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Publication date:
2014

Document Version
Peer reviewed version

Citation (APA):

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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

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The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The model is recognized as a slow-fast system.

INTRODUCTION

The L- and H-modes\textsuperscript{1} are confinement states of a toroidal plasma, referring to states of low and high confinement, respectively. The transition from the L- to the H-mode is called the L-H transition. The T-mode is a transient, intermediate mode between the L- and H-mode, characterized by an oscillatory behavior. The L-H transition still lacks a first principle explanation. Modeling the L-H transition might contribute to a better understanding of the underlying mechanisms.

THE 3-ODE L-H TRANSITION MODEL

We consider the minimal 3-ODE predator-prey type L-H transition model suggested by Kim and Diamond \textsuperscript{2, 3}. The model ignores spatial dependencies. The dependent variables in the model are:

- \( \mu \): the drift wave turbulence level \( \varepsilon \),
- \( v \): the shear of the zonal flow \( V_\parallel \), and
- \( w \): the gradient of the ion pressure \( \nabla N \).

The model can be formulated as

\[
\begin{align*}
\frac{d\mu}{dt} &= \mu \left( \frac{N}{V_\parallel} - N_{\text{H}} \right)
+ \mu \left( 1 + \frac{V_\parallel}{V_\parallel} \right) \nabla N
+ \mu \left( 1 + \frac{N}{V_\parallel} \right) \nabla V_\parallel,
\end{align*}
\]

where \( a, b, c, i = 1, 2, 3 \) are parameters and \( Q \) is the heating power.

Introducing new variables and time,

\[
\begin{align*}
\alpha &= a_1 c_1 c_2 \varepsilon, \\
\beta &= a_2 c_1 c_2 V_\parallel, \\
\gamma &= a_2 c_1 c_2 N,
\end{align*}
\]

results in the non-dimensionalized system

\[
\begin{align*}
\frac{d\mu}{dt} &= \mu \left( \frac{N}{V_\parallel} - N_{\text{H}} \right)
+ \mu \left( 1 + \frac{V_\parallel}{V_\parallel} \right) \nabla N
+ \mu \left( 1 + \frac{N}{V_\parallel} \right) \nabla V_\parallel,
\end{align*}
\]

Here, \( \mu_i, i = 1, \ldots, 5 \) are new parameters and \( \sigma \) is the rescaled heating power.

BIFURCATION ANALYSIS

The nullclines are

\[
\begin{align*}
N_1 &= \{ u = 0 \} \cup \{ u = w (1 - w^3) \}, \\
N_2 &= \{ v = 0 \} \cup \{ v = \mu_2 (1 + \mu w) \}, \\
N_3 &= \{ w = (1 + \mu w) \}.
\end{align*}
\]

Stability of the equilibrium points:

- \( L \) is a stable node when it is below \( N_r \).
- \( H \) is always a saddle (unstable).
- \( T \) is a focus point. Stability depends on the value of \( \sigma \).
- \( QH \) is stable for \( \sigma > 1 \).

THE 3 TRANSITION TYPES

The bifurcation diagram structure depends on \( \mu_i, i = 1, \ldots, 5 \). By varying \( \mu_2 \) and \( \mu_3 \), the three different transition types are observed.

DIMENSION REDUCTION WITH GSPT

Put \( \mu_2 = \frac{1}{\varepsilon} \), where \( 0 < \varepsilon \ll 1 \).

- \( u \) and \( v \) are slow variables,
- \( w \) is a fast variable.

For \( \varepsilon > 0 \), but sufficiently small, solutions converge to the slow manifold,

\[
\mathcal{M}_s = \mathcal{M}_0 + \varepsilon \mathcal{M}_1 + \varepsilon^2 \mathcal{M}_2 + \cdots
\]

The reduced system of the flow is found by taking the limit \( \varepsilon \to 0 \):

\[
\begin{align*}
\frac{du}{dt} &= u \left( w - u - v - w^4 \right), \\
\frac{dv}{dt} &= \mu v \left( \frac{u}{1 + \mu w} - \mu_2 \right), \\
\frac{dw}{dt} &= \mu_2 \left( \sigma - w (1 + \mu w) \right).
\end{align*}
\]

The reduced system contains the same dynamics as the full system.

CONCLUSION

Kim and Diamond’s 3-ODE L-H transition model was investigated with bifurcation theory.

- The model contains three types of transitions: an oscillating, a sharp with hysteresis, and a smooth transition.
- The system can be reduced to a 2-ODE system

A spatio-temporal L-H transition model has been proposed by Miki et al. \textsuperscript{4}.

References: