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An overview of data for wake model evaluation in the Virtual Wakes Laboratory

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Abstract

Improved prognostic models of wind turbine wakes are essential to improved design of offshore wind farms to maximized power production and minimize turbine fatigue loading. This paper describes the Virtual Wake Laboratory, an online open access resource that provides meteorological and wind farm data for use in wake characterization and wake model evaluation exercises. Specific examples of the types of analyses that can be conducted using these data are given, along with recommendations regarding appropriate metrics to be applied and considerations for wake model evaluation.

1 Introduction

Development of offshore wind farms containing dozens to hundreds of wind turbines, means that very few turbines experience free-stream or ambient conditions and the majority have to operate in wakes from upstream turbines and thus experience complex wind and turbulence climates,
leading to reduced power production and increased fatigue loading. However, it is difficult to fully optimize turbine layouts based on power output and load minimization because there are significant uncertainties in the physical understanding of wakes and thus wake characterization used in wind farm models.

Improved prognostic models of wind turbine wakes are thus an essential component of the drive towards optimizing multi-megawatt wind farm layouts to both maximize power production [1] and minimize turbine loading [2, 3]. Total power output from existing large wind farms is suppressed by 5-20% due to wake losses depending on turbine spacing/placement and site climatology [4]. There is a demonstrable benefit, therefore, to be gained by improving pre-construction wake prediction and wind farm design optimization within the constraints arising from space and infrastructure (e.g. cable layouts and access roads). Although recent increases in wind farm size [5] have driven a need to understand multiple wake interactions, modeling individual wakes correctly is still a challenging issue, especially in complex terrain [6], and remains a key step towards improved understanding of wake behavior as discussed herein.

A number of field studies have been conducted to provide observational data necessary to develop improved physical understanding of the parameters that control the evolution and interactions of single or multiple wakes within small wind farms. The earliest studies used measurements from one or more meteorological masts placed in a position that will experience a direct wake from a nearby wind turbine e.g. [7, 8]. In those early studies measurements were typically taken with standard cup anemometers and wind vanes deployed at more than one height. Some of the most well-known studies of this type include those from the Nibe turbine [9, 10] and Vindeby wind farm [11, 12]. In addition to meteorological wind speed/turbulence measurements, some studies included higher time resolution measurements with sonic
anemometers such as those at Nørrkær Enge [13] and for larger turbines at the ECN test farm [14]. Further studies included instrumentation of turbines with strain gauges to measure forces on the blades and/or tower resulting from wakes e.g. [10, 15]. More recent measurement campaigns have deployed sodar placed at different distances from the turbine both on-land [8] and offshore [16]. Understanding of the integrated effects of total wind farm wakes resulting from larger wind farms has used both meteorological masts deployed at large downwind distances [17] and remote sensing instrumentation deployed on satellite platforms [18, 19], but probably the most significant advance in terms of the evolution of measurement technologies is application of lidar for both single turbine wake [20] and wind farm characterization [21]. The need to develop observations of whole wind farm effects on downstream flow has prompted use of a range of innovative remote sensing measurement approaches including satellite borne radiometers [22], radars [23] and lidar [24].

During the 1980’s a number of wake models were developed and evaluated, mostly for small numbers of turbines [25], and most current generation wind farm wake models still leverage that work (see the review by Crespo et al. [26]). An array of models are currently available, including those that start by treating each wake with relatively simple analytical approaches (e.g. WAsP [27]) where the initial wake velocity deficit is calculated as a function of the turbine thrust coefficient, and wake expansion at downwind distances is described using a coefficient which is a function of the roughness/turbulence regime. More complex models such as the eddy viscosity wake model (e.g. [28]) are based on numerical solutions to Navier-stokes equations applied with assumptions regarding an axisymmetric velocity deficit and disregarding pressure variations. This latter constraint means these models do not treat the near-wake but are applicable to wake development from a distance of about three wake diameters downstream from the rotor. This
approach has been incorporated into a range of wind farm models including Windfarmer [29]
and FlaP [30]. A less parameterized model [31] was shown to give good agreement with
observations in the far wake and was used as the basis for WAKEFARM [32]. Computational
fluid dynamics (CFD) models have more recently been applied to simulation of wake case
studies in detail [33-40] including wake simulations using large eddy simulation (LES) for both
single [41] and multiple wakes [42, 43].

Most prognostic wind farm models for power production forecasting from large wind farms with
many turbines (up to hundreds) utilize approaches that were developed for single wakes and then
rely on fairly simple methods for combining multiple wakes, with or without parameterizations
for unresolved interactions (e.g. WAsP does not account for turbine-added turbulence in wakes).
These include ‘the sum of squares’ approach used in the WAsP model [44] that calculates the
combined velocity deficit in merging turbine wakes as the square root of sum of the velocity
deficits squared. Windfarmer uses a modified ‘nearest neighbor’ approach where the velocity
deficit and wake width is determined by the nearest wind turbine in the direction of the flow.

