DTU Informatics
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Uncertainty Quantification on High-speed Railway Dynamics

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Introduction

Modern developments in railway vehicle dynamics are boosting the idea of running faster trains with improved safety. Important contributions to these developments are being given by precise numerical approximations. The Newton-Euler formulation of vehicle models always contain nonliearities due to the wheel-rail contact forces and components in the suspension system. These make the solution of these systems computationally challenging [4].
On the other hand, the nonlinear terms in these systems have a critical influence on the dynamics and cannot be neglected. Model parameters of the vehicle as well as track irregularities are often known within a certain degree of accuracy due to measurement errors. It is then fundamental to try and detect the parameters that can be critical for the riding safety. The goal of this work is to employ modern uncertainty quantification techniques (e.g. generalized Polynomial Chaos) in order to obtain estimates of the sensitivity of the dynamics to such parameters which can then be taken into account in vehicle designs.

The model

The study focuses first on a simplified model [3] (see fig. 1) formed by a wheel set connected to the car body by a spring, with stiffness K_s, and a dry friction damper, with static and dynamic friction coefficients v and μ. The model runs on an irregular straight track, where the irregularities are described by a sinusoidal function. The investigation is restricted to the lateral (x_1) and the yaw (x_3) dynamics, that can be expressed by the nonlinear system in (1).
A complete realistic railway wagon model [1] will be used for more complex investigations. The model considers 35 degrees of freedom (66 nonlinear ODEs).

$$\begin{cases} \dot{x}_1 = \tilde{x}_2 \\ \dot{x}_2 = \frac{F_{in}+F_d}{m} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{-2AF_y}{I} \end{cases} \qquad (1)$$

$$\tilde{x}_1 = x_1 + a\sin\left(\frac{2\pi V}{l}t\right), \quad \tilde{x}_2 = x_2 + a\frac{2\pi V}{l}\cos\left(\frac{2\pi V}{l}t\right)$$

$$F_{in} = -2F_x - 2K_s\tilde{x}_1, \quad F_{sl} = -\mathrm{sign}\,(\tilde{x}_2)\mu_d N$$

$$F_{st} = \begin{cases} -\mathrm{sign}\,(F_{in})\mu_s N & |F_{in}| > \mu_s N \\ -F_{in} & |F_{in}| \leq \mu_s N \end{cases}$$

$$F_d = F_{st}\mathrm{sech}\,(\alpha\tilde{x}_2) + F_{sl}(1 - \mathrm{sech}\,(\alpha\tilde{x}_2))$$

$$F_x = \frac{\xi_x F_r}{\Psi\xi_r}, \quad F_y = \frac{\xi_y F_r}{\Phi\xi_r}, \quad F_r = \begin{cases} \xi_r C\left(1 - \frac{C\xi_r}{3\mu_t} + \frac{C^2\xi_r^2}{27\mu_t^2}\right) & \text{if } C\xi_r < 3\mu_t \\ \mu_t & \text{otherwise} \end{cases}$$

$$\xi_x = \frac{\tilde{x}_2}{V} - x_3, \quad \xi_y = \frac{Ax_4}{V} + \frac{\lambda\tilde{x}_1}{r_0}, \quad \xi_r = \sqrt{\left(\frac{\xi_x}{\Psi}\right)^2 + \left(\frac{\xi_y}{\Phi}\right)^2}$$

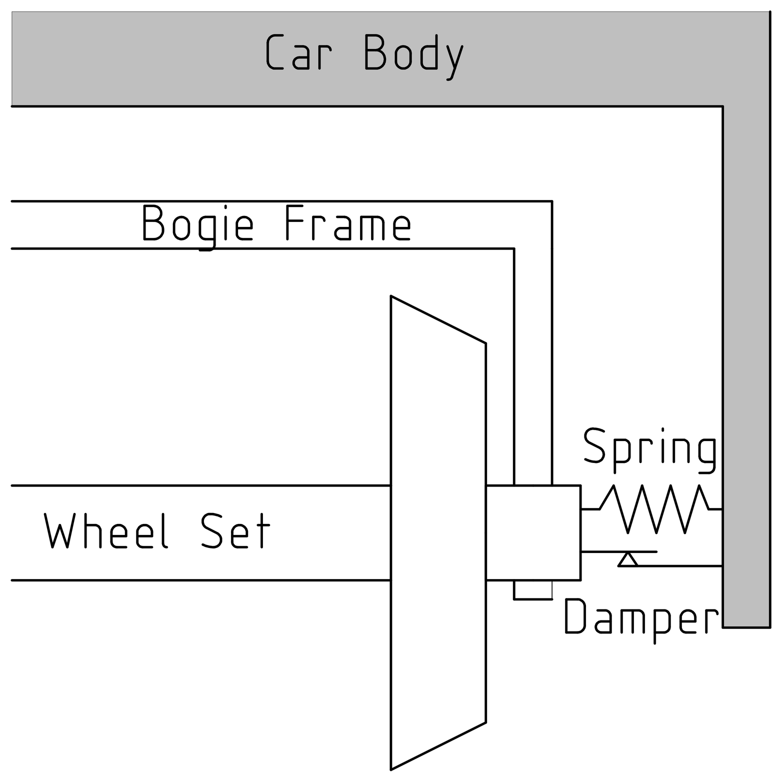
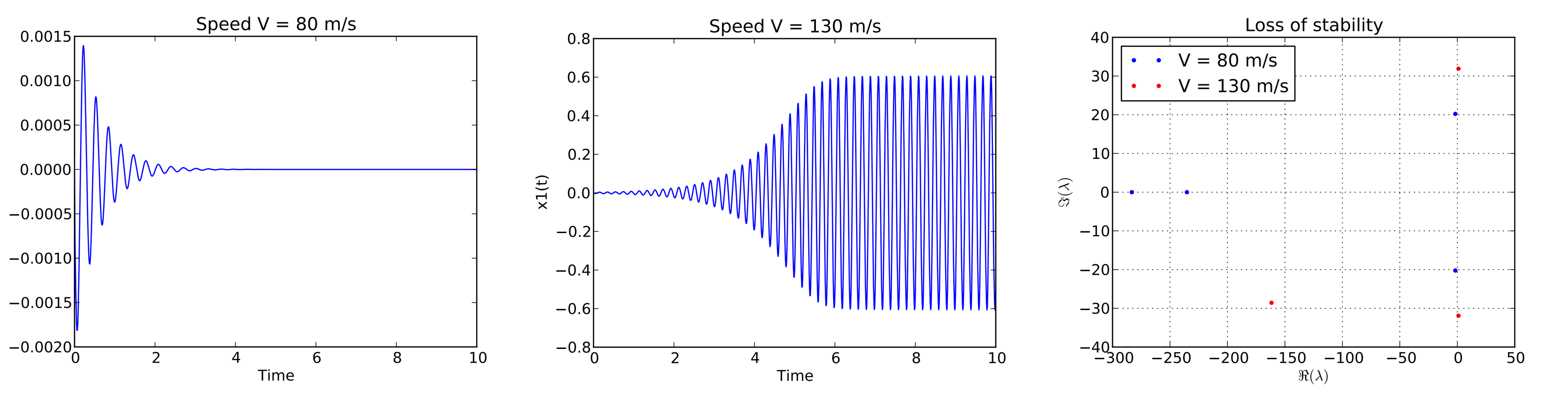
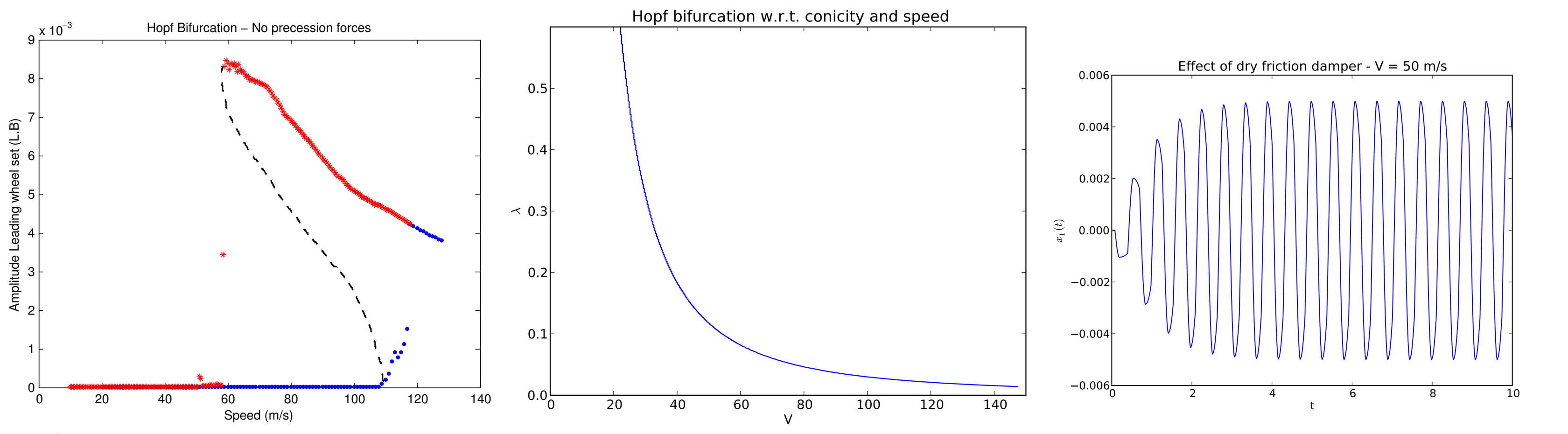


Figure 1: Simplified model.



(a) Stable solution (b) Hunting (c) Migration of the eigenvalues of the Jacobian matrix

Figure 2: Loss of stability of the centered solution above the linear critical speed (Hopf bifurcation) on the simplified model. Here, the dry friction damper and the sinusoidal irregularities of the track are disregarded.



(a) Subcritical Hopf Bifurcation of a complete vehicle on straight track. (b) Linear critical speed with respect to the conicity of the wheel set. (c) Destabilizing effect due to the dry friction damper.

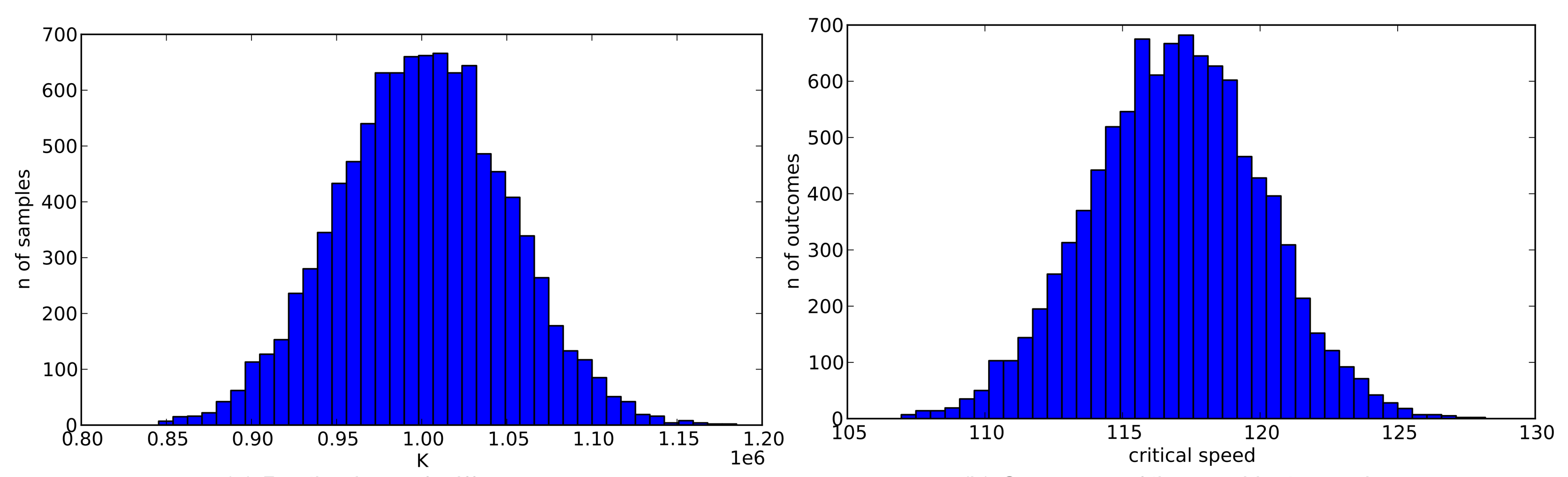
Figure 3: Nonlinear dynamics effects

Nonlinear Dynamics

Railway vehicles are known to develop lateral oscillations, even on a straight smooth track, when running at high speeds [2]. This is due to the loss of stability of the centered solution. This can result in the starting of periodic motion through a super/subcritical Hopf Bifurcation (see fig. 2) and the development of chaotic motion.
Figure 3a shows that it is not sufficient to identify the speed at which the Hopf bifurcation takes place, but also to determine the shape of the hysteresis that can be present. The linear critical speed can be identified as the velocity at which the centered solution loses stability. The nonlinear critical speed is instead identified as the speed above which the system can have multiple solutions.
The identification of bifurcation points and hysteresis is a computationally expensive task, due to the presence of non-smoothnesses in the system that slow down the convergence of conventional ODE solvers.
The dynamics of the system can change drastically depending on the values of its parameters. Figure 3b shows an example of such aspect when the conicity of the wheel set is modified.
Also, the presence of dry friction dampers can have destabilizing effects on the dynamics as shown in figure 3c.

Uncertainty Quantification

In the vehicle industry all the parameters of the components of the suspension system are known within certain tolerances, due to measurement errors as well as wear. In the railway industry the irregularities of the rails should be considered as well. In the simple model described by (1) these deformations were described by sinusoidal functions for simplicity. In reality track irregularities have a more complex pattern that can be described by stochastic processes.
The figures in 4 show the Monte Carlo approach in the detection of the linear critical speed with respect to the stiffness value of the lateral spring. The detection of linear critical speed is not computationally expensive and can be done observing the migration of the eigenvalues of the Jacobian as shown in fig. 2c, so the slow convergence of the Monte Carlo approach is acceptable. On the contrary, the nonlinear critical speed can only be obtained calculating all the dynamics with respect to the speed, thus alternative techniques are required for the computation of the statistics.



(a) Realizations of stiffness K_s. (b) Outcomes of linear critical speed.

Figure 4: Detection of the linear critical speed of the simplified model using Monte Carlo method.

References

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