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# COMPUTER ANALYSIS AND OPTIMAL DESIGN OF MACHINE ELEMENTS

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## 1 INTRODUCTION

The subject machine elements is traditional a research field where standards have a significant influence on the design, i.e. the keyway in a shaft has a given shape and size dictated by a standard. This is important for the interaction between different elements produced at different locations, other examples of standardized components are bolts and gears. The standard designs are typically very good, many years of experience have gone into the design process. The design consists in most cases of straight lines and circular arches, not including e.g. the involute shape of gears. The standard cutting rack for the involute gear is however made from straight lines and circular arches.

The optimization objective is strength and the focus is on fatigue. The designs are improved by lowering the stress concentrations that controls the strength. In the lecture the following components are discussed; bolts-plate connections, shaft-hub connections and gears. Overall the different components are improved by shape optimization using the finite element method for analysis. The optimality criteria states that the designs are optimal when the stress along the boundary is constant if possible.

For bolts it is shown how design changes, that does not influence the function, can greatly improve the strength. The function is unchanged because even though the thread design is modified it can still be combined with the standard design. For gears the contacting surface has the involute shape and should not be changed, instead the design changes are made to the gear root where there is no contact and therefore total design freedom. The final machine elements that are presented are shaft-hub connections, i.e. keyways and splines, also for these elements design changes that does not influence the function are shown to have a large impact on the fatigue strength.

For machine elements it is important that all design changes made by shape optimization are practical, i.e. that the shape can be communicated to the users and subsequent produced. It is therefore essential that the parametrization of the shape to be designed is simple. In most of the cases presented a super elliptical parametrization is used, sometimes in the standard form but typically also distorted such that slope constraints at the end point of the design domain can be meet.

## 2 BOLTS

The most common design of bolts and nuts are controlled by the ISO metric thread design. The strength of the connection is controlled by many factors. The overall goal, as presented in<sup>1</sup> and<sup>2</sup>, is to optimized the state of stress in bolts and nuts. Assuming that the failure mode is fatigue then the strength is primarily controlled by the preload and the position and size of the external load together with the stress concentrations. The maximum stress in bolts are found at three points; at the fillet under the head, at the thread start, or at the thread root.

In the two papers shape optimization is applied to; the thread root, the nut, or to the transition from bolt shank to the thread. Changing only the shape of the thread root leads typically to a reduction of the maximum stress by 7% to 8% see<sup>1</sup>. A larger reduction is found if we optimize the nut face and the transition from bolt shank to the thread. This is performed in<sup>2</sup> where stress reduction of up to 34% is found, still with the standard ISO thread.

The design changes suggested in the papers has in some cases also the positive effect of reducing the joint stiffness factor, leading to an even stronger design. In Figure 1 the normalized stress of part of an axisymmetric finite element model is shown, it should be noted that the maximum stress is found simultaneously at three points; at the thread root, in the shank to thread transition, or in the cut in nut.

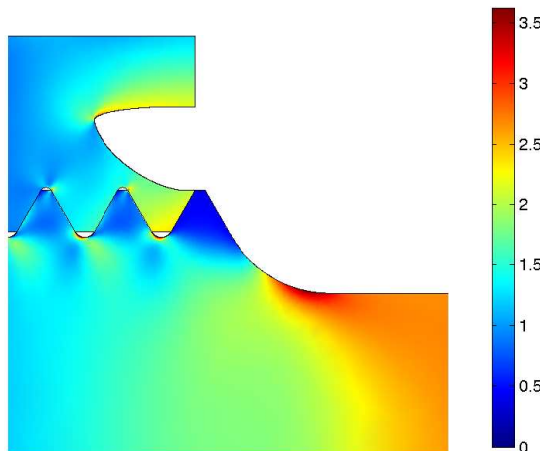


Figure 1: Zoom of the first 2 thread roots and the shank fillet of optimized design. Normalized von Mises stress surface colour plot, i.e. indicating the stress concentration factor. Taken from<sup>2</sup>.

## 3 GEARS

For gears the strength is typically defined relative to pitting or tooth-breakage. Pitting is related to the contact between the gears, and if we keep the involute shape the only way to reduce pitting is to increase the contact angle, this can be done using e.g. asymmetric gears<sup>3</sup>. For the tooth-breakage the strength is controlled by the stress concentration at the root, there is no contact between the gears here, so we can

design the shape to minimize the maximum stress.

The shape of gears is defined by standards (ISO standard), and the manufacturing is done by two types of tools; a rack tool, or a gear tool. In Figure 2 the stress along the root of a gear tooth is shown for the ISO tooth and the shape optimized tooth. It should be noted for the optimized tooth root how the stress is constant over a large part of the root, indicating that the design is optimized.