There is an additional optional correction for large offshore wind farms based on reduced wind
speed over the area of the wind farm from applying additional roughness elements [29]. Some
models are intended to incorporate physical interactions between wakes and the atmospheric
boundary layer in a pseudo-explicit manner [45, 46] accounting for the change of wake
expansion in different regimes of the wind farm while others use empirical corrections for larger
wind farms based on limited available data. The current state of the art with respect to the
application of CFD to entire wind farms in wake studies [47] may require simplifying
assumptions such as homogeneity in flow conditions across the wind farm [48], and thus neglect
wind speed gradients across the wind farm [49].
A universal issue in wake modeling is access to wind farm or other relevant observational data sets for model evaluation and development. Some authors argue that data from wind tunnel experiments are the most appropriate for evaluation of near-wake models [50]. Wind tunnel experiments offer the ability to provide a highly controlled environment, and greater flexibility in terms of the position of both upstream and in-wake observations, particularly at very high frequency [51]. However, at least some of the limitations of field measurements can be overcome by the use of scanning and nacelle-based lidar [52, 53]. Furthermore, some early studies illustrated that even single operating turbines introduce properties in the wake flow that are difficult to represent at the correct scales in wind tunnel data. These include turbulence added by the wind turbine [54] and stratification related to the vertical temperature gradient both of which impact wake development [8]. Given current interest is focused on large wind farms with multiple wakes that likely impact both the overlying and downwind atmosphere over distances at least twice the length of the wind farm [13, 19, 55], wind tunnel studies with their inherent scale limitations are likely less valid than studies using ambient data. Even given the inherent variability issues of field-based data and the difficulties of both making measurements in wind farms and/or accessing and using wind farm data, they are still required. Access to, and analysis of, high-quality ambient in situ data from operational wind farms remains central to efforts to improve both understanding of the meteorological variables that determine wind turbine wakes and physical descriptions thereof, and evaluation of wind farm (and specifically) wind turbine wake models. This paper focuses on in situ field data available from the free and open access Virtual Wakes Laboratory (VWL) hosted by Indiana University (http://mypage.iu.edu/~rbarthel/Welcome.pdf). To gain access to the site potential users are required to register but thereafter access to all data is provided freely.
The structure of this paper is as follows. In Section 2 the types of data included in the VWL are documented, their advantages and disadvantages are described and examples of their use for wake characterization are provided. In Section 3, issues and challenges pertaining to wind farm wake model evaluation are described and a variety of model skill metrics are presented which can be applied to improve detailed evaluation of models not only in a statistical sense but also in terms of a process-level perspective. This is necessary in order to transition from characterizing average wake losses to modeling of wake dynamics in specific events. Example criteria used for model evaluation of single wakes are given, and additional approaches for multiple wakes assessment are described. In section 4 concluding remarks are provided.

2 Data types available in the Virtual Wakes Laboratory (VWL)

Table 1 shows the data currently (2012) contained with the VWL. These data and the types of analyses they are best suited to are described below.

2.1 SCADA data

Supervisory Control and Data Acquisition (SCADA) systems are used to collect a range of data from each individual wind turbine deployed in operational wind farms. These data are time synchronized and, in the context of using them for wake analysis, typically include key parameters such as; power production, wind speed from the nacelle mounted anemometer, turbine yaw angle, and a turbine status signal. SCADA data are particularly useful because they are continuous through time and are present for every turbine location in an array. These data (that are typically made available with 10 minute averaging periods) can therefore provide a degree of spatial information in wind farms that is very difficult to obtain from other methods, except perhaps scanning lidar [21], and provide a unique opportunity to assess wind farm performance over time. Apart from obvious issues regarding data confidentiality, the primary
drawbacks of analyzing SCADA data for wake studies relate to the size of the databases that usually require extensive pre-processing, and the time averaging that is typically applied to the data prior to public release. Nevertheless, SCADA data are a direct representation of turbine operation and power production.

A number of SCADA data sets are available in the Virtual Wakes Laboratory (see Table 1) including data for 2001-2004 inclusive from Middelgrunden wind farm in Denmark [56]. The wind farm has a particularly interesting design with 20 turbines laid out in an arc, with turbine to turbine spacing along the arc of 2.4 rotor diameters (D) (see Figure 1). Data from this site are thus extremely useful for conditional sampling based on wind direction to select whether the turbines are operating in ‘no wake’, single wake, partial wake, or multiple wake conditions. As shown in Figure 1, the mean power output at each turbine is strongly determined by both its position in this wind farm and by the wind direction, which dictate whether individual wind turbines are operating in wake conditions. A key issue in analysis of wind turbine SCADA data for wake characterization and wake model evaluation pertains to whether it is preferable to use output from the anemometer mounted on the nacelle to represent the inflow wind speed, or whether it is better to derive the inflow wind speed from the turbine power production data and the turbine power curve [57]. Use of the power curve for a given turbine and power production data to calculate the wind speed introduces a level of uncertainty since typically only the manufacturer’s power curve will be available rather than an individual onsite measured power curve and there may be discrepancies arising from the onsite conditions. However, the power output from a turbine represents an integrated wind signal across the area of the rotor rather than at one location just above the nacelle [53]. As has been reported by many studies, the flow around the nacelle is extremely complex and reported errors in wind speed are both large [58]
and dependent on both wind speed and power output [59]. However, a study on an Australian wind farm found that “nacelle-based wind speed observations are more accurate than the on-site met mast wind speed observations” [57]. Figure 2 shows an example of the discrepancy in derived freestream inflow wind speed from the nacelle-mounted anemometer and the power production data using SCADA data from the VWL for Middelgrunden data. Wind speeds are derived from a polynomial fit to the manufacturer’s power curve between 4 and 15 ms⁻¹ using the power output from turbine 1 (see Figure 1 for a site map) and selecting only conditions when it was operating in freestream conditions. The correlation coefficient between the wind speed from the nacelle anemometer and the wind speed derived from the power curve is greater than 0.985. However, on average the power curve derived wind speeds are 3% higher than those measured by the nacelle anemometer, and the root mean squared difference (i.e. a measure of the dispersion of the data cloud) is 0.48 ms⁻¹ (relative to a mean wind speed of 8.1 ms⁻¹), which implies the selection of the power production or nacelle anemometer can have a substantial impact on the implied inflow wind speed for any given 10 minute period. The discrepancies illustrated in Figure 2 may reflect physical causes (e.g. flow around the nacelle), integration of the flow field across the rotor diameter in determining power production (v. a point measurement), discrepancies in the power curve (relative to the manufacturers power curve), or instrument bias/uncertainty.

2.2 Meteorological masts

Until relatively recently mast-based instrumentation was the only viable method of obtaining meteorological measurements from wind farms [8, 60]. Use of data from instrumented meteorological masts for wake studies also has a number of advantages and disadvantages. The disadvantages derive principally from three sources; (i) the meteorological masts are in fixed
positions and thus a truly 3-dimensional depiction of flow within the farm and its variation over
time cannot be produced, (ii) data quality is a strong function of the quality of instruments
deployed and maintenance there-of, in addition to the suitability of the mast structure and
instrument booms [61] and (iii) the cost of installation of tall masts increases non-linearly with
height, thus as turbine hub-heights have increased measurements from masts do not always
extend across the rotor plane. Nevertheless, many prospective wind farm installation locations
are equipped with meteorological masts for site characterization activities. Presuming those
masts are maintained post construction, and have at least one free-stream sector remaining after
the wind farm is built, the resulting data can be analyzed to assess wind farm performance over
time, and used in wake studies, although obviously this depends on the position of the mast in
relation to the wind turbines/wind farm [62]. Meteorological masts have some advantages in
wake studies in that masts can be instrumented at many heights and, if carefully installed and
maintained they can provide a long-term record that includes vertical profiles of wind speed,
turbulence and temperature.

Data from high-grade meteorological instrumentation deployed on meteorological masts in
offshore wind farms are also provided in the VWL. For example, data from three meteorological
masts installed close to the Vindeby wind farm in Denmark (see Figure 3) are available from the
VWL continuously for 1993-2002 with only one major change in 1996 when one of the offshore
masts was closed (mast SMS in Figure 3) and some of the measurement heights were changed.
Selection of meteorological mast type and instrumentation, in addition to the placement of the
meteorological masts at Vindeby was carefully chosen to augment wake studies. The mast
located southwest of the turbines (mast SMW) was placed in the position of a third row giving
measurement of two direct double wakes, while the mast located south of the turbines (mast
SMS) was placed in the sixth position of the first row giving a single direct wake and a multiple wake with five turbines (see Figure 3). A further mast was located on the coastline 1.5 km to the south of the wind farm (mast LM in Figure 3). This data set has been extensively analyzed [11, 12, 63] and used for model evaluation [64], and remains the longest publically available time series of wake measurements. One issue with this (and other) datasets is that turbine power production data were not collected concurrently with that from instruments deployed on the meteorological mast, thus careful time synchronization between meteorological and SCADA data must be carried out. However, availability of such long data sets facilitates a wide array of statistical analyses and options for conditional sampling to resolve the physical processes controlling wind turbine wake behavior [64]. For example, Figure 4 shows the average ratio of wind speeds as measured by mast-mounted anemometers at hub-height (38 m) for mast SMS to those at mast LM. The presence of the coastline leads to quite complex flows across the wind farm and spatial gradients of wind speed at Vindeby. This is evident from the high ratios of wind speed associated with wind directions from the east and the south-southwest, when the mast SMS has a relatively long offshore fetch, but flow to mast LM is over land. However, the presence and magnitude of the wind turbine wakes are also clearly evident as the minima in the wind speed ratios associated with wind directions when the mast LM was experiencing free-stream (undisturbed) flow and the mast SMS was in the wake of the wind farm. For example, the mast SMS is clearly impinged upon by a single wake (i.e. a wake deriving from only one upstream turbine) for wind directions of about 22°. That is, for this wind direction, data from the mast SMS indicate lower wind speeds and higher turbulence (as manifest as increased standard deviation of the wind direction) relative to the mast LM. Conditions in a multiple wake condition are evident at the mast SMS for wind directions ~320°.
2.3 Remote sensing data

The obvious advantages of moving from in situ instrumentation to ground-based remote sensing for quantifying characteristics of wind turbine wakes are that i) measurements can be made to turbine height and beyond and ii) lidar and sodar are more mobile than traditional masts and thus can be deployed at specific downwind distances from turbines. The disadvantages largely derive from the much larger power demands for operation of the systems (relative to standard meteorological equipment), the challenges in analysis of the resulting large and complex data sets, the precision and accuracy of the measurements and difficulties in mounting these instruments on moving platforms (e.g. ships) for offshore measurements. Nevertheless, a boat-mounted sodar was able to capture and quantify wind turbine wakes at the Vindeby wind farm [16, 65]. The experimental design of this campaign included a series of prescribed measurements in which during a period of forecast wind speed stationarity, a turbine(s) was selectively halted and the sodar was deployed to measure ‘freestream’ conditions, and then restarted so the sodar could measure the wake at an identical location.

Data are presented in the Virtual Wake Laboratory for a number of sodar experiments conducted at the Vindeby wind farm. During these experiments the sodar was operated on a ship moored at distances between 1.4 and 7.1 rotor diameters (D) from the wind turbines [16]. The resulting data can be used to calculate the velocity deficit profile with unprecedented vertical detail. For the case shown in Figure 5 the sodar was deployed at a downwind distance of 2.8 D. In this case the near-wake classic ‘double bell’ shape is clearly visible with maximum relative velocity behind the turbine close to hub-height (38 m) and wake expansion to just over 50 m diameter ($D_w$) from the initial turbine diameter ($D_0$) of 35.5 m.
Following the equation given by Jensen [44] (and used in the WAsP program) the wake diameter $D_w$ at any distance downstream of a turbine comprises the initial rotor diameter $D_0$ plus expansion $D_E$ [27]:

$$D_w = D_0 + D_E$$  \hspace{1cm} (1)

Where:

$D_E$ is given in WAsP by:

$$D_E = 2kX$$  \hspace{1cm} (2)

where

$k$ is the wake expansion coefficient (0.05 is suggested for offshore [66]).

$X$ is the distance downstream from the turbine.

Following equation [2] the wake expansion coefficient ($k$) for the example shown in Figure 5 is calculated to be 0.07, which is higher than the value suggested for offshore of 0.04-0.05 [66] that is likely due to slightly higher turbulence intensity at Vindeby due to its proximity to the coast.

Since predicting loads in wakes is also of interest, a tremendous advantage of remote sensing techniques is that they are also able to quantify the profiles of turbulence intensity. Figure 5 shows profiles of turbulence intensity (calculated here as the standard deviation of the measured horizontal wind speed divided by the wind speed at the same height) from the sodar measurements at Vindeby for this case study. Measurements of ambient turbulence intensity profiles for typical turbine hub- and tip heights generally indicate decreasing values with height (Figure 5) due to increasing wind speed with height and the decreasing impact of surface
roughness [67]. When turbine-induced turbulence is superimposed on the ambient values, turbulence intensity profiles vary according to whether they are measured in the near-wake (where a non-Gaussian (double-bell) wind profile is present [16]) or in the far wake where the wake is broadly Gaussian in form. Model and wind tunnel results [50] suggest that the peak in turbulence intensity in the near-wake is evident at heights 0.5 D above hub-height, but this feature is less pronounced in the sodar observations (Figure 5). Although there remain challenges in accurately estimating turbulence intensity from sodar measurements, the wake profile clearly has higher turbulence intensity than the non-wake profile. Applying the Frandsen model [68]:

\[
I_{total}^2 = I_{ambient}^2 + I_{turbine}^2
\]  

(3)

Where:

- \(I_{total}\) is the total turbulence intensity
- \(I_{ambient}\) is freestream turbulence intensity
- \(I_{turbine}\) is the turbulence intensity added by the turbine

To the sodar observations, and integrating \(I_{total}\) across the rotor plane (20-70 m) indicates the wake influenced \(I_{total}\) is 0.197, the freestream value \((I_{ambient})\) is 0.100, thus the turbine-added turbulence \((I_{turbine})\) is 0.169. Added turbulence intensity at hub-height for this case was previously estimated to be 0.087 (see Case 4 in Table 3 in [16]), which is due to the fact that the turbine-added turbulence is not maximized at hub-heights in the near-wake.

These examples illustrate the utility of data such as those contained in the Virtual Wakes Laboratory for examining characteristics of wind turbine wakes, but the majority of current users of VWL access the data for model evaluation. Use of appropriate metrics to further wake model
evaluation and improvement are discussed in the next section again using examples from the VWL.

3 Tools for, and approaches to, evaluation of wake models

3.1 Wake model evaluation approaches and metrics

Much of the data presented in the Virtual Wakes Laboratory have previously been used in evaluation of wake and wind farm models of varying complexity. There is likely no one metric or variable that can serve to assess whether wake models are performing adequately, but a series of quantitative metrics should serve as a basis for assessing whether model performance is improving. Below we briefly describe the types of metrics and evaluation approaches that have been adopted to date, and their relative strengths and weaknesses.

As the demands for improved accuracy of wake models has increased and the need for characterization of conditions beyond the mean hub-height wind speed have become obvious, the level of sophistication of these wake model evaluation exercises has also increased. Nevertheless, the majority of wake model evaluation exercises that have been conducted to date have tended to focus on comparison of centerline velocity deficits at hub-height and, where quantitative evaluation has been attempted, have used metrics such as mean absolute error or root mean square error model v. observations [65]. However, this approach reduces the evaluation of model skill to comparison of a single point on the wake profile for a single metric (wind speed). As shown in Figure 6, beyond the near-wake region (i.e. for downstream distances beyond approximately 3-4 D), wakes are typically near-Gaussian in shape. While most wake models use a Gaussian or near-Gaussian distribution to depict the wake [69], some (e.g. WAsP) use a top-hat distribution, hence while the integrative momentum deficit in the wake may be
correct in WAsP simulations, the maximum velocity deficit should be systematically underestimated relative to observations [65]. Thus, other studies have compared integrated variables across the rotor plane from models and observations. For example, boat-mounted sodar data at the Vindeby wind farm were used to evaluate range of models of varying complexity using a range of diagnostics including the relative velocity deficit and the cumulative velocity deficit over the wake profile [65]. Despite some obvious differences in the model types, this evaluation of model predictions for single wakes did not find any systematic bias for any particular model, regardless of its complexity. However, the absolute discrepancies between the models and the measurements were large. For the six wake models evaluated [65], the mean absolute error (expressed as a fraction of the free-stream wind speed) at hub-height between the predictions and the observations for single wakes in the six wake cases at downwind distances of 1.7 to 7.1 D ranged from 11-17%. Other studies have evaluated wakes models in terms of the power output from individual turbines. However, given power production is determined by the integrated wind speed across the rotor plane [70], the accuracy of the turbine power curve is critical to use of modeled wind speeds to predicted power output.

A key issue in all model evaluation studies is the relatively small signal to noise ratio in wake detection. For the generation of sodar used in the Vindeby experiment contained in VWL and with the confounding influence of corrections for boat motion, the 30-minute average measurements of wind speed and turbulence intensity were estimated to have accuracy of ±20%. In Figure 5, the average wake wind speed from 20-65 m height is 3.88 ms⁻¹. Measurement uncertainty quantified as the average standard deviation of consecutive measurements within the 30 minute periods is 0.80 ms⁻¹. The wake wind speed at 2.8 D is 66% of the freestream wind
speed, thus the reduction in velocity in the turbine wake is larger than the measurement uncertainty.

It is important to note that irrespective of the specific metric used to evaluate wake model performance for individual or multiple wakes a key source of uncertainty pertains to defining the free-stream wind speed and turbulence profiles (see section 2) [65]. Even if accurate free-stream wind and turbulence profiles can be established, a second issue in comparing models and measurements is to reconcile the fundamental differences in the physical variables being measured and observed. As discussed in the following section, wake width can be objectively determined from observations for single and double wakes. It provides a useful tool for evaluating models (and quantifying the role of wake expansion v. wake meandering), but also illustrates the challenges of make model-observations comparisons. Focusing on the single wake at Vindeby shown in Figure 6, the center of the wake is evident for wind directions of ~22° where the measured velocity at mast SMS is ~0.9 of the free-stream velocity. This wind direction is thus associated with direct flow from the upstream wind turbine to the meteorological mast. However, for slight variations in wind direction from 22°, the meteorological mast is impinged upon by a peripheral part of the wake. Thus a plot, like that shown in Figure 6, can be developed by conditionally sampling the wind speed data for slight variations in the wind direction. Using an arbitrary threshold that the wind speed must exceed 0.99 of the freestream to represent the edges of the wake, the wake width using all observations at this downstream distance of 8.6 D is estimated to be 16° (which equates to a distance of 2.4 D or 86 m, see Table 2). Note that a challenge of applying the assumption of axisymmetric wakes to infer wake vertical profiles from wake widths (as determined herein) is that the vertical profiles are strongly impacted by the non-uniform inflow vertical wind speed profile.
For multiple wakes, and indeed wake effects at the wind farm scale, the model evaluation process becomes more complex. As indicated in [4], as wakes move through wind farms they are subject to inhomogeneity in the flow, and wakes from individual turbines impact the ground, impinge upon downstream turbines and merge with wakes from laterally displaced turbines. For sufficiently large wind farms the behavior of wind turbine wakes is additionally controlled by the characteristics of the whole boundary-layer (e.g. atmospheric stability, turbulence intensity, boundary-layer height, and wind shear). The VWL also provides case study data from the large offshore wind farms at Horns Rev and Nysted that can be used to quantify these effects and have already been used for wake model evaluation [48, 71]. Wind farm efficiency is one integrative metric that can be used to evaluate model performance at the wind farm scale and is readily expressed using a range of error statistics [71].

The studies and approaches described above assume that power output is the desired quantity. If fatigue loads are to be modeled then the demands on model performance are even more stringent given the difficulties in measuring and modeling turbulence intensity [72]. The need to integrate power output and fatigue loading for wind farm optimization has prompted a more dynamic, case-study approach, with only limited evaluation of model skill [73].

3.2 Wake width as a metric of model performance

The actual width of a wind turbine wake is a function of three components;

(i) the turbine rotor diameter (this determines the volume of air from which momentum was extracted, $D_0$),

(ii) the wake expansion due to mixing in of higher momentum air at the lateral boundaries of the wake, and
meandering of the wake due to variations in the wind direction over the measurement period.

However, deconvoluting these terms has proved challenging and represents a major source of uncertainty in comparing observations and models. Using equation [1], and assuming a wake expansion coefficient of 0.05, the wake width at 8.6 D is predicted to be $1.86D_0$. From simple geometry we can convert this to a width in degrees ($\sim 12^\circ$). As indicated above this is similar to, but smaller than, the wake width derived from the observations. However, the observationally derived estimate is for all 10-minute data periods without sub-selection for the degree of meteorological stationarity and thus the degree of wake meandering due to variations in the wind direction. Based on the discussion above, the wake width at a specific distance downstream $D_w$ can be assumed to consist of three additive components by expanding Equation 1. As shown in Figure 7, the total wake width includes not just the initial rotor diameter ($D_0$) and the expansion ($D_E$ where $D_E=2kX$ as shown in equation 2) but also an additional component that arises from the directional variability ($D_M$) that is also called wake meandering and arises from the variability of direction in the freestream flow:

$$D_W = D_0 + D_E + D_M$$ (4)

Using sub-sets of the Vindeby observations, selected with smaller standard deviations of wind direction, the width of the wake reduces because $D_M \rightarrow 0$. When the data are conditionally sampled to select only periods in which the standard deviation of wind direction $< 5^\circ$, the wake is approximately $9^\circ$ wide. For the single wake case at Vindeby, at a downwind distance of 300 m, the original rotor diameter $D_0$ (35.5 m) corresponds to an angle $\alpha = 3.35^\circ$ that would be observed as a wake width of $6.7^\circ$ (Figure 6). Assuming that the case in which the wind
directional variability $< 5^\circ$ represents the case where the wake expansion component from meandering is negligible (i.e. $D_M=0$) for $D_W=9^\circ$, then from above $D_0=6.8^\circ$ and $D_E=2.2^\circ$ (Figure 6). Further, assuming that $D_E$ is invariant with the standard deviation of direction (although wake expansion depends on stability and turbulence [74]), the relative contribution of $D_M$ can be estimated (using the wake width for all observations $D_W=16^\circ$) from Equation 4 $D_M = 7^\circ$. In other words, assuming the assumption of linear expansion is approximately correct, for the single wake case at Vindeby, $D_M$ makes a larger contribution to total wake expansion than $D_E$. Thus, comparing observations that include both $D_E$ and $D_M$ with models that do not include variability in the wind direction (and thus set $D_M=0$) may erroneously lead to the inference that $D_E$ is under-estimated.

Ainlie [28] proposed the following approximation to determine the contribution of wake meandering to the centerline wake velocity deficit:

$$v_c = v_0 \left[ 1 + 7.12 \left( \frac{X \sigma_\theta}{D_w} \right)^2 \right]^{-0.5}$$  \hspace{1cm} (5)

Where:

$v_c$ is the velocity deficit including wake meandering

$v_0$ is the velocity deficit without meandering

$\sigma_\theta$ is the standard deviation of wind direction

$X$ is downwind distance
Here, we invoke the assumption of a Gaussian wake shape to evaluate this approximation in terms of the implied meandering contribution to wake, based on the following:

The standard deviation of wind direction ($\sigma_\theta$) at Vindeby is 4-7° (Figure 4) if calculated using [75]:

$$\sigma_\theta = \sin^{-1}(\varepsilon)[1 + b\varepsilon^3] \quad (6)$$

Where:

$$b = \left(\frac{2}{\sqrt{3}}\right) - 1$$

$$\varepsilon = \sqrt{1 - \sin^2 \theta - \cos^2 \theta}$$

When expressed in radians $\sigma_\theta$ is comparable to values for turbulence intensity calculated as the ratio of the standard deviation of wind speed to freestream wind speed (0.07 to 0.16) for different heights and surface types [28], thus validating one of the assumptions employed by Ainslie [28].

Based on the observations from Vindeby summarized in Figure 6, $D_w = 2.4 \; D_0, \; X = 8.6 \; D$, the centerline velocity deficit including meandering (expressed as SMS/LM) is 0.88. The increase in velocity deficit is 5% when determined from equation (5) for a case in which the meandering component is minimized (i.e. the maximum deficit in that case would equate to SMS/LM = 0.84). Using a Gaussian distribution to describe the wake form:

$$f(x) = \left[\frac{1}{\sigma(2\pi)^{0.5}}\right] \exp \left[\frac{-(x-x_0)^2}{2\sigma^2}\right] \quad (7)$$
an increase in $f(x)$ computed at $x =$ centerline (i.e. the wake depth) as a result of setting $D_M=0$, can be mapped onto the necessary reduction in wake width ($\sigma$) since the momentum deficit must be conserved.

