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Benchmarking of optimization methods for topology optimization problems

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Benchmarking of optimization methods for topology optimization problems

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DTU Wind Energy
Department of Wind Energy

Why?

- Asses general purpose 2nd order optimization methods in topology optimization problems.

Why?

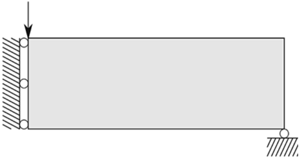
- Asses general purpose 2nd order optimization methods in topology optimization problems.

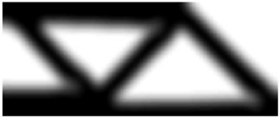
Main results from the Benchmarking

- GCMMA outperforms MMA.
- GCMMA and MMA tend to obtain a design with large KKT error. 
- The performance of GCMMA and MMA do not highlight respect to other solvers. 
- The interior-point solver IPOPT, when the exact Hessian is used (IPOPT SAND), produces the best designs using few number of iterations 
- IPOPT SAND is the most robust solver in the study. 
- The SAND formulation requires lot of memory and computational time. 

Topology optimization problems

- **Goal**: Obtain optimal design of a structure with given loads.





- **Model as an optimization problem**

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g(x) \leq 0 \\ & h(x) = 0. \end{aligned}$$

Topology optimization formulations

SAND formulation:

- Minimum compliance

$$\begin{array}{ll} \underset{\mathbf{t},\mathbf{u}}{\text{minimize}} & \mathbf{f}^T\mathbf{u} \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- Minimum volume

$$\begin{array}{ll} \underset{\mathbf{t},\mathbf{u}}{\text{minimize}} & \mathbf{a}^T\mathbf{t} \\ \text{subject to} & \mathbf{f}^T\mathbf{u} \leq C \\ & \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- Compliant mechanism design

$$\begin{array}{ll} \underset{\mathbf{t},\mathbf{u}}{\text{minimize}} & \mathbf{l}^T\mathbf{u} \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- $\mathbf{f} \in \mathbb{R}^d$ the force vector.
- $\mathbf{a} \in \mathbb{R}^n$ the volume vector.
- $V > 0$ is the upper volume fraction.
- $C > 0$ the upper bound of the compliance.
- $\mathbf{l} \in \mathbb{R}^d$ vector that indicates the output displacement.

Topology optimization formulations

NESTED formulation:

- Minimum compliance

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{u}^T(\mathbf{t})\mathbf{K}(\mathbf{t})\mathbf{u}(\mathbf{t}) \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- Minimum volume

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{a}^T\mathbf{t} \\ \text{subject to} & \mathbf{u}^T(\mathbf{t})\mathbf{K}(\mathbf{t})\mathbf{u}(\mathbf{t}) \leq C \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- Compliant mechanism design

$$\begin{array}{ll} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{l}^T\mathbf{u}(\mathbf{t}) \\ \text{subject to} & \mathbf{a}^T\mathbf{t} \leq V \\ & \mathbf{0} \leq \mathbf{t} \leq \mathbf{1}. \end{array}$$

- $\mathbf{u}(\mathbf{t}) = \mathbf{K}^{-1}(\mathbf{t})\mathbf{f}$.
- $\mathbf{f} \in \mathbb{R}^d$ the force vector.
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- $V > 0$ is the upper volume fraction.
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Considerations on the problem formulation

- Use only one external static load.
- Linear elasticity in the equilibrium equation.
- Assume $\mathbf{K}(\mathbf{t}) \succ 0$ to avoid ill-conditioning.
- Use continuous density variables.
- Use SIMP penalization and a density filter.

Bendsøe, M. P and Sigmund, O. Material interpolation schemes in topology optimization. *Archive of Applied Mechanics*,69:635–654,1999.

Bourdin, B. Filters in topology optimization. *International Journal for Numerical Methods in Engineering*,50(9):2143–2158, 2001.

Optimization methods

Topology optimization problem

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.

Andreassen, E and Clausen, A and Schevenels, M and Lazarov, B. S and Sigmund, O. Efficient topology optimization in MATLAB using 88 lines of code. *Structural and Multidisciplinary Optimization*, 43(1): 1–16, 2011.

Svanberg, K. The method of moving asymptotes a new method for structural optimization. *International Journal for Numerical Methods in Engineering*, 24(2): 359–373. 1987.

Svanberg, K. A class of globally convergent optimization methods based on conservative convex separable approximations. *SIAM Journal on Optimization*, 12(2): 555-573, 2002.

Optimization methods

Topology
optimization
problem

⇓

non-linear
problem

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.

- **FMINCON**: Interior-point MATLAB. Use exact Hessian.
- **SNOPT**: Sequential quadratic programming. BFGS approximations.
- **IPOPT**: Interior-point software. Exact Hessian in the SAND formulation, BFGS in the NESTED formulation.

Gill, P. E and Murray, W and Saunders, M. A. SNOPT: An SQP Algorithm for Large -Scale Constrained Optimization. *SIAM Journal on Optimization*, 47(4):99–131, 2005.

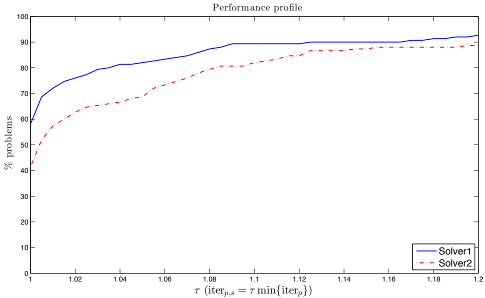
Wächter, A and Biegler, L. T. On the implementation of an interior point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, 2006.

Benchmarking in topology optimization

- **How?** Using performance profiles.
  - Evaluate the cumulative ratio for a performance metric.
  - Represent for each solver, the percentage of instances that achieve a criterion for different ratio values.

$$\rho_s(\tau) = \frac{1}{n}\text{size}\{p \in P : r_{p,s} \leq \tau\},$$

$$r_{p,s} = \frac{\text{iter}_{p,s}}{\min\{\text{iter}_{p,s} : s \in S\}}.$$



Dolan, E. D and Moré, J. J. Benchmarking optimization software with performance profiles. *Mathematical Programming*,91:201–213, 2002.

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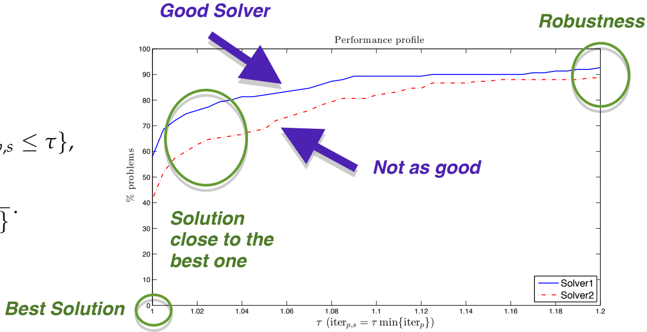

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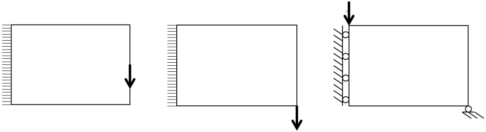
Benchmark set of topology optimization problems

Minimum compliance /minimum volume

- Michell, Cantilever and MBB domains, respectively.

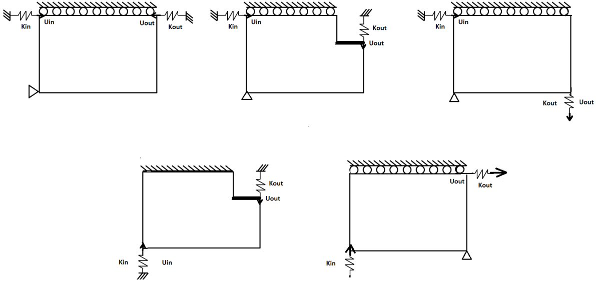


- Length ratio: Michell: 1×1, 2×1, and 3×1. Cantilever: 2×1, and 4×1. MBB: 1×2, 1×4, 2×1, and 4×1.
- Discretization: 20, 40, 60, 80, 100 elements per ratio.
- Volume constraint: $0.1 - 0.5$.
- Compliance constraint: 1, 1.25, $1.5 \times C$. Where $C = \mathbf{f}^T \mathbf{K}^{-1}(\mathbf{t}_0)\mathbf{f}$.
- Total Problems Compliance: 225.
- Total Problems Volume: 135.