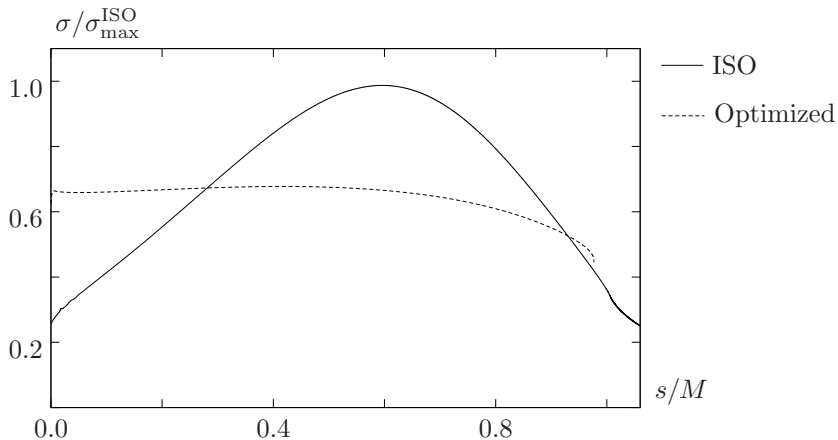


Figure 2: Von Mises stress at the root of the tooth for a gear with 17 teeth. Stresses are normalized with the maximum stress for the ISO tooth and the arc length is normalized by the module. Taken from<sup>4</sup>.

In<sup>4</sup> it was shown how the shape optimization can be made directly on the final gear. The optimized shape can then be used to find the cutting tool (from the gear envelope) that can create this optimized gear shape. It is important to note that the optimization is performed so that the original gear involute shape is unchanged.

#### 4 SPLINES AND KEYWAYS

If shaft-hub connection should have the possibility of disassembly then either keyways or spline are typically used, spline are applied when larger torque capacity is needed. The design of these elements are given in different standards, and very few papers deals with improving the design of splines/keyways, although they are commonly used in practical design. Optimization of keyways or splines can be found in e.g.<sup>5</sup> and<sup>6</sup>.

In<sup>5</sup> a new standard is proposed for a wide range of shaft diameters. The improvement of the design using this new standard is of course slightly smaller than if the shape optimization is performed specifically on a given shaft size. The stress concentration factor for the new standard relative to the original design is presented in Figure 3.

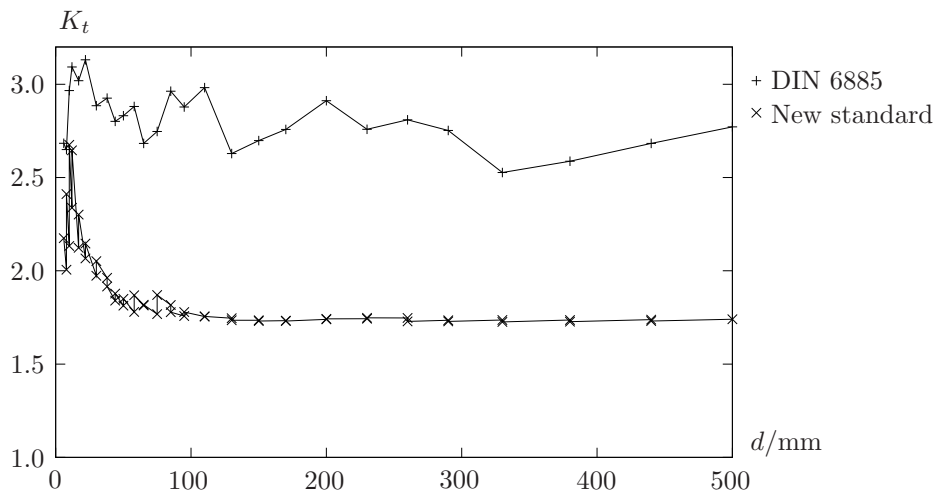


Figure 3: The stress concentration factor for the prismatic part of a keyway in pure torsion as a function of the diameter. Taken from<sup>5</sup>.

## 5 CONCLUSION

The findings of the referenced papers and references therein are that it is possible to improve many of the designs for standard machine elements. In doing the optimization of strength it is important that the focus is on simplicity. The parametrization of the shape has in all cases been made with very few design parameters, typically less than five. The designs can in many cases be improved further if a more involved parametrization is used however this will come at a cost of less simplicity. In all cases the degree to which the design is optimal can be controlled by how constant the stress is along the designed boundary.

## REFERENCES

- [1] Pedersen, N. L. Optimization of bolt thread stress concentrations. *Archive of Applied Mechanics* **83**, 1–14 (2013).
- [2] Pedersen, N. L. Overall bolt stress optimization. *Journal of Strain Analysis for Engineering Design* **48**, 155–165 (2013).
- [3] Pedersen, N. L. Improving bending stress in spur gears using asymmetric gears and shape optimization. *Mechanism and Machine Theory* **45**, 1707–1720 (2010).
- [4] Pedersen, N. L. Minimizing tooth bending stress in spur gears with simplified shapes of fillet and tool shape determination. *Engineering optimization* **47**, 805–824 (2015).
- [5] Pedersen, N. L. Stress concentrations in keyways and optimization of keyway design. *Journal of Strain Analysis for Engineering Design* **45**, 593–604 (2010).
- [6] Pedersen, N. L. Optimization of straight-sided spline design. *Archive of Applied Mechanics* **81**, 1393–1407 (2011).