Using this approach (see Figure 6 and Table 2), the implied change in wake width due to wake meandering is smaller than the value for $D_M$ inferred from the observations. However, there are a number of caveats to this analysis and the inferences drawn there-from. This analysis is illustrative of a specific downwind distance (8.6 D), naturally, as the downwind distance increases, the impact of directional variability increases. Wake expansion ($D_E$) is strongly determined by the wind turbine thrust coefficient (and thus incident wind speed) and the prevailing atmospheric stability and turbulence (which may manifest themselves as increased directional variability). $D_M$ is determined in the empirical analysis as a residual term and thus is subject to considerable uncertainty. Nonetheless, this case study illustrates an additional reason why it is difficult to find an equal basis on which to compare models and measurements.

Empirical wake models use expansion coefficients based on observations and these include elements of both wake expansion and meandering due to turbulence on all scales while most CFD (and LES) models include inflow turbulence that imparts only some part of directional variability [42].

Many comparisons of observations and model predictions assume that both are exactly observing/predicting the wake center and thus the maximum velocity deficit. In reality, a wider sector width has to be used to determine the wake velocity deficit from observations, in part because of observational uncertainty and atmospheric non-stationarity. The observational uncertainty derives from issues such as bias or offset in the wind direction measurement due to the initial calibration of the instrument (that is notoriously difficult to perform in the field) or
drift. Either of these will result in an offset between the observed and modeled wake center. In addition, wind directional variability in the atmosphere is known to be related to stability conditions, indeed the standard deviation of wind direction is used in some versions of stability classifications, particularly in plume dispersion modeling [76]. Thus selection of the wind direction sector for conditionally sampling the observational data to determine parameters such as the total momentum deficit, or wake width, or the wake intensity (i.e. wake depth in terms of the velocity deficit) is also a critical determinant of model/measurement comparisons. For the example in Figure 8, and following the logic expressed above (equation 5), assuming a narrow central direction sector (i.e. $22^\circ \pm 0.5^\circ$) captures the largest velocity deficit, while expanding the directional sector (i.e. conditionally sampling over wind direction of 20-24°) decreases the size of the maximum velocity deficit derived from the observations. That is, if the velocity deficits are averaged over a larger wind direction sector, this leads to a smaller derived average velocity deficit. Assuming the wake conforms to a Gaussian distribution, approximately 68% of the wake lies within one standard deviation of the wake centerline (i.e. the wind direction is within $\pm 5^\circ$), 95% of the total wake is within two standard deviations and 99.7% within three standard deviations. Thus an alternative to using a fixed width approach to defining sectors for model validation is to use one standard deviation of the observed wind direction measurements or a fraction of the standard deviation to include a prescribed amount of the wake width. This is illustrated in Figure 8 which shows the single wake at Vindeby for a downwind distance of 8.6 D marking the area for one and two standard deviations from the mean. Obviously the width of the wake varies with the distance from the turbine but so does the velocity deficit. As shown in Figure 8, the average wind speed ratio (wake /freestream) at the center of the wake is 0.88 if the narrowest possible directional sector is chosen. Increasing the number of direction bins to $22^\circ \pm 1^\circ$
increases the implied wind speed ratio to 0.90 and so on until including a wind direction sector of
22° ± 15° increases the ratio to 0.97 (i.e. if the data are conditionally sampled to center on a wind
direction of 22° but include directions ±15°, the wake wind speed is 97% of the incident
freestream wind speed). It is common to use conditional data sampling applying a screening
threshold of ±2°, ±5° or ±10° to the wind direction sector included. Use of these data screening
tools would decrease the wake wind speed to freestream ratio from 0.907 (in a sector with wind
directions ±2° around the centerline) to 0.933 and 0.955, respectively. As indicated in Figure 8,
in this case choosing a wind direction ±5° is close to ± one standard deviation and ±10° close to
two standard deviations. Thus, wake model evaluation studies must incorporate consideration of
the wake shape, treatment of wind directional variability and incorporate consideration of the
width of the direction sector used to compute the observed wake characteristics.

4 Concluding remarks

Accurate prediction of power losses from wind turbine wakes in large wind farms remains
elusive. At present there are a number of approaches to modeling power losses due to wind
turbine wakes in large wind farms that range in complexity, but all require further evaluation and
refinement. Significant issues limiting the improvement of models are the lack of appropriate
data for evaluation, limited application of evaluation metrics to quantify model skill and very
limited attempts at attribution of model error. This paper outlines some of the issues involved in
using different types of data for evaluating wind turbine wake modeling and shows how these
data can be used to characterize wake properties using examples based on data from the open
access Virtual Wakes Laboratory. The paper also describes some of the issues involved in using
data to evaluate models, with a focus on wake width.
5 Acknowledgements

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6 References


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<th>Brief description</th>
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<td>30 minute average time series of multiple variables from 2 meteorological masts and limited turbine power output</td>
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<td>2001</td>
<td>Case studies of wake measurements by sodar at Vindeby</td>
<td>ENDOW project Operated by Risø National Laboratory. Data credit to DTU Wind</td>
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Table 2. Wake widths for the case study from Vindeby shown in Figure 6 expressed in degrees, meters and rotor diameters. Note these measurements pertain to a single wake case at Vindeby in which the distance between the turbine and the measurements is 8.6 D.

<table>
<thead>
<tr>
<th>Description</th>
<th>Width in degrees</th>
<th>Width in m</th>
<th>Width in rotor diameters D</th>
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<tr>
<td>Turbine rotor diameter $D_0$</td>
<td>6.7</td>
<td>35.5</td>
<td>1.0</td>
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<td>Wake diameter with expansion ($D_0+D_E$) but limited meandering</td>
<td>9</td>
<td>47.2</td>
<td>1.3</td>
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<td>(wake width when standard deviation of direction &lt;5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wake diameter including expansion and meandering ($D_0+D_E+D_M$) (i.e. total wake width measured using all observations)</td>
<td>16</td>
<td>84</td>
<td>2.4</td>
</tr>
<tr>
<td>Ainslie wake diameter for $D_0+D_E+D_M$</td>
<td>15</td>
<td>79</td>
<td>2.2</td>
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<tr>
<td>Ainslie wake diameter excluding meandering</td>
<td>12</td>
<td>63</td>
<td>1.8</td>
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7 Figure captions

Figure 1. Schematic of the layout of the Middelgrunden wind farm and mean power produced by each turbine for 2001-2004 (left) and mean total power produced at each turbine as a function of incident wind direction (right).

Figure 2. Top. Comparison of wind speeds derived from the manufactures power curve ($U_p$) and from the nacelle anemometer ($U_a$) from turbine 1 at Middelgrunden based on data from 2001-2004. The red line is the best fit where $U_p = U_a * 1.03$, the gray dashed line is the 1:1 line. Bottom left: Power output plotted against $U_p$. Bottom right: Power output plotted against $U_a$.

Figure 3. Schematic of the layout of the Vindeby wind farm (turbine locations shown by triangles) and meteorological masts (solid squares). Distance between the turbines in each row is 300 m (8.6 D) and the distance between the two rows is also 300 m. Also shown are the wind directions associated with direct single, double and multiple wake measurements at the masts.

Figure 4. Ratio of the wind speed at 38 m (i.e. hub-height) at the mast SMS relative to the mast LM for the Vindeby wind farm as a function of wind direction. The locations of the mast are as shown in Figure 3. The solid black lines show the mean ratio in each 1° wind direction bin as a running average. The gray shading shows the standard deviation of the ratio around the mean. Also shown is the mean standard deviation of the wind direction (black triangles).

Figure 5. Wind speed and turbulence profiles from an experiment at Vindeby using boat mounted sodar [16]. Turbulence intensity is calculated as the standard deviation of the horizontal wind speed divided by the horizontal wind speed in a 30-minute period. The solid symbols denote the wake condition (at a distance of 2.8 D) while the open symbols show the free-stream...
values (both half hour averages). The gray bar on the left shows the original turbine hub-height and blade tips.

Figure 6. The ratio of wind speed at Vindeby mast SMS to mast LM by direction. The colored symbols show averages of wind speed ratio in 1° direction bins while solid lines are running means. The color coding shown in the legend denotes the ratios conditionally sampled by the standard deviation of observed wind direction i.e. the black line and symbols show the observed ratios and the running means for all observations while the red symbols and line show those conditionally sampled for standard deviations less than ±5°. The number in brackets in the legend is the calculated standard deviation of direction for the sample shown i.e. using all observations the standard deviation of direction is 4.88°, while the standard deviation of direction for observations conditionally sampled for standard deviations of less than ±5° is 3.88°. Once the standard deviation of the direction is less than ±5° there are too few observations in each 1° direction bin to define the wake width. The lines at the top of the graph indicate the approximate contributions of $D_0$, $D_E$ and $D_M$ to the total wake width $D_W$ as indicated from Equation 4. Also shown (as grey lines) on the graph are the wake profiles determined using the approximation of Ainslie [28] for a case with and without wake meandering.

Figure 7. Illustration of the components comprising the wake width at a distance $X$ from the turbine.

Figure 8. Top: Wake width at Vindeby showing observations of the wind speed ratio (gray squares and line) compared with a fitted Gaussian distribution (black line). The area in red shows the observations that are included if the mean direction ± one standard deviation of direction in the calculation of the wind speed ratio, the area in blue includes observations included with the
mean ± two standard deviations of direction. Below: The average wind speed ratio for the center of the wake (22°) ± the number of direction bins shown. The red line indicates the average at ± one standard deviation, the blue line the average at ± two standard deviations. The error bars on the ratio are ± one standard deviation (black dashed lines).
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.