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Modeling and Solution Methods for the Energy-Efficient Train Timetables Problem

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Abstract

Timely recuperation of energy through regenerative braking is crucial in order to ensure energy efficient railway timetables. This requires a careful synchronisation of train departures such that high energy peaks, as a result of simultaneously accelerating trains, can be avoided. In this report we consider a variant of this problem as presented in the FAU Open Research Challenge in Discrete Optimization. We propose a mixed integer linear programming formulation (MILP) together with a number of heuristics based on this model. We show that the MILP can obtain optimal solutions to most of the instances proposed as part of the challenge, and that the matheuristics can find good solutions in short computation times.

1 Introduction

Rail is one of the major consumers of energy, and there is a desire for railway operators to be energy-efficient in their daily operations. Towards this aim a significant component of a train operating company's electricity bill usually depends on the highest peak of energy usage within a given period, a measure imposed to encourage operators to maintain a balanced distribution of power consumption throughout their operations [2].

Compared to the case where, for example, trains are billed individually based on their power consumption, this measurement results in more complex timetabling problems since the energy peaks are highly dependent on the interactions between trains. Synchronisation becomes important in order to ensure that energy regenerated (when trains are braking) is efficiently recuperated by accelerating trains.

In this report we consider the Energy-Efficient Train Timetables Problem (EETTP), which comprises the construction of a timetable such that the period with the highest peak power usage is minimised. The problem was presented and instances were provided by the Chair of Economics, Discrete Optimization and Mathematics at Friedrich-Alexander-University Erlangen-Nürnberg (FAU), in the Discrete Optimization part of their *Open Research Challenge*¹.

In this paper we present a mixed-integer programming formulation (MILP) together with a number of heuristic approaches based on this formulation. Our approach is based on *dwelt time control*, *i.e.* deciding only the departure times of trains at stops while assuming fixed running times. Albrecht [1], for example, considers the same problem, but uses an approach for train running time control instead of train dwell time control. Sansó & Girard [6] considers the problem with dwell time control within the context of the Montreal metro system, and present a formulation in some ways similar to the one presented in this paper. However, due to the fact that we consider a more complex network structure, we model safety (headway) constraints differently and also include constraints for maintaining passenger connections which they do

¹See <https://openresearchchallenge.org/discreteOptimization/ChairofEconomics/The+Challenge>.

not. A general overview of energy efficiency in railways is given by Albrecht [2]. For a general overview of railway timetabling see Cacchiani & Toth [4].

We note that the problem under consideration in this report is similar to the Resource Leveling Problem (RLP) in project scheduling [5]. Arrivals and departures of trains may be viewed as project activities, while headway, runtime, dwell time and connection constraints may be viewed as precedence relations between activities. Moreover, electricity may be viewed as a single resource, the usage of which needs to be flattened out (levelled) over the scheduling horizon. The RLP and EETTP differ mostly with respect to the objective function, since resource levelling problems usually focus on specific objective functions that differ from the one considered in this paper. The RLP has been well studied in the literature, and an overview of exact methods for both continuous and discrete time variants is given by Rieck & Zimmermann [5].

The presented problem has a nice structure that can easily be modelled using a linear program with binary variables. We therefore investigate the potential of solving such a model with existing state-of-the-art solvers. The problem is fairly constrained, and our intuition is that variable/constraint preprocessing and constraint propagation may be key in solving the problem. Furthermore, an exact approach may provide bounds on optimality that can be used to evaluate the effectiveness of heuristic approaches. Finally, we also investigate matheuristic approaches as they are able to combine strengths of both worlds (exact and heuristic approaches).

This paper is organised as follows. In Section 2 we give an informal problem description, followed by a more formal description in terms of a MILP in Section 3. We propose a number of matheuristics in Section 4 and we compare all methods with respect to the provided instances in Section 5. We conclude the report with a summary in Section 6 and ideas for future research in Section 7.

2 Problem description

In input we are given a set of trains and a railway network. For each train we are given a route in the network, which is a sequence of legs connecting a sequence of stops. For each leg a departure window is given together with a power profile for traversing the leg. Here, a power profile defines the power phases of a train on a trip, *i.e.*, when a train is consuming or regenerating power during the trip. The problem then amounts to deciding the times of the departures of all legs such that the highest peak of electrical power usage is minimised.

The choice of departure times for the trains is governed by minimum dwell-time constraints at platforms and headway time constraints between trains travelling on the same track in the same direction. Further constraints include a fixed order in which trains are to traverse each track in the network, and the fact that passenger connections between certain trains at certain stations need to be maintained. Here, a passenger connection is defined whenever the difference in time between an arrival and departure at the same station falls within a 5–15 minute interval in the original timetable. We consider a discretization of time such that trains can only depart on whole minutes, and we assume fixed running times and single-direction tracks.

Given a solution which specifies the departure times for all legs, the power profiles are used to calculate the net power usage of all trains for each *second* of the considered time horizon. Within a power profiles each second can be either positive (acceleration) or negative (regeneration). For consecutive periods of 15 minutes, covering the entire considered time horizon, the average power consumption is calculated for each period by only taking into account seconds with a positive net power usage (the assumption is thus made that power regenerated by a braking train that is not consumed by an accelerating train in the span of the same second is lost). The objective is to minimise the 15-minute period with the highest average power consumption.

3 MILP formulation

We are given a set of legs \mathbb{L} together with a list of consecutive possible departure times $\mathbb{T}_i \subseteq \mathbb{T}$ for each leg $i \in \mathbb{L}$, where \mathbb{T} is the set of discrete time instants in the considered time horizon. We consider the problem of deciding at which time instant each leg should depart so as to minimize peaks in energy consumption, subject to constraints ensuring that runtimes, dwell times, headways and connections are respected. To this aim we define a binary decision variable x_t^i which assumes a value of 1 if leg $i \in \mathbb{L}$ departs at time $t \in \mathbb{T}_i$ and 0 otherwise. First we impose constraints to select exactly one of the possible departures:

$$\sum_{t \in \mathbb{T}_i} x_t^i = 1 \quad i \in \mathbb{L}. \quad (1)$$

The track headway, passenger connections, dwell time, and runtime constraints share a common *precedence* structure, which is why we model time using the same notation. In order to model the constraints on departure times we consider a directed graph $G = (\mathbb{L}, \mathbb{A})$ where $(i, j) \in \mathbb{A} \subseteq \mathbb{L} \times \mathbb{L}$ denotes a *precedence constraint* on the departure time of j with respect to the departure time of i . Each precedence constraint $(i, j) \in \mathbb{A}$ is associated with a set of *conflicting* pairs of time instants $\mathbb{C}_{ij} \subseteq \mathbb{T}_i \times \mathbb{T}_j$, where $(t, t') \in \mathbb{C}_{ij}$ indicates that a conflict will arise if leg i departs at t and leg j departs at time t' . The precedence constraints may then be modelled by one of the following two inequality sets:

$$\sum_{t' \in \mathbb{T}_j: (t, t') \in \mathbb{C}_{ij}} x_{t'}^j + x_t^i \leq 1 \quad (i, j) \in \mathbb{A}, t \in \mathbb{T}_i. \quad (2)$$

$$\sum_{t' \in \mathbb{T}_i: (t, t') \in \mathbb{C}_{ij}} x_{t'}^i + x_t^j \leq 1 \quad (i, j) \in \mathbb{A}, t \in \mathbb{T}_j. \quad (3)$$

Note the subtle difference between the two inequality sets, and the fact that using only one set is sufficient. Including both sets can strengthen the LP relaxation, but in our experiments we found this to be of no significant benefit.

In the subsection that follows we discuss in more detail the construction of the sets \mathbb{A} and \mathbb{C}_{ij} , and in the section after that we discuss the objective function.

3.1 Precedence constraints

First of all, the fixed runtime and dwell time (at the destination) of each leg should be respected. Let r_i denote the running time and d_i the dwell time of leg $i \in \mathbb{L}$. In input we have for each leg a *train successor* leg which represents the next movement of the same train. If j is the train successor leg of i , then $(i, j) \in \mathbb{A}$ and

$$\mathbb{C}_{ij} = \{(t, t') \in \mathbb{T}_i \times \mathbb{T}_j \mid t + r_i + d_i > t'\}.$$

Secondly, headways need to be respected between trains entering the same track. Note, we assume that no track is used in both directions. In input we have for each leg a *track successor* leg that will follow it on the same track (recall that the order in which trains traverse the tracks cannot be altered). If j is the track successor leg of i , then $(i, j) \in \mathbb{A}$ and

$$\mathbb{C}_{ij} = \{(t, t') \in \mathbb{T}_i \times \mathbb{T}_j \mid t + h_{ij} > t'\},$$

where h_{ij} denotes the required headway between the consecutive legs i and j .

Finally, connections need to be maintained. In input we have for each leg a *connection successor* leg, that has to maintain a connection with it within a specified time window. If j is the connection successor leg of i , then $(i, j) \in \mathbb{A}$ and

$$\mathbb{C}_{ij} = \{(t, t') \in \mathbb{T}_i \times \mathbb{T}_j \mid t + r_i + c_{ij}^{\min} > t' \vee t + r_i + c_{ij}^{\max} < t'\},$$

where c_{ij}^{\min} and c_{ij}^{\max} denote the minimum and maximum time j should wait after the arrival of i .

3.2 Objective function

The set \mathbb{T} will be a discretization of time in minutes, but for the calculation of the objective function we consider a finer discretization of time in seconds, represented by the set \mathbb{S} .

Let $\pi_{st}^i \in \mathbb{R}$ denote the power usage² during second $s \in \mathbb{S}$ by leg $i \in \mathbb{L}$, given that it departs at time $t \in \mathbb{T}_i$ (calculated from its power profile given in input). Let $\pi_s \in \mathbb{R}_0^+$ be a decision variable representing the total power consumption during second $s \in \mathbb{S}$, realised by the constraint set

$$\sum_{i \in \mathbb{L}} \sum_{t \in \mathbb{T}_i} \pi_{st}^i x_t^i \leq \pi_s \quad s \in \mathbb{S}. \quad (4)$$

Note that if the left hand side of the above inequality is negative, this represents lost energy. Consequently, $\pi_s = 0$ in this case, as π_s is defined as a non-negative continuous variable.

As stated before, the objective is to minimise the 15 minute time window with the highest average power consumption. Towards this end, let \mathbb{P} be the set of consecutive 901 second periods that covers the considered time window, where any two consecutive periods overlap in exactly one second. Let f_p and ℓ_p denote the first and last second in period $p \in \mathbb{P}$, respectively, and let Π be the total energy consumption of the period with the highest consumption. This value is realised by the constraint set

$$\frac{1}{2} \sum_{s=f_p}^{\ell_p-1} (\pi_s + \pi_{s+1}) \leq \Pi \quad p \in \mathbb{P}, \quad (5)$$

using the trapezoidal rule for approximating integrals.

Finally, the MILP formulation is given by

$$\begin{aligned} & \text{minimize } \Pi \\ & \text{subject to (1)–(5)}. \end{aligned}$$

4 Heuristic approaches

In this section we describe two heuristic approaches to the problem, both making use of variants of the MILP presented in the previous section in different ways. A third heuristic will also be used, namely imposing a time limit on the solution of the MILP presented in the previous section.

4.1 Heuristic approximation of Π

The calculation of the objective function requires a large amount of variables to be included in the MILP model, namely a variable π_s for each second $s \in \mathbb{S}$ in the considered time horizon. This is necessary to ensure that a negative net power usage during any second results in energy lost to the system.

We consider an aggregation of these variables, thereby obtaining a relaxation of the problem by assuming that regenerated energy is not lost and can be recuperated at any other point in time. This is, of course, not a realistic assumption, but it results in a MILP with less variables and constraints that can be solved in considerably less time. Full accuracy is lost because the power at every second is no longer restricted to be non-negative.

In order to achieve this, we define, for each leg $i \in \mathbb{L}$, time instant $t \in \mathbb{T}$ and period $p \in \mathbb{P}$, the quantity

$$\pi_{pt}^i = \frac{1}{2} \sum_{s=f_p}^{\ell_p-1} (\pi_{st}^i + \pi_{s+1t}^i)$$

²By *power usage* we mean either consumption ($\pi_{st}^i > 0$), generation ($\pi_{st}^i < 0$), or that the leg simply does not cover second s if it departs at time t ($\pi_{st}^i = 0$).

i.e. the total power contribution of leg i in period p , given that i departs at time t . Note, that π_{st}^i and π_{pt}^i are not variables but coefficients in the model.

Now constraint sets (4) and (5) may be removed and replaced by

$$\sum_{i \in \mathbb{L}} \sum_{t \in \mathbb{T}_i} x_t^i \pi_{pt}^i \leq \Pi \quad p \in \mathbb{P}. \quad (6)$$

Note that the solution given by the MILP in this case will both give a lower bound to the optimal solution of the original MILP, as well as a heuristic solution to the problem.

4.2 Rolling horizon matheuristic

In this section we propose an iterative matheuristic, which, during each iteration, attempts to improve the incumbent solution by focussing on the period with the highest average power consumption in the incumbent, henceforth referred to as the *focus period*. The matheuristic considers a restricted time window that includes the focus period together with a certain number of periods around it (governed by an input parameter), and solves the EETTP inside this window using a local search heuristic. The local search heuristic itself uses the MILP to make small changes to the incumbent solution as long it improves the objective function value. This MILP is a restricted version of the presented MILP in Section 3, where a selected set of departures (around the focus period) are only allowed to be slightly shifted in time with respect to the incumbent (*i.e.* the departure windows are trimmed).

In order to impose a rolling horizon, we restrict the considered time horizon to some window $[T_{\min}, T_{\max}] \subseteq \mathbb{T}$, and only allow legs with departure windows completely inside this window to be altered, while all other departures stay fixed. The set of fixed legs are then given by

$$\hat{\mathbb{L}} = \{i \in \mathbb{L} : \mathbb{T}_i \cap [T_{\min}, T_{\max}] \subset \mathbb{T}_i\},$$

and we impose the additional constraints

$$x_t^i = \hat{x}_t^i, \quad i \in \hat{\mathbb{L}}, \quad (7)$$

given a solution $\hat{\mathbf{x}} = [\hat{x}_t^i]$ to (1)–(2). In other words, all departure decisions outside the considered horizon are fixed to match the incumbent.

In order to trim the departure windows according to $\hat{\mathbf{x}}$ for the purpose of applying the local search heuristic, we impose the constraints

$$\sum_{t \in \mathbb{T}_i(\hat{\mathbf{x}})} x_t^i = 1, \quad i \in \mathbb{L}, \quad (8)$$

where

$$\mathbb{T}_i(\hat{\mathbf{x}}) = \left\{ t \in \mathbb{T}_i : \left| \sum_{t' \in \mathbb{T}_i} t' \hat{x}_{t'}^i - t \right| \leq \tau \right\},$$

for some parameter τ . In other words, departure times in the new solution can differ in at most τ minutes from the corresponding times in the incumbent.

The matheuristic is formally described in Algorithm 1. The algorithm starts from an initial solution³ $\hat{\mathbf{x}}$ and initial restricted time window of 1, which indicates the number of periods on either side of the focus period to include in the rolling horizon. The next rolling horizon window is determined in Steps 5–7 and the local search is performed in Steps 8–11. During each iteration the local search solves the MILP including the constraints for the rolling horizon as well as for trimming the departure windows. It continues as long as it can find improved solutions for the

³The provided instances all came with initial feasible solutions, and we used these solutions as starting points for the matheuristic.

current horizon. The stretch stays fixed as long as the local search can find improved solutions, after which the stretch is increased and the process repeated. The entire process is repeated until the stretch reaches a maximum value, and then the matheuristic terminates.

In what follows we will use the notation $T\tau A\delta$ to denote the specific configuration of the rolling horizon matheuristic where the parameter τ has the same meaning as above and where the horizon $[T_{\min}, T_{\max}]$ covers δ periods completely, with the focus period in the middle.

Algorithm 1 Rolling horizon matheuristic

Input: Instance of the EETTP, trim size τ , max window stretch n_{\max}
Output: A solution \hat{x} to the instance

- 1: Generate initial solution \hat{x}
- 2: $n \leftarrow 1$
- 3: **while** $n \leq n_{\max}$ **do**
- 4: **while** improvement found **do**
- 5: $p^* \leftarrow$ period with the highest average power consumption
- 6: $T_{\min} \leftarrow T_{p^*} - 900n$
- 7: $T_{\max} \leftarrow T_{p^*} + 900n$
- 8: **while** improvement found **do**
- 9: $\tilde{x} \leftarrow$ minimize Π subject to (1)–(8)
- 10: **if** $\Pi_{\tilde{x}} < \Pi_{\hat{x}}$ **then**
- 11: $\hat{x} \leftarrow \tilde{x}$
- 12: $n \leftarrow n + 1$

5 Results

The complete MILP model and a set of matheuristic have been tested using the provided instances, and in this section we report the results. To our surprise the complete MILP model appears to be quite solvable, and produces superior results. The heuristic approaches can be faster but never better. We show the resulting solution quality that can be achieved using heuristics. Note that the MILP used in the implementation of the matheuristics is programmed to be as efficient as possible; it does not contain variables with fixed values nor constraints without variables.

The best results (which also are the submitted results) were produced by solving the complete MILP model using CPLEX 12.6. An overview of the statistics are shown in Table 1. In all cases, except the first, we were able to obtain solution with a proven gap less than 0.5%. In 8 of the 11 cases we have result that are prove to less than 0.0001%, thereafter the search terminates. All produced results are obtained within a maximal runtime of 24 hours. Instance 7 and 10 run out of memory (24GB) before 24 hours are reached, and the solutions for both instances are therefore obtained in less time. In contrast to our expectations, the first instance proves to be hard to solve. We believe that the strength of the MILP model is largely due to preprocessing and constraint propagation; experimental runs with finding the LP relaxation turns out to be more time-consuming than running the MILP formulation (Table 3)

