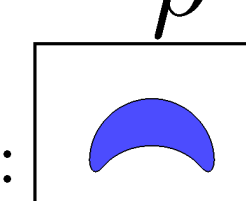
## Robust design

Simulates eroded, intermediate and dilated design realisations [5]

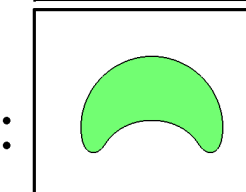
Gradient constrained design field



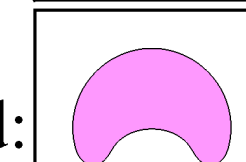
Over-etched:



Normal-etched:



Under-etched:



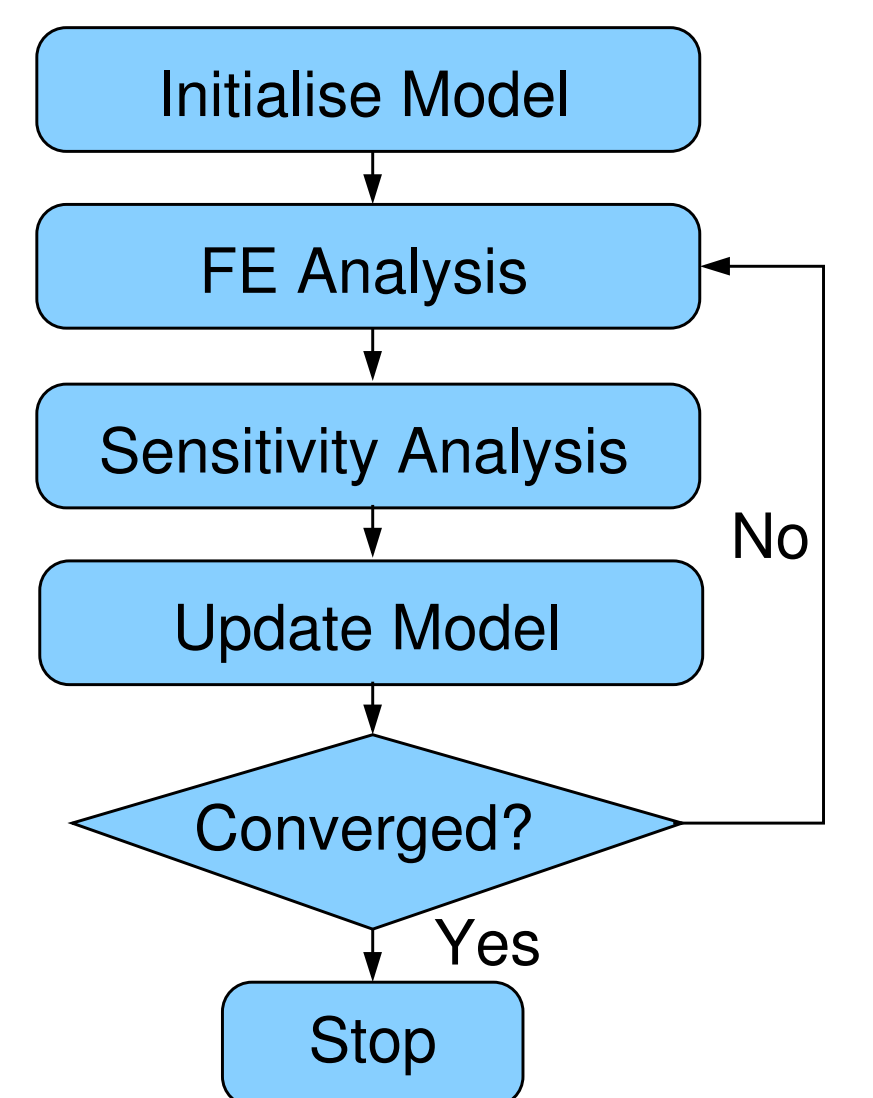
## Topology optimisation

$$\begin{aligned} \min_{\rho_j} : \max_q : \max_i : \{h_i((\bar{\rho}_j)^q)\}, \quad 0 < \rho_j \leq 1, \\ \text{s.t.} : (\mathbf{K}_i^q - \omega^2 \mathbf{M}_i^q) \mathbf{u} = \mathbf{f}_i, \quad h_i = \{T \text{ or } R\}, \\ (\hat{\mathbf{K}}_i - \lambda_1^q \hat{\mathbf{M}}_i) \mathbf{x} = 0, \quad \lambda_1^q > \delta, \quad q = \{e, i, d\}, \\ i = 1, \dots, N_{\omega, \theta, \text{pol}}, \quad j = 1, \dots, N_{\text{el}} \end{aligned}$$

