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Mirzaei, Mahmood; Poulsen, Niels Kjølstad; Niemann, Hans Henrik

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Robust Model Predictive Control of a Wind Turbine

Mahmood Mirzaei, Niels Kjølstad Poulsen and Hans Henrik Niemann

***Abstract*— In this work the problem of robust model predictive control (robust MPC) of a wind turbine in the full load region is considered. A minimax robust MPC approach is used to tackle the problem. Nonlinear dynamics of the wind turbine are derived by combining blade element momentum (BEM) theory and first principle modeling of the turbine flexible structure. Thereafter the nonlinear model is linearized using Taylor series expansion around system operating points. Operating points are determined by effective wind speed and an extended Kalman filter (EKF) is employed to estimate this. In addition, a new sensor is introduced in the EKF to give faster estimations. Wind speed estimation error is used to assess uncertainties in the linearized model. Significant uncertainties are considered to be in the gain of the system (B matrix of the state space model). Therefore this special structure of the uncertain system is employed and a norm-bounded uncertainty model is used to formulate a minimax model predictive control. The resulting optimization problem is simplified by semidefinite relaxation and the controller obtained is applied on a full complexity, high fidelity wind turbine model. Finally simulation results are presented. First a comparison between PI and robust MPC is given. Afterwards simulations are done for a realization of turbulent wind with uniform profile based on the IEC standard.**

I. INTRODUCTION

A. Wind turbine control

In recent decades there has been an increasing interest in green energies, of which wind energy is one of the most important. Wind turbines are the most common wind energy conversion systems (WECS) and are hoped to be able to compete economically with fossil fuel power plants in near future. However this demands better technology to reduce the price of electricity production. Control can play an essential part in this context because, on the one hand, control methods can decrease the cost of energy by keeping the turbine close to its maximum efficiency. On the other hand, they can reduce structural fatigue and therefore increase the lifetime of the wind turbine. There are several methods for wind turbine control ranging from classical control methods [1] which are the most used methods in real applications, to advanced control methods which have been the focus of research in the past few years [2]; gain scheduling [3], adaptive control [4], MIMO methods [5], nonlinear control [6], robust control [7], model predictive control [8], μ-Synthesis design [9] are just a few. Advanced model based control methods are thought to be the future of wind turbine control as they can conveniently employ new generations of sensors on wind turbines (e.g. LIDAR [10]), new generation of actuators (e.g. trailing edge flaps [11]) and also treat the turbine as a MIMO system. The last feature seems to be becoming more important than before, as wind turbines are becoming bigger and more flexible. This trend makes decoupling different modes, specifying different objectives and designing controllers based on paired input/output channels more difficult. Model predictive control (MPC) has proved to be an effective tool to deal with multivariable constrained control problems [12]. As wind turbines are MIMO systems [5] with constraints on inputs and outputs, using MPC seems to be effective.

Nominal MPC proved to give satisfactory results for offshore wind turbine control [13] and trailing edge flap control [14]. However these works have not taken into account uncertainty in the design model and this problem has been bypassed by trial-error and extensive simulations to get the best performance from the controllers. Based on this argument extending nominal MPC of wind turbines to robust MPC and including model uncertainties in the design seems to be natural.

The wind turbine in this paper is treated as a MIMO system with pitch (θ_{in}) and generator reaction torque (Q_{in}) as inputs and rotor rotational speed (ω_r), generator rotational speed (ω_g) and generated power (P_e) as outputs. This paper is organized as follows: In section II modeling of the wind turbine including modeling for wind speed estimation, linearization and uncertainty modeling are addressed. In section III robust MPC design is explained. Finally in section IV simulation results are presented.