Granted, it is hard to improve upon the result produce by solving the complete MILP model. However, shorter runtimes are in some cases important. Furthermore, in face of even larger instances, the MILP may become impractical. Four different heuristic variants are chosen, the results are shown in Table 2. The first two methods are *T1A5* and *T3A9* (using the notation introduced in Section 4.2). The third heuristic is the one presented in Section 4.1 that uses a heuristic approximation of the objective function, while the forth method is the full MILP together with a time limit of 15 minutes. We observe that the last two methods produce the best overall results, and the amount of time to solve the problem is relatively low. The results

Instance	Cost	Gap	Runtime	Variables	Constraints	Nodes
1	1.09	15.61%	86 410	16 737	16 763	117 817
2	2.94	0.00%	2 681	16 981	16 959	1 833
3	15.18	0.00%	435	17 748	17 835	6 271
4	14.97	0.00%	1 361	20 008	20 795	23 421
5	19.42	0.00%	1 499	21 306	21 786	12 870
6	20.43	0.00%	5 687	20 265	20 574	138 556
7	16.32	0.11%	28 825	25 927	29 366	62 280
8	2.42	0.00%	7 092	41 163	114 433	8 996
9	98.14	0.00%	18 003	27 907	30 413	61 451
10	68.12	0.01%	43 220	33 702	38 849	127 267
259.03						

Table 1: Overview of best obtained results

of the other approaches show that the methods are able to produce solutions within short time using the heuristic described. To our surprise, the matheuristics presented in Section 4 did not prove to be more effective than the MILP in section 3, even with if used with a heuristic objective.

Instance	T1A5		T3A9		Heuristic Objective		MILP 15min	
	Cost	Runtime	Cost	Runtime	Cost	Runtime	Cost	Runtime
1	1.20	53	1.17	876	1.46	6	1.17	901
2	3.44	49	3.17	169	3.77	1	2.99	901
3	19.50	29	17.92	131	15.29	2	15.18	366
4	19.00	30	17.51	397	15.33	3	14.97	901
5	25.84	65	23.82	405	20.06	4	19.42	901
6	25.81	43	22.92	673	20.92	49	20.45	902
7	20.53	109	16.47	5 781	18.23	492	16.86	902
8	2.68	18	2.67	62	2.71	29	2.61	902
9	107.98	188	101.81	5 298	98.99	63	98.26	904
10	79.25	17	68.61	11 000	68.14	581	68.72	903
305.23		601.00	276.08	24 791	264.90	1 230	260.64	8 484

Table 2: Overview of four different heuristic solution methods.

6 Conclusion

We have presented several solution methods for solving the EETTP. One exact and several heuristic approaches have been proposed and benchmarked.

The exact method, based on a MILP model, performs very well. The best found solutions are obtained with this method. Even if a short time limit is given, *e.g.* 15 minutes per instance, the exact method provides superior solutions. Using a heuristic objective good results can be obtained faster. The matheuristic approaches show a general lack of strength. The solution quality is poor compared to the MILP model with a 15 minute time limit, and given more flexibility the runtime becomes too high.

To our surprise, the exact MILP model approach is found to be superior to the other considered approaches. In addition to the high quality solution obtained, this exact method is also

Instance	Objective	Runtime	Variables	Constraints
1	735.34	143	16 737	16 844
2	2 640.36	30	16 981	17 069
3	13 600.85	195	17 748	18 110
4	13 411.34	228	20 008	21 830
5	17 350.00	259	21 306	22 640
6	18 317.91	141	20 265	21 284
7	14 607.51	2 013	25 927	33 507
8	2 032.33	97 767	41 163	188 728
9	88 100.69	2 163	27 907	34 276
10	61 044.77	2 934	33 702	45 560

Table 3: Results of solving the LP relaxations, using both (2) and (3) in the MILP.

able to provide optimality bounds. The quality of any solutions found by heuristic approaches will always be unproven without good bounds.

7 Future research

As mentioned in the introduction, the precedence constraints considered in this report is similar to precedence constraints in project scheduling problems, and can be modelled using similar techniques. Artigues *et al.* [3] present a large variety of MILP models for the *resource constrained project scheduling problem*, where they refer to the time-indexed variables that we have used in our model as “pulse” start variables. They presented a number of different time-indexed variables — such as whether or not a specific activity starts before a specific point in time (start “step” variables), or whether or not a specific activity is in progress at a specific point in time (on/off variables) — and these ideas could lead to a number of alternative formulations for the EETTP. As future research we propose further investigations into alternative MILP approaches inspired by these ideas.

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