### Design sensitivity

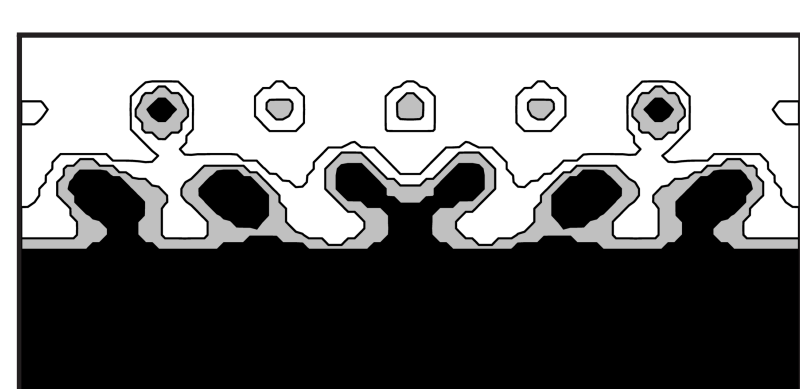
$$\frac{d\Phi}{d\rho_i} = 2\Re \left( \lambda^T \frac{\partial \mathbf{S}}{\partial \rho_i} \mathbf{u} \right)$$

$$\mathbf{S}^T \boldsymbol{\lambda} = \sum_{e \in e_{\text{in}}} \frac{i}{4\omega\mu} \mathbf{Q}^e \bar{\mathbf{u}}_e$$

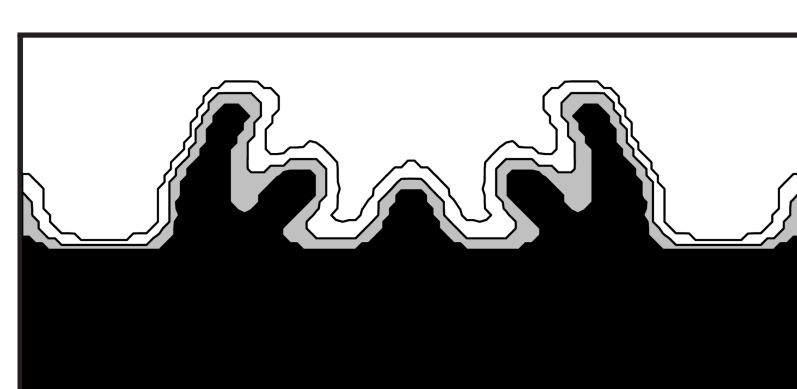


## Numerical examples

### Max transmittance - multiangles



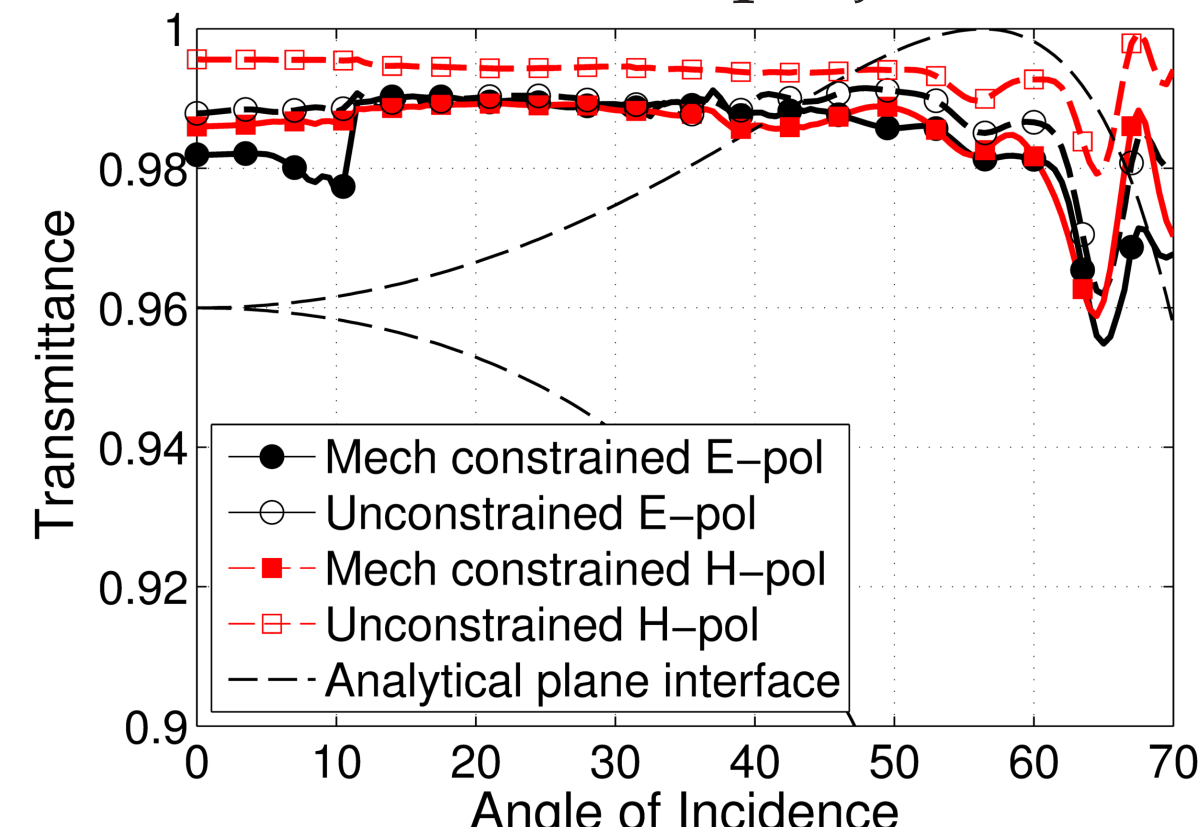
No constraints



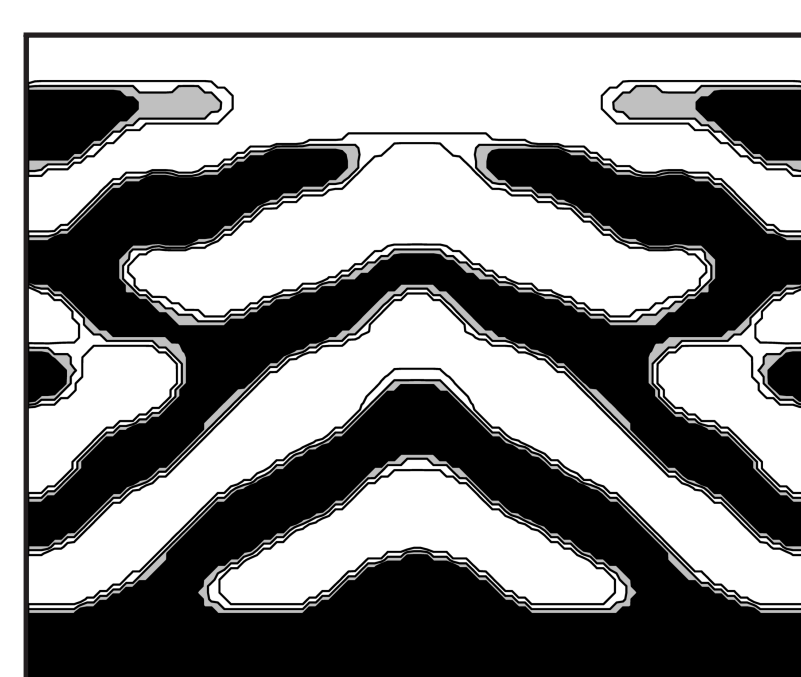
Mechanically constrained

#### Angle sweep

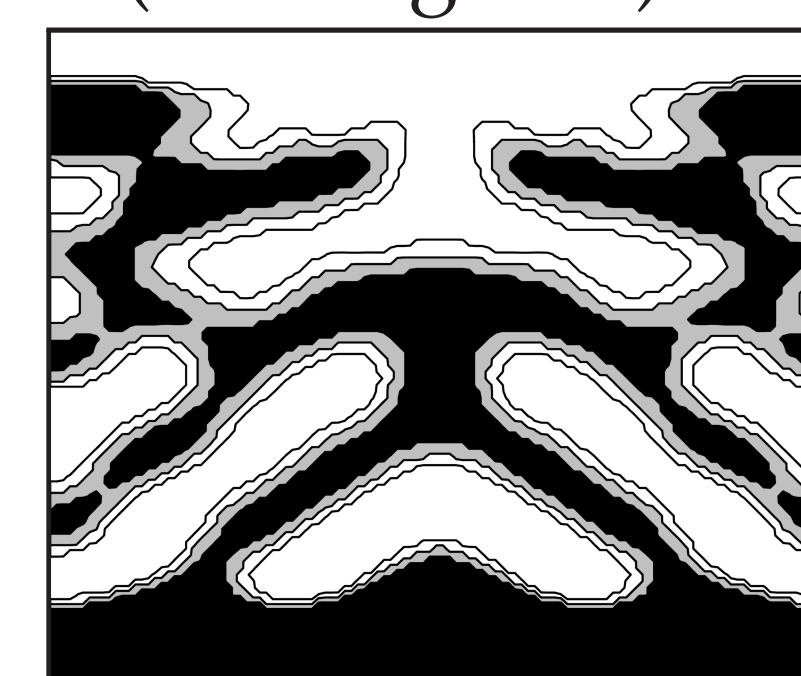
Intermediate projection



### Maximise reflectance - multifrequency



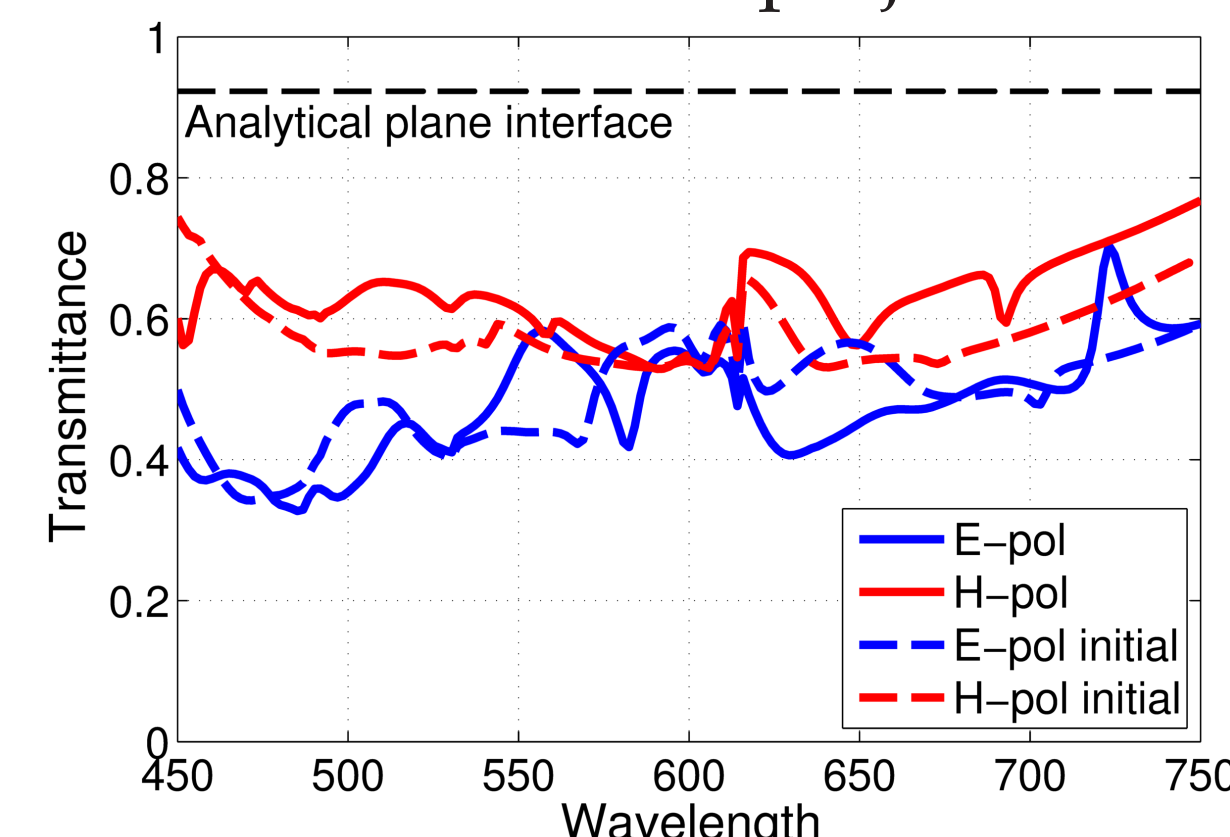
No constraints (initial guess)



Mechanically constrained

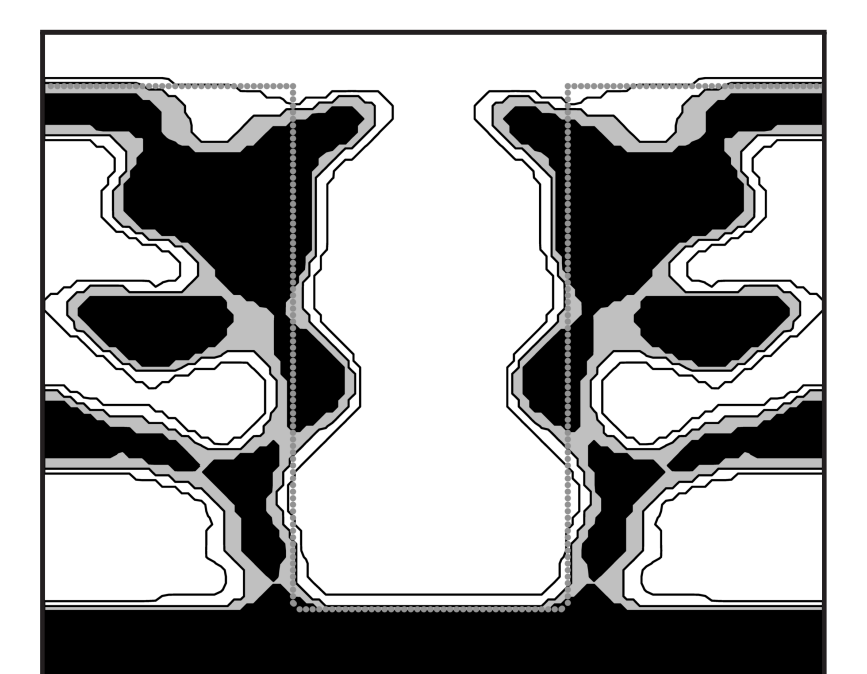
#### Wavelength sweep

Intermediate projection



Surface design build by intermediate projection

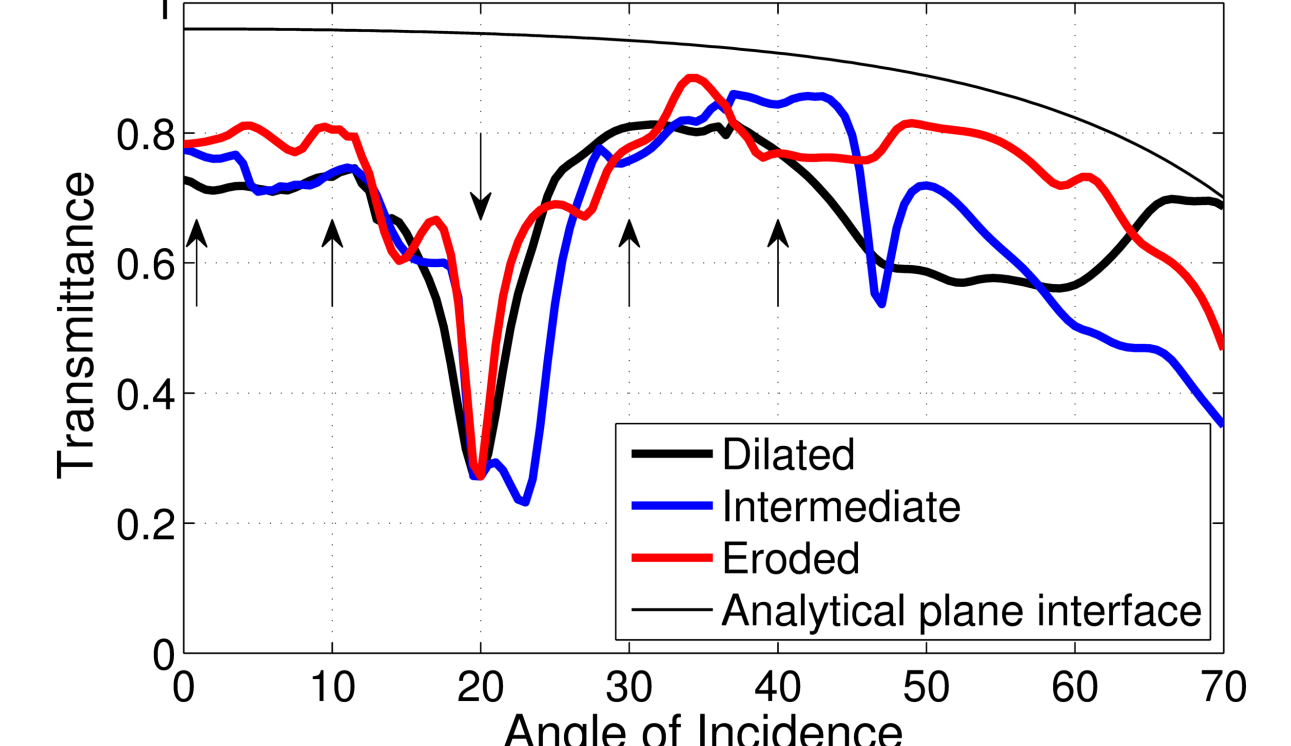
### Selective reflectance



Mechanically constrained

#### Angle sweep

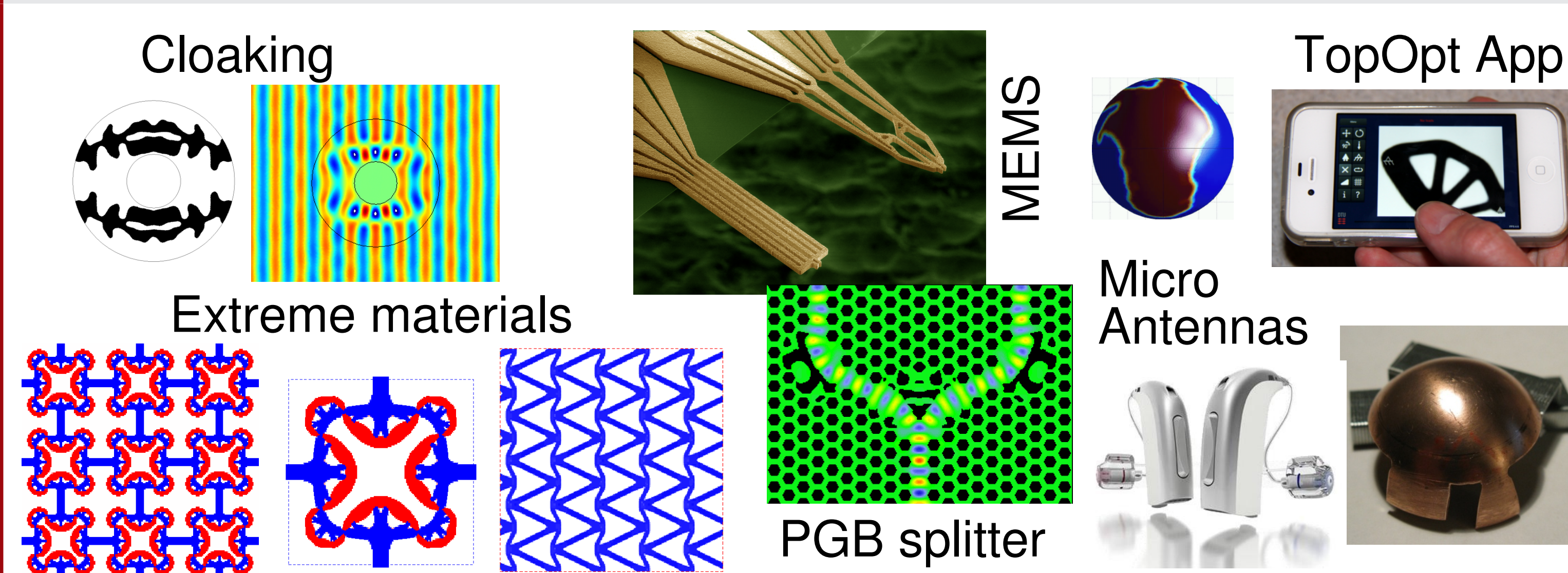
E-polarisation



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## Previous examples



## TopOpt App