II. MODELING

A. Wind model

Wind can be modeled as a complicated nonlinear stochastic process. However for practical control purposes it could be approximated by a linear model [15]. In this model the wind has two elements, mean value term (v_m) and turbulent term (v_t): $v_e = v_m + v_t$. The turbulent term could be modeled by the following transfer function:

$$v_t = \frac{k}{(p_1 s + 1)(p_2 s + 1)} e; \quad e \in N(0,1)$$

And in the state space form:

$$\begin{pmatrix} \dot{v}_t \\ \ddot{v}_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{p_1 p_2} & -\frac{p_1+p_2}{p_1 p_2} \end{pmatrix} \begin{pmatrix} v_t \\ \dot{v}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k}{p_1 p_2} \end{pmatrix} e \qquad (1)$$

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M. Mirzaei is with the Department of Informatics and Mathematical Modeling, Technical University of Denmark, 2800 Kongens Lyngby, Denmark mmir@imm.dtu.dk

N. K. Poulsen is with the Department of Informatics and Mathematical Modeling, Technical University of Denmark, 2800 Kongens Lyngby, Denmark nkp@imm.dtu.dk

H. H. Niemann is with the Department of Electrical Engineering, Technical University of Denmark, 2800 Kongens Lyngby, Denmark hhn@elektro.dtu.dk

The parameters p_1, p_2 and k are found by second order approximation of the wind power spectrum [15] and they depend on the mean wind speed v_m. For wind speed estimation, a one degree of freedom (DOF) nonlinear model of the wind turbine is augmented with the wind model given above. An extended Kalman filter uses this model to estimate the effective wind speed. This wind speed is used to find the operating point of the wind turbine and consequently calculate appropriate control signals.

B. Nonlinear model

For modeling purposes, the whole wind turbine can be divided into 4 subsystems: aerodynamics subsystem, mechanical subsystem, electrical subsystem and actuator subsystem. The aerodynamic subsystem converts wind forces into mechanical torque and thrust on the rotor. The mechanical subsystem consists of the drivetrain, tower and blades. The drivetrain transfers rotor torque to the electrical generator. The ower holds the nacelle and withstands the thrust force and the aerodynamically shaped blades transform wind speed into torque and thrust. The generator subsystem converts mechanical energy to electrical energy and finally the blade-pitch and generator-torque actuator subsystems are part of the control system. To model the whole wind turbine, models of these subsystems are obtained and at the end they are connected together. A wind model is obtained and augmented with the wind turbine model to be used for wind speed estimation. The dominant dynamics of the wind turbine come from its flexible structure. Several degrees of freedom could be considered to model the flexible structure, but for control design just a few important degrees of freedom are usually considered.

In this work we only consider two degrees of freedom, namely the rotational DOF and the drivetrain torsion.

Nonlinearity of the wind turbine mostly comes from its aerodynamics. Blade element momentum (BEM) theory [16] is used to calculate aerodynamic torque and thrust on the wind turbine. This theory explains how torque and thrust are related to wind speed, blade pitch angle and rotational speed of the rotor. In steady state, i.e. disregarding dynamic inflow, the following formulas can be used to calculate aerodynamic torque and thrust.

$$Q_r = \frac{1}{2}\frac{1}{\omega_r}\rho\pi R^2 v_e^3 C_p(\theta, \omega, v_e) \tag{2}$$

$$Q_t = \frac{1}{2}\rho\pi R^2 v_e^2 C_t(\theta, \omega, v_e) \tag{3}$$

In which Q_r and Q_t are aerodynamic torque and thrust, ρ is the air density, ω_r is the rotor rotational speed, v_e is the effective wind speed, C_p is the power coefficient and C_t is the thrust force coefficient.

The absolute angular position of the rotor and generator are of no interest to us, therefore we use $\psi = \theta_r - \theta_g$ instead which is the drivetrain torsion. Having aerodynamic torque and modeling the drivetrain with a simple mass-spring-damper, the whole system equation with two DOFs becomes:

$$J_r\dot{\omega}_r = Q_r - c(\omega_r - \frac{\omega_g}{N_g}) - k\psi \tag{4}$$

$$(N_g J_g)\dot{\omega}_g = c(\omega_r - \frac{\omega_g}{N_g}) + k\psi - N_g Q_g \tag{5}$$

$$\dot{\psi} = \omega_r - \frac{\omega_g}{N_g} \tag{6}$$

$$P_e = Q_g \omega_g \tag{7}$$

In which J_r and J_g are rotor and generator moments of inertia, ψ is the drivetrain torsion, c and k are the drivetrain damping and stiffness factors, respectively lumped in the low speed side of the shaft, and P_e is the electrical power generated. For numerical values of these parameters and other parameters given in this paper, refer to [17].

C. Uncertain Linear Model

1) Linearized model: As mentioned in the previous section, wind turbines are nonlinear systems. A basic approach to design controllers for nonlinear systems is to linearize them around some operating points. For a wind turbine, the operating points on the quasi-steady C_p and C_t curves are nonlinear functions of rotational speed ω_r, blade pitch θ and wind speed v. To get a linear model of the system we need to linearize around these operating points. Rotational speed and blade pitch are measurable with enough accuracy, however this is not the case for the effect of wind on the rotor. Wind speed changes along the blades and with the azimuth angle (angular position) of the rotor. This is because of wind shear and tower shadow as well as the stochastic spatial distribution of the wind field. Therefore a single wind speed does not exist which can be used and measured for finding the operating point. We bypass this problem by defining a fictitious variable called effective wind speed (v_e), which shows the effect of wind in the rotor disc on the wind turbine.

In our two DOFs model only the aerodynamic torque (Q_r) and electric power (P_e) are nonlinear and Taylor expansion is used to linearize them. For the sake of simplicity in notations we will use Q_r, P_e, θ, ω and v_e instead of ΔQ_r, ΔP_e, $\Delta\theta$, $\Delta\omega$ and Δv_e around the operating points from now on. Using the linearized aerodynamic torque, the two DOFs linearized model becomes:

$$\dot{\omega}_r = \frac{a-c}{J_r}\omega_r + \frac{c}{J_r}\omega_g - \frac{k}{J_r}\psi + \frac{b_1}{J_r}\theta + \frac{b_2}{J_r}v_e \tag{8}$$

$$\dot{\omega}_g = \frac{c}{N_g J_g}\omega_r - \frac{c}{N_g^2 J_g}\omega_g + \frac{k}{N_g J_g}\psi - \frac{Q_g}{J_g} \tag{9}$$

$$\dot{\psi} = \omega_r - \frac{\omega_g}{N_g} \tag{10}$$

$$P_e = Q_{g_0}\omega_g + \omega_{g_0} Q_g \tag{11}$$

2) The uncertain model: As mentioned previously, effective wind speed is not measurable and we need to use an estimation of it instead. An extended Kalman filter (EKF) is used to estimate the wind speed, for more details see section IV-A. The estimated v_e has uncertainties and as we use this estimation to linearize the aerodynamics of the wind turbine, we end up with an uncertain linear model. The uncertainty

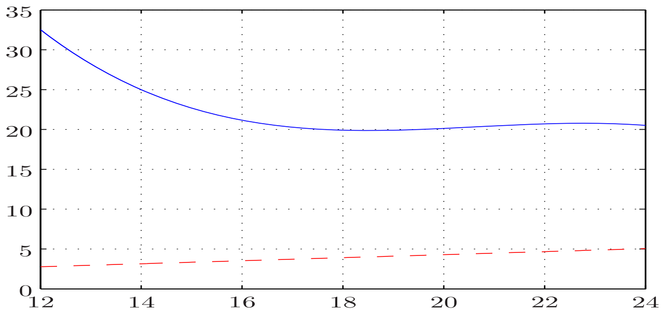

Fig. 1: Relative uncertainties of the parameters (percent), α solid-blue and β_1 red-dashed

is only in the equation (8). The uncertain linear model could be written as:

$$\dot{\omega}_r = \alpha(\delta_1)\omega_r + \frac{c}{J_r}\omega_g - \frac{k}{J_r}\psi + \beta_1(\delta_2)\theta + \beta_2(\delta_3)v_e$$

In which $\alpha(\delta_1), \beta_1(\delta_2)$ and $\beta_2(\delta_3)$ could be written as:

$$\alpha(\delta_1) = \bar{\alpha}(1 + p_1\delta_1) \qquad |\delta_1| \leq 1 \tag{12}$$
$$\beta_1(\delta_2) = \bar{\beta}_1(1 + p_2\delta_2) \qquad |\delta_2| \leq 1 \tag{13}$$
$$\beta_2(\delta_3) = \bar{\beta}_2(1 + p_3\delta_3) \qquad |\delta_3| \leq 1 \tag{14}$$

$\bar{\alpha}, \bar{\beta}_1$ and $\bar{\beta}_2$ are nominal values and p's show relative uncertainties. To get numerical values for relative uncertainty variables (p_1, p_2 and p_3) we have assumed $1m/s$ wind speed estimation error. Figure 1 shows a mapping from this estimation error to errors in the parameters of the linearized model (α and β_1) for different wind speeds. It can be seen in figures 1 that wind speed estimation error gives less than 5% error in α, however this value is more than 20% for β_1. Using this argument and in order to simplify the optimization problem we neglect uncertainty in the dynamics of the system (which is determined by α) and consider the uncertainty only to be in the gain of the system. Collecting all the discussed models, matrices of the state space model become:

$$A = \begin{pmatrix} \frac{a-c}{J_r} & \frac{c}{J_r} & -\frac{k}{J_r} \\ \frac{c}{N_g J_g} & -\frac{c}{N_g^2 J_g} & \frac{k}{N_g J_g} \\ 1 & -1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} \frac{b_1(\delta_2)}{J_r} & 0 \\ 0 & -\frac{1}{J_g} \\ 0 & 0 \end{pmatrix} \tag{15}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & Q_{g_0} & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \omega_{g_0} \end{pmatrix} \tag{16}$$

In which $x = \begin{pmatrix}\omega_r & \omega_g & \psi\end{pmatrix}^T$, $u = \begin{pmatrix}\theta & Q_g\end{pmatrix}^T$ and $y = \begin{pmatrix}\omega_r & \omega_g & P_e\end{pmatrix}^T$ are states, inputs and outputs respectively. In the matrix B, parameter b_1 is uncertain.

III. Control

A. Control objectives

The most basic control objective of a wind turbine is to maximize captured power during the life time of the machine. This means trying to maximize captured power when wind speed is below its rated value which is called maximum power point tracking (MPPT). Rated wind speed is a value where the turbine starts to operate at its rated speed and power. When the wind speed is above rated, the control objectives become regulation of the outputs around their rated values while trying to minimize dynamic loads on the structure. These objectives should be achieved against fluctuations in wind speed which acts as a disturbance to the system. In this work we have considered operation of the wind turbine in the above rated wind speed (full load region). Therefore we try to regulate rotational speed and generated power around their rated values and remove the effect of wind speed fluctuations.

B. Minimax MPC formulation

MPC uses a model of the system (to be controlled) to predict its future behavior. In nominal MPC the prediction of the output ($\hat{y}_{k+N|k}$) is a single value and it is calculated based on one model. However in robust MPC because the model is uncertain, this prediction is no longer a unique value but it is a set instead. An approach to tackle the problem with an uncertain model is to try to consider the most pessimistic situation with respect to uncertainties. This means maximizing the cost function on the uncertainty set. After maximization, we minimize the obtained cost function over control inputs as we do in nominal MPC. This approach is called minimax MPC and it is a common solution to robust MPC problems [18]. As explained in II-C.2 the model obtained from our system only has uncertainties in the B matrix. The special structure of our problem can help us in simplification of the minimax MPC problem. Therefore we formulate robust MPC of the wind turbine in the form of minimax MPC of a system with uncertain gain [18]:

$$x_{k+1} = Ax_k + B(\Delta_k)u_k \tag{17}$$
$$y_k = Cx_k + Du_k \tag{18}$$

Polytopic uncertainty and additive disturbances are common ways to include uncertainties in robust MPC formulation [12]. However here we have employed norm-bounded uncertainty to model our system [19]:

$$B(\Delta_k) = B_0 + B_p\Delta_k C_p, \quad \Delta_k \in \mathbf{\Delta} \tag{19}$$
$$\mathbf{\Delta} = \{\Delta : \|\Delta\| \leq 1\} \tag{20}$$

With norm-bounded uncertain model of the system, we formulate the minimax MPC with quadratic performance and soft constraints. In order to simplify notations, we use stacked variables from now on and we define the following matrices:

$$\Phi_x = \begin{pmatrix} CA & CA^2 & \dots & CA^{N-1}\end{pmatrix}^T \tag{21}$$

$$\Gamma = \begin{pmatrix} CB(\Delta_1) & D & \dots & 0 \\ CAB(\Delta_2) & CB(\Delta_1) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B(\Delta_N) & CA^{N-2}B(\Delta_{N-1}) & \dots & D \end{pmatrix} \tag{22}$$

By using these matrices, the predicted output vector could be written as:

$$Y = \Phi_x \hat{x}_{k|k-1} + \Gamma U \tag{23}$$

$$\Gamma = \Gamma_0 + \Gamma_\Delta(\Delta^N) \tag{24}$$

$$\Delta^N = \begin{pmatrix} \Delta_1 & \Delta_2 & \dots & \Delta_N \end{pmatrix}^T \tag{25}$$

And the minimax optimization problem becomes:

$$\min_U \max_{\Delta^N} \quad Y^T Q Y + U^T R U + \Upsilon^T S_1 \Upsilon + \Xi^T S_2 \Xi \tag{26}$$

$$\text{subject to} \quad U \leq U_{max} + \Upsilon \tag{27}$$

$$U \geq U_{min} - \Upsilon \tag{28}$$

$$\Delta U \leq \Delta U_{max} + \Xi \tag{29}$$

$$\Delta U \geq \Delta U_{min} - \Xi \tag{30}$$

$$\Upsilon \geq 0 \tag{31}$$

$$\Xi \geq 0 \tag{32}$$

We use $\Delta U = \Psi U - I_0 u_{k-1}$ to rewrite constraints on ΔU in the form of constraints on U in which:

$$\Psi = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \qquad I_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{33}$$

Now we use semidefinite relaxation and apply the Schur complement to the optimization problem to get the following optimization problem with LMI constraints:

$$\min_{t,U} \quad t \tag{34}$$

$$\text{s.t.} \begin{pmatrix} t & Y^T & U^T & \Upsilon^T & \Xi^T \\ \star & Q^{-1} & 0 & 0 & 0 \\ \star & \star & R^{-1} & 0 & 0 \\ \star & \star & \star & S_1^{-1} & 0 \\ \star & \star & \star & \star & S_2^{-1} \end{pmatrix} \succeq 0 \tag{35}$$

$$\begin{pmatrix} I \\ -I \\ \Psi \\ -\Psi \end{pmatrix} U - \begin{pmatrix} U_{max} + \Upsilon \\ -U_{min} + \Upsilon \\ \Delta U_{max} + I_0 u_{k-1} + \Xi \\ -\Delta U_{min} - I_0 u_{k-1} + \Xi \end{pmatrix} \leq 0 \tag{36}$$

$$\Upsilon \geq 0 \qquad \Xi \geq 0 \tag{37}$$

However in the above formulation Y is of the form:

$$Y = \Phi_x \hat{x}_{k|k-1} + \Gamma_0 U + \sum_{j=1}^{N} V_j \Delta_j W_j U, \ \Delta_j \in \boldsymbol{\Delta}$$

$$V_1 = \begin{pmatrix} B_p & AB_p & \dots & A^{N-1}B_p \end{pmatrix}^T$$

$$V_2 = \begin{pmatrix} 0 & B_p & AB_p & \dots & A^{N-2}B_p \end{pmatrix}^T$$

$$\vdots$$

$$V_N = \begin{pmatrix} 0 & 0 & 0 & \dots & B_p \end{pmatrix}^T$$

$$W_1 = \begin{pmatrix} C_p & 0 & \dots & 0 \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 0 & C_p & \dots & 0 \end{pmatrix}$$

$$\vdots$$

$$W_N = \begin{pmatrix} 0 & 0 & \dots & C_p \end{pmatrix}$$

and it contains uncertain elements. Based on results from [18], we use the following theorem to eliminate uncertainties [20].

Theorem 1: robust satisfaction of the uncertain LMI

$$F + L\Delta(I - D\Delta)^{-1}R + R^T(I - \Delta^T D^T)^{-1}\Delta^T L^T \succeq 0$$

is equivalent to the LMI

$$\begin{bmatrix} F & L \\ L^T & 0 \end{bmatrix} \succeq \begin{bmatrix} R & D \\ 0 & I \end{bmatrix}^T \begin{bmatrix} \tau I & 0 \\ 0 & -\tau I \end{bmatrix} \begin{bmatrix} R & D \\ 0 & I \end{bmatrix}$$

$$\tau \geq 0$$

Now we pull out the first uncertain element (Δ_1) from Y in the LMI constraint. To do so we define the following variable:

$$\gamma_i = \Phi_x \hat{x}_{k|k-1} + \Gamma_0 U + \sum_{j=i}^{N} V_j \Delta_j W_j U, \quad i = 1, \dots, N \tag{38}$$

Using theorem 1, and collecting the matrices on the left hand side we get the following LMI:

$$\begin{pmatrix} t & \gamma_2^T & U^T & U^T W_1^T & \Upsilon^T & \Xi^T \\ \star & Q^{-1} - \tau_1 V_1 V_1^T & 0 & 0 & 0 & 0 \\ \star & \star & R^{-1} & 0 & 0 & 0 \\ \star & \star & \star & \tau_1 I & 0 & 0 \\ \star & \star & \star & \star & S_1^{-1} & 0 \\ \star & \star & \star & \star & \star & S_2^{-1} \end{pmatrix} \succeq 0 \tag{39}$$

We pulled out Δ_1, and now we repeat the same procedure until we pull out all the uncertainties Δ_i for $i = 2, \dots, N$. Afterwards we apply the Schur complement to write the final LMI in the form of smaller LMIs. Finally the optimization problem can be written in the following form:

$$\min_{t,\tau,U} \quad t_x + t_u + t_\upsilon + \sum_{j=0}^{N-1} t_j \tag{40}$$

$$\text{subject to} \quad \begin{pmatrix} t_x & \hat{x}_{k|k-1}^T \Phi_x^T + U^T \Gamma_0^T \\ \star & Q^{-1} - \sum_{j=0}^{N-1} \tau_j V_j V_j^T \end{pmatrix} \succeq 0 \tag{41}$$

$$\begin{pmatrix} t_u & U^T \\ \star & R^{-1} \end{pmatrix} \succeq 0 \ \begin{pmatrix} t_\upsilon & \Upsilon^T \\ \star & S^{-1} \end{pmatrix} \succeq 0 \ \begin{pmatrix} t_j & U^T W_j^T \\ \star & \tau_j I \end{pmatrix} \succeq 0 \tag{42}$$

$$\begin{pmatrix} I \\ -I \\ \Psi \\ -\Psi \end{pmatrix} U - \begin{pmatrix} U_{max} + \Upsilon \\ -U_{min} + \Upsilon \\ \Delta U_{max} + I_0 u_{k-1} + \Xi \\ -\Delta U_{min} - I_0 u_{k-1} + \Xi \end{pmatrix} \leq 0 \tag{43}$$

$$\tau_j \geq 0 \qquad \Upsilon \geq 0 \qquad \Xi \geq 0 \tag{44}$$

We have used SeDuMi [21] to solve this optimization problem. SeDuMi is a program that solves optimization problems with linear, quadratic and semidefinite constraints.

C. Offset free reference tracking and constraint handling

Persistent disturbances and modeling error can cause an offset between measured outputs and desired outputs. To avoid this problem, we have employed an offset free reference tracking approach (see [22] and [23]). Our RMPC solves the regulation problem around the operating point.

However we regulate around the operating points extracted from wind speed estimation which might be erroneous and results in offset from desired outputs. Besides, the difference between linear model and nonlinear model accounts for some of the differences between the measured outputs and the desired outputs as well. To avoid this problem, in our control algorithm we shift origin in our regulation problem to new operating points which ensures offset free reference tracking. It is clear that they should be included in the constraints of the robust MPC formulation.

