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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

Magnus Dam1, Morten Brøns1, Jens Juul Rasmussen2, Volker Naulin3, and Guosheng Xu4

1DTU Compute, Technical University of Denmark, Kgs. Lyngby, Denmark; 2Association Euratom-DTU, DTU Physics, Technical University of Denmark, Kgs. Lyngby, Denmark; 3Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, China. Email: magnusd@dtu.dk

The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The model is recognized as a slow-fast system.

INTRODUCTION

The L- and H-modes are confinement states of a toroidal plasma, referring to states of low and high confinement, respectively. The transition from the L- to the H-mode is called the L-H transition. The T-mode is a transient, intermediate mode between the L- and H-mode, characterized by an oscillatory behavior. The L-H transition still lacks a first principle explanation. Modeling the L-H transition might contribute to a better understanding of the underlying mechanisms.

THE 3-ODE L-H TRANSITION MODEL

We consider the minimal 3-ODE predator-prey type L-H transition model suggested by Kim and Diamond [2, 3]. The model ignores spatial dependencies. The dependent variables in the model are:

- the drift wave turbulence level \( \varepsilon \),
- the shear of the zonal flow \( V_z \), and
- the gradient of the ion pressure \( N \).

The model can be formulated as

\[
\frac{d}{dt} \varepsilon = \mathcal{E} (N - a_i \varepsilon - a_i \varepsilon^2 N^i - a_i V_z^2),
\]

\[
\frac{d}{dt} V_z = V_{zd} \left( \frac{b_i \varepsilon}{1 + b_i \varepsilon N^i} - b_i \right),
\]

\[
\frac{d}{dt} N = Q(t) - N (c_1 \varepsilon + c_2),
\]

where \( a_i, b_i, c_i, i = 1, 2, 3 \) are parameters and \( Q \) is the heating power. Introducing new variables and time,

\[
a = a_1 a_i^{1/2} c_i^{1/2} \varepsilon, \quad v = a_1 a_i^{1/2} c_i^{1/2} V_z^2,
\]

\[
w = a_1 a_i^{1/2} c_i^{1/2} N,
\]

results in the non-dimensionalized system

\[
w = u \left( u - u - v - w^2 \right),
\]

\[
\frac{d}{dt} u = \mu_1 v \left( \frac{u}{1 + \mu_1 u} - \mu_1 u \right),
\]

\[
\frac{d}{dt} v = \mu_2 (\sigma - w(1 + \mu_2 w)).
\]

Here, \( \mu_i, i = 1, \ldots, 5 \) are new parameters and \( \sigma \) is the rescaled heating power.

BIFURCATION ANALYSIS

The nullclines are

\[
N_u = \{ u = 0 \} \cup \{ u = w(1 - w^2) - v \},
\]

\[
N_v = \{ v = 0 \} \cup \{ u = \mu_2 (1 + \mu_2 w) \},
\]

\[
N_w = \{ w(1 + \mu_2 w) = 0 \}.
\]

Stability of the equilibrium points:

- \( L \) is a stable node when it is below \( N_u \).
- \( H \) is always a saddle (unstable).
- \( T \) is a focus point. Stability depends on the value of \( \sigma \).
- \( Q \) is stable for \( \sigma > 1 \).

CONCLUSION

Kim and Diamond’s 3-ODE L-H transition model was investigated with bifurcation theory.

- The model contains three types of transitions: an oscillating, a sharp with hysteresis, and a smooth transition.
- The system can be reduced to a 2-ODE system

A spatio-temporal L-H transition model has been proposed by Miki et al. [4].

REFERENCES: