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Publication date: 2014

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):

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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

Magnus Dam, Morten Brøns, Jens Juul Rasmussen, Volker Naulin, and Guosheng Xu

The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The model is recognized as a slow-fast system.

INTRODUCTION

The L- and H-modes are confinement states of a toroidal plasma, referring to states of low and high confinement, respectively. The transition from the L- to the H-mode is called the L-H transition. The T-mode is a transient, intermediate mode between the L- and H-mode, characterized by an oscillatory behavior. The L-H transition still lacks a first principle explanation. Modeling the L-H transition might contribute to a better understanding of the underlying mechanisms.

THE 3-ODE L-H TRANSITION MODEL

We consider the minimal 3-ODE predator-prey type L-H transition model suggested by Kim and Diamond [2, 3]. The model ignores spatial dependencies. The dependent variables in the model are:

- the drift wave turbulence level \( \varepsilon \),
- the shear of the zonal flow \( V_{z,z} \), and
- the gradient of the ion pressure \( \nabla N \).

The system can be formulated as

\[
\frac{dE}{dt} = E(N - a_2E - a_3E^2N^3 - a_4V_z^2),
\]

\[
\frac{dV_z}{dt} = V_z(\frac{b_1E}{1 + b_2E^2} - b_3),
\]

\[
\frac{dN}{dt} = Q(t) - N(c_1E + c_2),
\]

where \( a_i, b_i, c_i \), \( i = 1, 2, 3 \) are parameters and \( Q \) is the heating power. Introducing new variables and time:

\[
\alpha = \frac{1}{\sqrt{a_2}}, \quad \beta = \frac{1}{a_3}, \quad \gamma = \frac{1}{a_4},
\]

\[
\omega = \frac{1}{\sqrt{b_1}} \quad \tau = \frac{1}{c_1}, \quad \nu = \frac{1}{c_2},
\]

results in the non-dimensionalized system

\[
\frac{d\hat{u}}{dt} = \hat{u} \left( \hat{w} - \hat{u} - \hat{v} - \hat{w}^2 \right),
\]

\[
\frac{d\hat{v}}{dt} = \mu_v \left( \hat{u} - \mu_v \hat{w} - \mu_u \right),
\]

\[
\frac{d\hat{w}}{dt} = \mu_w (\hat{v} - \hat{w} - \hat{w}^2).\]

Here, \( \mu_i, i = 1, \ldots, 5 \) are new parameters and \( \sigma \) is the rescaled heating power.

BIFURCATION ANALYSIS

The nullclines are

\[
N_0 = \{ u = 0 \} \cup \{ u = w(1-w^2)-v \},
\]

\[
N_v = \{ v = 0 \} \cup \{ v = \mu_v(1+\mu_u)w \},
\]

\[
N_w = \{ w = 0 \} \cup \{ w = 1+\mu_u \}.
\]

Stability of the equilibrium points:

- \( L \) is a stable node when it is below \( N_v \).
- \( H \) is always a saddle (unstable).
- \( T \) is a focus point. Stability depends on the value of \( \sigma \).
- \( QH \) is stable for \( \sigma > 1 \).

THREE TRANSITION TYPES

The bifurcation diagram structure depends on \( \mu_i, i = 1, \ldots, 5 \). By varying \( \mu_2 \) and \( \mu_3 \), the three different transition types are observed.

DIMENSION REDUCTION WITH GSPR

Put \( \mu_2 = \frac{1}{\varepsilon} \), where \( 0 < \varepsilon \ll 1 \).

- \( u \) and \( v \) are slow variables,
- \( w \) is a fast variable.

For \( \varepsilon > 0 \), but sufficiently small, solutions converge to the slow manifold,

\[
\mathcal{M} = \mathcal{M}_0 + \varepsilon \mathcal{M}_1 + \varepsilon^2 \mathcal{M}_2 + \ldots
\]

The reduced system of the flow is found by taking the limit \( \varepsilon \to 0 \):

\[
\frac{du}{dt} = u \left( w - u - v - w^2 \right),
\]

\[
\frac{dv}{dt} = \mu_v \left( \frac{u}{\sigma} - \mu_v w - \mu_u \right),
\]

\[
\frac{dw}{dt} = \mu_w \left( \frac{v}{\sigma} - 1 - \mu_u \right).
\]

The reduced system contains the same dynamics as the full system.

CONCLUSION

Kim and Diamond’s 3-ODE L-H transition model was investigated with bifurcation theory.

- The model contains three types of transitions: an oscillating, a sharp with hysteresis, and a smooth transition.
- The system can be reduced to a 2-ODE system

A spatio-temporal L-H transition model has been proposed by Miki et al. [4].