Benchmark set of topology optimization problems

Compliant mechanism design

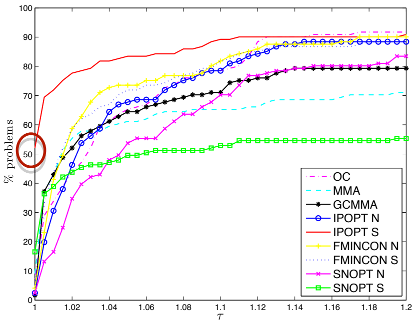
- Force inverter, Compliant gripper, Amplifier, Compliant lever, and Crimper domain examples, respectively.



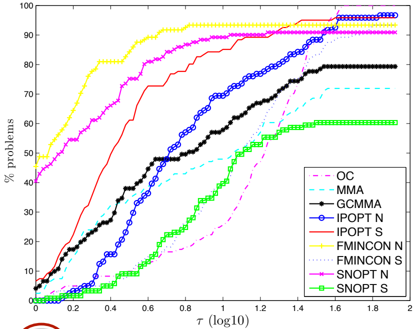
- Length ratio: 1×1 and 2×1.
- Volume constraint: $0.2 - 0.4$
- Discretization: $20, 40, 60, 80, 100$ elements per ratio.
- Total Problems Mechanism Design: 150.

Performance profiles for minimum compliance problems

Objective function value



Number of iterations

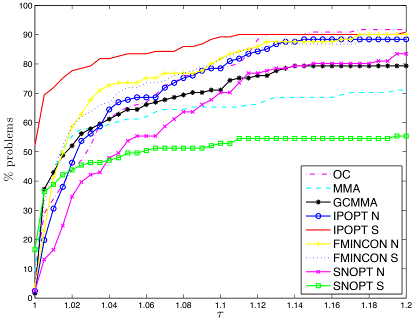


Performance profiles in a reduce test set of 121 instances.

Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for minimum compliance problems

Objective function value



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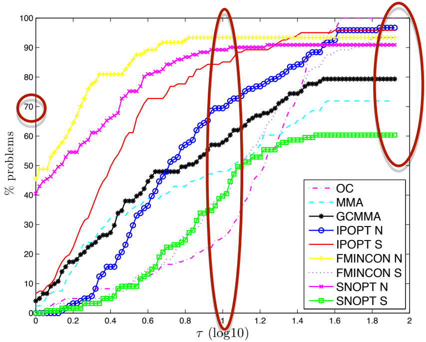

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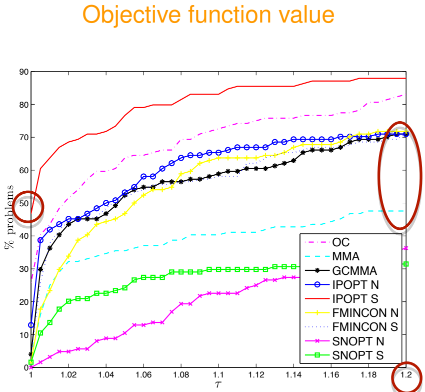
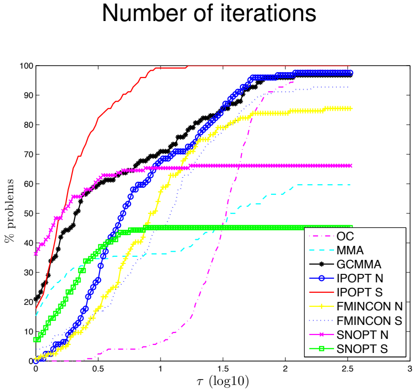
Performance profiles for compliant mechanism design problems



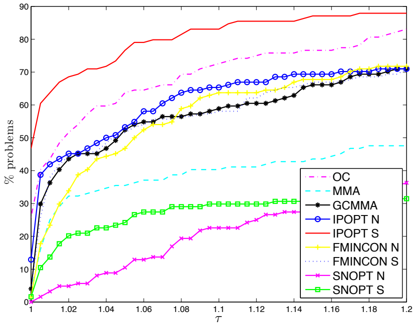


Performance profiles in a reduce test set of 124 instances.

