Meta-Logical Reasoning in Higher-Order Logic

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Abstract

The semantics of first-order logic (FOL) can be described in the meta-language of higher-order logic (HOL). Using HOL one can prove key properties of FOL such as soundness and completeness. Furthermore, one can prove sentences in FOL using the formalized FOL semantics. To aid in the construction of the proof an interactive proof assistant like Isabelle can be used. The proof assistant can even automate simple proofs using the formalized FOL semantics.

Introduction

In textbooks the language of first-order logic (FOL) is usually presented in English. FOL is the object language, since it is the logic that is described, and English is the meta-language, since it is the language that describes the object language. However, instead of using English as meta-language we can also use a meta-logic. We will show how higher-order logic (HOL) can be used as a meta-logic to describe and reason about FOL, which is possible as HOL is much more expressive than FOL, cf. Farmer (2008).

Logic itself is about formalizing which arguments are valid. Thus in FOL we have a clear definition of which theorems are valid and which are not. However, it is also interesting to prove theorems about FOL, for instance the soundness and completeness of a proof system for FOL. By using HOL as meta-language we can ensure that there is also a clear definition of which theorems about FOL are valid, cf. Harrison (1998). Furthermore, we can show the theorems about FOL to be valid by proving them in a sound proof system for HOL.

Proof systems for HOL have been implemented in interactive proof assistants which are computer programs that can help their users in proving theorems. In the following we will use the Isabelle proof assistant, cf. Nipkow (2002). In addition to helping the users to construct correct proofs, the proof assistants can in some cases even do the proofs automatically.

Formalization in Isabelle

We consider a formalization in Isabelle of FOL with only binary predicates:

theory Semantics imports Main begin

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