IV. SIMULATIONS

In this section firstly wind speed estimation is explained. Afterwards simulation results for the obtained controllers are presented. The controllers are implemented in MATLAB and are tested on a full complexity FAST [24] model of the reference wind turbine [17]. Simulations are done with realistic turbulent wind speed using the Kaimal turbulence model [25]. TurbSim [26] is used to generate a time marching hub-height wind profile. In order to stay in the full load region, a realization of turbulent wind speed is used from category C of the turbulence categories of the IEC 61400-1 [25], with $18m/s$ as the mean wind speed.

A. Wind speed estimation

Wind speed estimation is essential in our control algorithm and in order to get a faster estimator we have introduced a sensor that measures rotor acceleration. This could be done using rotor speed and generator speed measurements [27]. A one DOF model of the wind turbine, including only rotor rotational degree of freedom is used for wind speed estimation. The first order nonlinear equations used in the extended Kalman filter are:

$$\dot{\omega} = \frac{1}{J_r} Q_r(\omega, \theta, v_e) - \frac{1}{J_r} Q_g \tag{45}$$

$$y = \begin{pmatrix} \omega & P_e & Q_r - Q_g \end{pmatrix}^T \tag{46}$$

Using the nonlinear equations above and wind model (1) an extended Kalman filter is designed to estimate the effective wind speed. Figure 2 shows wind speed and its estimation.

B. Stochastic simulations

In this section simulation results for a stochastic wind speed are presented. Control inputs, which are pitch reference θ_{in} and generator reaction torque reference Q_{in} along with system outputs, which are rotor rotational speed ω_r and electrical power P_e, are plotted in figures 3-6. The estimated wind speed is inaccurate and the controller is designed such that it can handle the uncertainties which arise from this inaccuracy. Simulation results show good regulations of generated power and rotational speed. Table I shows a comparison of the results between RMPC and a standard PI controller. The PI controller configuration and parameter values are taken from [17]. As could be seen from the table, the RMPC controller gives better regulation on rotational speed and generated power (smaller standard deviations) than the PI controller, while keeping the shaft moment less.

Parameters	RMPC	PI
ω_r standard deviation (RMP)	0.389	0.728
P_e standard deviation (Watts)	6.598×10^4	9.050×10^4
P_e mean value(Watts)	4.997×10^6	4.999×10^6
Pitch standard deviation (degrees)	10.261	8.623
Shaft moment standard deviation (N.M.)	0.840×10^3	2.376×10^3

TABLE I: RMPC and PI performance comparison

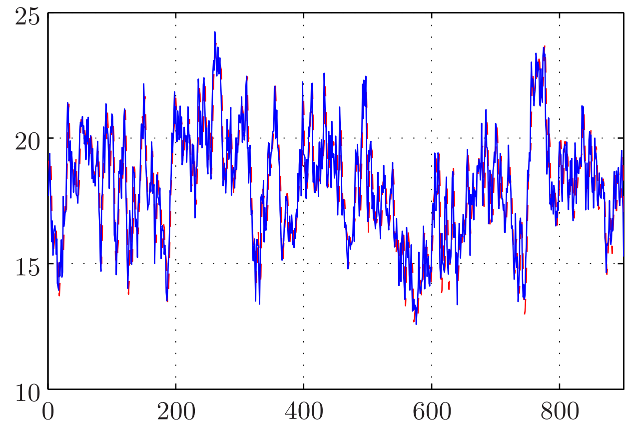

Fig. 2: Wind speed (blue-solid), Estimated wind speed (red-dashed) (m/s)

However when it comes to pitch activity (here we have used pitch standard deviation), it has more pitch activity.

V. CONCLUSIONS

In this paper we found a second order nonlinear model of a wind turbine, using blade element momentum theory (BEM) and first principle modeling of the drivetrain. Our control methodology is based on linear models, therefore we have used Taylor series expansion to linearize the obtained nonlinear model around system operating point. The operating point is a direct function of rotor rotational speed, pitch angle and wind speed. Wind speed estimation is used to find the operating point and we showed that this will result in an uncertain B matrix in our linear model. Special minimax model predictive control formulation was derived to take into account the assumed uncertainties. The final controller was

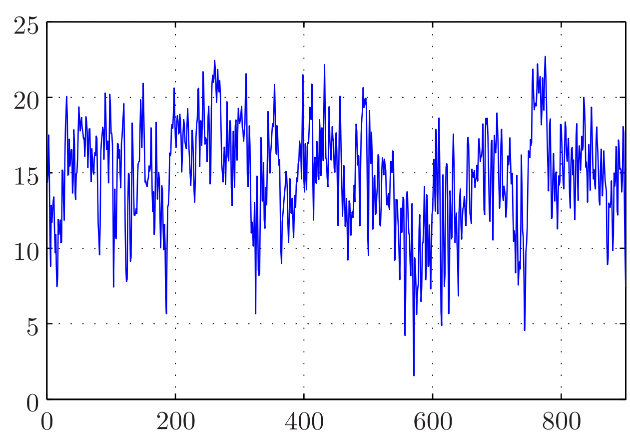

Fig. 3: Blade-pitch reference (degrees)

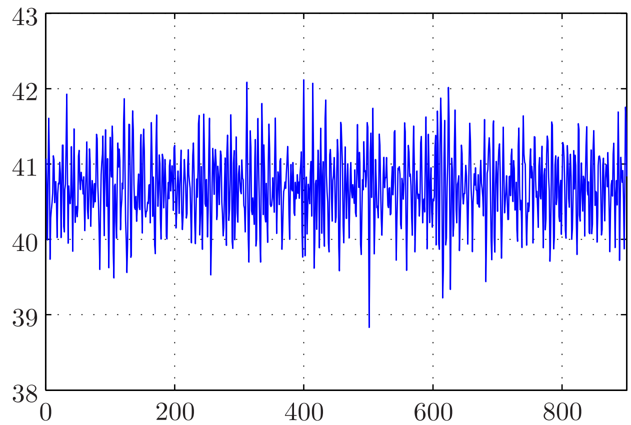

Fig. 4: Generator-torque reference (kilo N.M.)

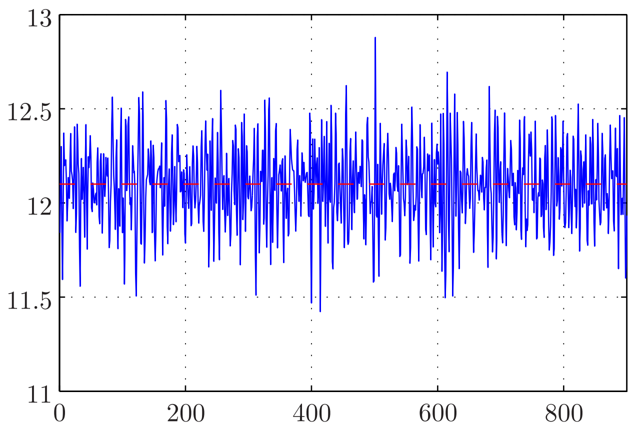

Fig. 5: Rotor rotational speed (ω_r) (rpm)

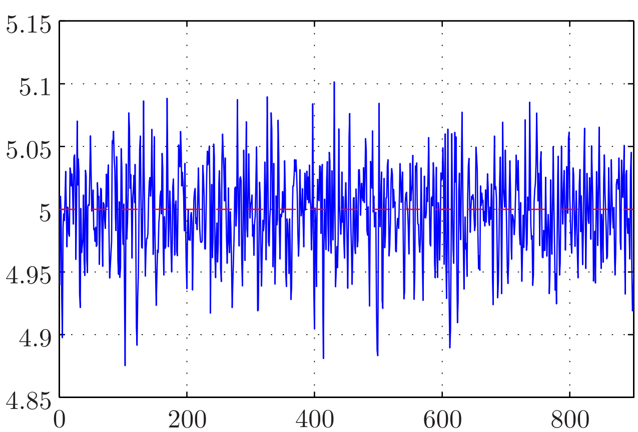

Fig. 6: Electrical power (mega watts)

applied on a full complexity FAST [24] model and compared with a standard PI controller.

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