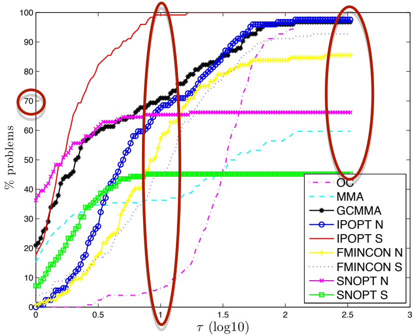
Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Performance profiles for compliant mechanism design problems

Objective function value



Number of iterations



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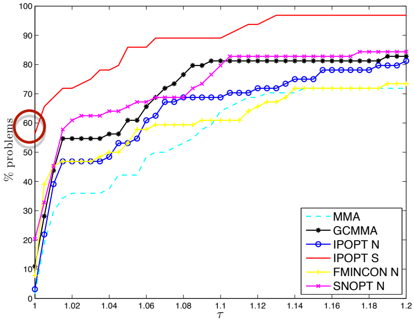
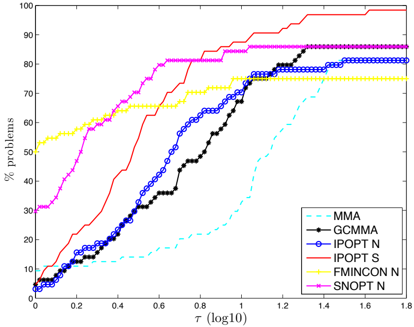
Penalization of problems with KKT error higher than $\omega = 1e-3$.

Performance profiles for minimum volume problems

Objective function value



Number of iterations

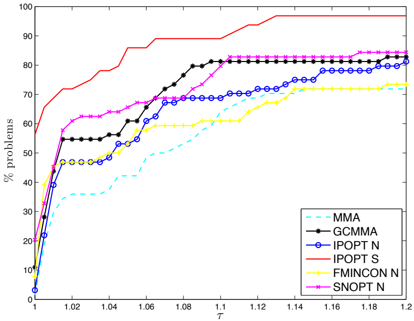


Performance profiles in a reduce test set of 64 instances.

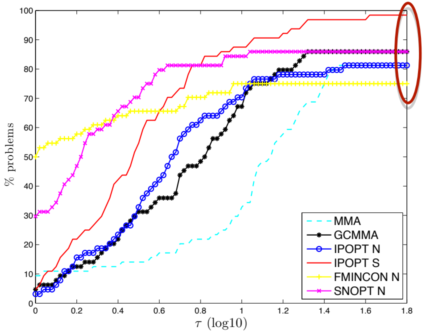
Penalization of problems with KKT error higher than $\omega = 1e-3$.

Performance profiles for minimum volume problems

Objective function value



Number of iterations



Performance profiles in a reduce test set of 64 instances.

Penalization of problems with KKT error higher than $\omega = 1e - 3$.

Conclusions

- **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

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- **What is missing?**
  - Large-scale problems, 3D domains, advance elements,...
  - Other regularization schemes.
  - Different formulations: Displacement constraint, stress constraints,...
  - More optimization solvers.
  - ...

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- **What can we conclude from the performance profiles?**
  - GCMMA outperforms MMA.
  - GCMMA and MMA tend to obtain a design with large KKT error.
  - IPOPT-S produces better designs using few number of iterations
  - IPOPT-S is the most robust solver in the study.
  - The SAND formulation requires lot of memory and computational time.

THANK YOU !!!

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