Strength and Biot's coefficient for high-porosity oil- or water-saturated chalk

Andreassen, Katrine Alling

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Strength and Biot's coefficient for high-porosity oil- or water-saturated chalk

Andreassen, K.A.

Technical University of Denmark, Kgs. Lyngby, Denmark

ABSTRACT: In the petroleum industry it is relevant to know the Biot coefficient for establishing the effective stresses present in both the overburden and for the reservoir interval. When depleting a reservoir it is important to estimate the settlement through the strain imposed by the effective stress. Also considerations for the size of the drilling window and the magnitude of the lateral stress involve the Biot coefficient. Additionally, the fluid effect of oil-saturated chalk behaving much stronger than water-saturated chalk affects geomechanical considerations related to e.g. water injection into a reservoir.

The Biot coefficient states the degree of cementation or how the pore pressure contributes to the strain resulting from an external load for a porous material. It is here calculated from dynamic measurements and correlated with the strength of outcrop chalk characterized by the onset of pore collapse during hydrostatic loading. The hypothesis is that the Biot coefficient and the theory of poroelasticity may cover the fluid effect by including the increased fluid bulk modulus from oil to water. A high number of test results for both oil- and water-saturated high-porosity outcrop chalk show correlation between the Biot coefficient and the strength.

1. INTRODUCTION

In the petroleum industry it is relevant to know the Biot coefficient for establishing the effective stresses present in both the overburden and for the reservoir interval. When depleting a reservoir it is important to estimate the settlement through the strain imposed by the effective stress. Also considerations for the size of the drilling window and the magnitude of the lateral stress involve the Biot coefficient. Additionally, the fluid effect of oil-saturated chalk behaving much stronger than water-saturated chalk affects geomechanical considerations related to e.g. water injection into a reservoir [1].

The dataset used here is from the Pasachalk project [2, 3]. It includes a large number of tests on outcrop chalk in oil or water saturated state with different loading paths. The chalk is Lixhe chalk from a quarry near Liège, Belgium, with a general porosity of 45%. A small number of the tests involve reloading branches. The applied loading rate ranges from 0.001 MPa/s to 0.004 MPa/s, with the majority performed at 0.001 MPa/s. Any effect on strength from a changing loading rate is considered as a second order effect and will not be dealt further with here. The average saturation is 0.98 and is thus close to full saturation.

Data from the Pasachalk project for both oil- and water-saturated chalk is given in Figure 1 with the onset of pore collapse, often termed yield strength, which marks the transition from elastic behaviour to plastic behaviour. The fluid effect with water weakening of the chalk is apparent as the data plot in two regions.

The porosity is observed to correlate with the stiffness (Engstrøm, 1992) and Figure 1 displays this for the present data set only within the oil- and water-saturated specimens. There do not seem to be a unified correlation between the porosity and onset of pore collapse.
In the following, I investigate the relation between the stiffness characterized by Biot’s coefficient and the theory of poroelasticity may cover the fluid effect by including the increased fluid bulk modulus from oil to water.

2. METHOD

The theory of poroelasticity can be stated as (Biot, 1941; Zimmerman, 2000a):

\[
\begin{align*}
\sigma_{xx} &= 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \alpha P_p, \\
\sigma_{yy} &= 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \alpha P_p, \\
\sigma_{zz} &= 2G\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \alpha P_p, \\
\sigma_{xy} &= 2G\varepsilon_{xy}, \quad \sigma_{xz} = 2G\varepsilon_{xz}, \quad \sigma_{yz} = 2G\varepsilon_{yz},
\end{align*}
\]

with stresses and strains positive for compression. Here \(\varepsilon_{ij}\) denotes the principal strain, \(\sigma_{ij}\) denotes the shear strain, \(\sigma_{ii}\) denotes the principal stress, \(G\) is the shear modulus, \(\lambda\) is Lamé’s parameter, \(P_p\) is the pore pressure and \(\alpha\) is Biot’s coefficient. For hydrostatic stress Biot’s coefficient is traditionally [1] 

\[ \alpha = 1 - \frac{K_{dry}}{K_{min}}, \quad (2) \]

although it may equally well be determined as proven by Alam et al. [2] by 

\[ \alpha = 1 - \frac{M_{dry}}{M_{min}}, \quad (3) \]

which is addressing the mineral p-wave modulus \(M_{min}\), relative to the drained p-wave modulus, \(M_{dry}\).

\[
M = \rho V_p^2, \quad (4)
\]

In this paper I will compare the results obtained by using Eq. (2) and (3) for the Biot coefficient.

From elastic wave propagation I determine the dynamic Biot coefficient by use of the isoframe model (Fabricius, 2003 – evt. Fabricius et al., 2007; 2010) as the Pasachalk data set contains dry and saturated p-wave measurements but not s-wave measurements and thus does not facilitate direct determination of Biot’s coefficient. The isoframe model is extended from upper Hashin-Shtrikman bounds and the isoframe values show how large part of the solid constitutes the load bearing skeleton,

\[
IF = \frac{(K_{min}+\frac{4}{3}G_{min})K_{fl}(K_{min}-K_{sat})+(K_{min}+\frac{4}{3}G_{min})\rho K_{sat}(K_{fl}-K_{min})}{(K_{sat}+\frac{4}{3}G_{min})\rho K_{min}(K_{min}-K_{sat})+(K_{min}+\frac{4}{3}G_{min})K_{fl}(K_{min}-K_{sat})} (1-\varphi) \]

with \(K_{min}\) the mineral bulk modulus, \(G_{min}\) the mineral shear modulus, \(K_{fl}\) the fluid bulk modulus, \(K_{sat}\) the saturated bulk modulus, and \(\varphi\) the porosity. This model is similar to a mixture of load bearing hollow spheres and suspended solid spheres. For a homogenous Reuss model the bulk modulus of the suspended particles is

\[
K_{sus} = \left(\frac{(1-\varphi)(1-IF)}{K_{min}} + \frac{\varphi}{K_{fl}}\right)^{-1}, \quad (6)
\]

then the Modified Upper Hashin–Shtrikman bound for the bulk modulus gives

\[
K_{HS+} = \frac{\varphi}{K_{sus}+\frac{4}{3}G_{sus}} + (1-\varphi)(1-IF) - \frac{4}{3}G_{min}, \quad (7)
\]

together with the shear modulus

\[
G_{HS+} = \left(\frac{\varphi}{\xi} + (1-\varphi)(1-IF) - \frac{(1-\varphi)IF}{G_{min}+\xi}\right)^{-1} - \xi, \quad (8)
\]

\[
\xi = \frac{G_{min}}{6} \left(\frac{9K_{min}+6G_{min}}{K_{min}+2G_{min}}\right). \quad (9)
\]

As the p-wave modulus is defined as

\[
M_{HS+} = K_{HS+} + \frac{4}{3}G_{HS+}. \quad (10)
\]

It follows that from equaling \(M_{HS+}\) to the measured p-wave modulus, \(M\), and fitting IF in the equations above the result is the Biot coefficient from the calculated dry bulk modulus

\[
K_{dry} = \frac{4IFK_{min}G_{min}(1-\varphi)}{(K_{min}+4G_{min})} - \frac{3IFK_{min}(1-\varphi)}{3}. \quad (11)
\]

From this the Biot coefficient is determined in Eq. (2). The mineral bulk modulus chosen for calcite is 70.8 GPa and the shear modulus is 30.3 GPa (Mavko et al., 2009). The fluid bulk modulus for water at 20ºC is 2.1913 GPa and for the oil (Soltril) it is 1.2919 GPa.
3. RESULTS

Table 1 and 2 give an overview of the dataset, which includes the dry bulk modulus, \( \rho_{\text{dry}} \), the saturated bulk modulus, \( \rho_{\text{bulk, sat}} \), the porosity, \( \phi \), the onset of pore collapse, \( \sigma_{\text{yield}} \), and the calculated Biot’s coefficient based on the procedure described above. The yield strength is determined as the first deviation from the linear elastic behaviour.

Table 1. Results for water-saturated Liège chalk. The Biot coefficient is calculated from the saturated state using IF modelling.

<table>
<thead>
<tr>
<th>No.</th>
<th>( \rho_{\text{dry}} ) kg/m(^3)</th>
<th>( \rho_{\text{bulk, sat}} ) kg/m(^3)</th>
<th>( V_{p,\text{dry}} ) m/s</th>
<th>( V_{p,\text{sat}} ) m/s</th>
<th>( \phi )</th>
<th>( \sigma_{\text{yield}} ) MPa</th>
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<td>5.8</td>
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<td>0.442</td>
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</table>

The water-saturated specimens have a general yield strength of 7.7 MPa and there is a correspondence between the p-wave velocity, porosity and the calculated Biot’s coefficient.

Table 2. Results for oil-saturated Liège chalk. The Biot coefficient is calculated from the saturated state using IF modelling.

<table>
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<tr>
<th>No.</th>
<th>( \rho_{\text{dry}} ) kg/m(^3)</th>
<th>( \rho_{\text{bulk, sat}} ) kg/m(^3)</th>
<th>( V_{p,\text{dry}} ) m/s</th>
<th>( V_{p,\text{sat}} ) m/s</th>
<th>( \phi )</th>
<th>( \sigma_{\text{yield}} ) MPa</th>
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<td>13.9</td>
</tr>
</tbody>
</table>

The oil-saturated specimens show a general yield strength of 18.3 MPa and here the correspondence between the p-wave velocity, porosity and the calculated Biot’s coefficient is also clear.

Figure 2 and 3 give a graphical overview the measured sound velocities.

![Fig. 2. Porosity and dry sound velocity data.](image-url)
Figures 2 to 4 illustrate the results from Table 1 and 2. Engstrøm (1992).

The porosity and Biot’s coefficient show a good correlation independent of fluid saturation. A higher porosity correlates with a higher Biot coefficient as expected.

As observed before by e.g. the porosity is correlating with the stiffness and thus also with the cementation and strength. Comparison of the data in Figure 2 with the trend found by Fabricius (2007) for a large number of North Sea chalk samples show the results from the Pasachalk dataset to lie lower than the reservoir data, see Figure 3.

This indicates that the Liège chalk behaves stiffer than expected.

From Figure 4 it is apparent that there is a correlation between the strength and Biot’s coefficient. As a consequence, lower Biot coefficient predicts higher strength as expected.

The fluid effect is found not to be covered by the Biot coefficient result through to the influence of the difference in fluid bulk modulus and bulk density of the specimens as predicted by using poroelastic theory. The scope of this could be verified by including results from chalk specimens with lower Biot’s coefficient, for reservoir chalk and for other pore fluids.

4. CONCLUSION

By use of the isoframe model the Biot coefficient is calculated from dynamic measurements for both oil- and water-saturated Liège outcrop chalk tested under hydrostatic stress.

The onset of pore collapse and the Biot coefficient calculated from dynamic measurements of p-wave velocity yields a correlation. The correlation between Biot’s coefficient and pore collapse is found not to include the fluid effect.

Verification of the correlation should be extended with data for materials with lower Biot’s coefficient and should also include reservoir rock or other pore fluids.

5. ACKNOWLEDGMENTS

I would like to thank Ida Fabricius for introducing the isoframe model to me. Also a thank goes to all the people involved in the Pasachalk project part 1 and 2 for producing the large amount of high quality laboratory data and giving access to it.

REFERENCES

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