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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

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The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The model is recognized as a slow-fast system.

INTRODUCTION

The L- and H-modes are confinement states of a toroidal plasma, referring to states of low and high confinement, respectively. The transition from the L- to the H-mode is called the L-H transition. The T-mode is a transient, intermediate mode between the L- and H-mode, characterized by an oscillatory behavior. The L-H transition still lacks a first principle explanation. Modeling the L-H transition might contribute to a better understanding of the underlying mechanisms.

THE 3-ODE L-H TRANSITION MODEL

We consider the minimal 3-ODE predator-prey type L-H transition model suggested by Kim and Diamond [2, 3]. The model ignores spatial dependencies. The dependent variables in the model are:

- the drift wave turbulence level \( \varepsilon \),
- the shear of the zonal flow \( V_{a} \), and
- the gradient of the ion pressure \( N \).

The model can be formulated as

\[
\begin{align*}
\frac{d}{dt}E &= E(N - a_{1}E - a_{2}c_{1}N^{4} - a_{3}V_{a}^{2}), \\
\frac{d}{dt}V_{a} &= V_{a}(b_{1}E + a_{2}c_{2}N^{4} - b_{3}), \\
\frac{d}{dt}N &= Q(t) - N(c_{1}E + c_{2}),
\end{align*}
\]

where \( a_{1}, b_{1}, c_{1}, c_{2} \), \( i = 1, 2, 3 \) are parameters and \( Q \) is the heating power. Introducing new variables and time,

\[
\begin{align*}
\sigma &= a_{1}^{1/3}c_{1}^{2/3}E, \\
\frac{1}{a_{1}^{1/3}c_{1}^{2/3}} &= v = a_{1}^{1/3}c_{1}^{2/3}V_{a}^{2}, \\
\frac{1}{a_{1}^{1/3}c_{1}^{2/3}} &= w = a_{1}^{1/3}c_{1}^{2/3}N.
\end{align*}
\]

results in the non-dimensionalized system

\[
\begin{align*}
\frac{dw}{dt} &= \mu_{i}(w - w - w^{3}), \\
\frac{dv}{dt} &= \mu_{i}v(v - \frac{w}{1 + \mu_{i}w} - \mu_{i}), \\
\frac{d\sigma}{dt} &= \mu_{i}(\sigma - w(1 + \mu_{i}w))
\end{align*}
\]

Here, \( \mu_{i}, i = 1, \ldots, 5 \) are new parameters and \( \sigma \) is the rescaled heating power.

BIFURCATION ANALYSIS

The nullclines are

\[
\begin{align*}
N_{u} &= \{ w = 0 \} \cup \{ w = w(1 - w^{3}) - v \}, \\
N_{c} &= \{ w = 0 \} \cup \{ w = \mu_{2}(1 + \mu_{3}w^{4}) \}, \\
N_{w} &= \{ w(1 + \mu_{4}w) = 0 \}.
\end{align*}
\]

Stability of the equilibrium points:

- \( L \) is a stable node when it is below \( N_{c} \),
- \( H \) is always a saddle (unstable),
- \( T \) is a focus point. Stability depends on the value of \( \sigma \),
- \( QH \) is stable for \( \sigma > 1 \).

CONCLUSION

Kim and Diamond’s 3-ODE L-H transition model was investigated with bifurcation theory.

- The model contains three types of transitions: an oscillating, a sharp with hysteresis, and a smooth transition.
- The system can be reduced to a 2-ODE system

A spatio-temporal L-H transition model has been proposed by Miki et al. [4].

References: