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RESEARCH ARTICLE

Probabilistic stability and “tall” wind profiles: theory and method for use in wind resource assessment

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ABSTRACT

A model has been derived for calculating the effects of stability and the finite height of the planetary boundary layer upon the long-term mean wind profile. A practical implementation of this probabilistic extended similarity-theory model is made, including its incorporation within the European Wind Atlas (EWA) methodology for site-to-site application. Theoretical and practical implications of the EWA methodology are also derived and described, including unprecedented documentation of the theoretical framework encompassing vertical extrapolation, as well as some improvement to the methodology. Results of the modelling are shown for a number of sites, with discussion of the models’ efficacy and the relative improvement shown by the new model, for situations where a user lacks local heat flux information, as well as performance of the new model using measured flux statistics. Further, the uncertainty in vertical extrapolation is characterized for the EWA model contained in standard (i.e. WAsP) wind resource assessment, as well as for the new model.

KEYWORDS

Atmospheric Stability; Monin-Obukhov similarity; Similarity theory; Wind profiles; Wind Extrapolation; Resource Assessment

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1. INTRODUCTION

In order to provide better estimates of wind energy production at heights above the atmospheric surface layer (z > ~50 m), the extrapolation of measured statistics requires a model for the wind profile that is applicable over typical turbine lifetimes.*

Starting with Monin-Obukhov (M-O) similarity theory [1], models for the wind profile that include the effects of atmospheric stability have been accepted and employed for several generations. However, wind profile forms based on M-O theory are not expressly derived for application to wind characterization over the long-term (i.e. years), nor for use above the atmospheric surface layer. Power-law formulations are sometimes used for both 10-minute and long-term use in wind energy (e.g. [2], and implicit in the IEC standard [3]). However, these power-law forms lack any systematic or universal description of the connection or difference between short- and long-term wind profiles, and have only just begun to have useful theoretical or physical connection to geostrophic theory [4] and stability measures [5]. Swift&Dixon [6] did examine power-law exponent variation and connection to the log-law over the ocean, with some consideration of the effect of a z-dependent power law upon the Weibull parameters, but this was focused on the sea-induced speed-dependent roughness and subsequent change in Weibull shape. Two-layer models (which generally patch an Ekman-layer form above to some surface-layer form) have existed for some time (e.g. [7], [8], [9]), but have been generally idealized and not applied in wind energy; however, recently Optis et al. [10] examined the mean performance of the two-layer model of

* Though commercially available LIDAR devices can record wind speeds at these heights, LIDAR’s costs still tend to be prohibitive, especially for longer (e.g. multi-year) measurement campaigns—which more reliably represent the local wind climate and reduce prediction uncertainties. Thus wind profile models are still needed for wind resource assessment, particularly on account of the ever-increasing hub heights (and rotor diameters) used in the wind industry.

The current de-facto standard for extrapolating measured winds for long-term wind energy predictions, contained within the European Wind Atlas (‘EWA’) [14] methodology implemented in software such as ‘WAsP’, does not explicitly specify a wind profile. Instead the EWA method calculates stability-induced deviations from the logarithmic wind profile, by applying perturbation theory to both the M-O form for the wind profile and to the geostrophic drag law. In effect the EWA framework for extrapolating winds includes a local or ‘micro-scale’ component, operating on wind statistics observed at a single location, as well as a larger-scale (geostrophic) part in its modeling. The EWA method uses this prescription for long-term wind variance also, to give a coupled formulation for the extrapolation of both wind speed and Weibull shape parameters—and subsequently wind energy density [15]—affected by geostrophic-scale stability perturbations.

In this paper we adapt and augment the long-term wind profile model of Kelly & Gryning [12] to function within the EWA framework [14]; i.e. we modify the presumably micro-scale profile to account for geostrophic-scale influences. We also show how the probabilistic profile of [12] (or potentially other scalar wind profile forms) can be made consistent with the geostrophic basis of the EWA, for practical modeling of the profiles of wind statistics. This includes elaboration of theoretical details of the EWA methodology as well as some further refinement of the method, outlining both theoretical and practical consequences of its use and adaptation.

2. THEORY, DEVELOPMENT, AND MODELING

We use the assumption that wind speed observations \( U \), when averaged over a period of 10–30 minutes (typical in wind energy) and taken over sufficiently narrow wind sectors (typically \( \leq 30^\circ \)) over one or more years, have a probability distribution function (PDF) described by a two-parameter Weibull form

\[
p(U) = \frac{k}{A} \left( \frac{U}{A} \right)^k \exp \left( -\left( \frac{U}{A} \right)^k \right)
\]

with scale parameter \( A \) and shape parameter \( k \). For a wind speed probability distribution of the Weibull form (1), the \( n \)th moment of the wind speed is given by

\[
\langle U^n \rangle (z) = [A(z)]^n \Gamma \left\{ 1 + n/k(z) \right\}
\]

where angle-brackets denote an average over one or more years, and \( \Gamma \{x\} \) is the Gamma function. So the profile of long-term mean wind speed is \( A(z) \Gamma \{1 + 1/k(z)\} \); its height dependence is predominantly contained in the scale parameter profile \( A(z) \), though a non-negligible contribution from \( k(z) \) can affect the vertical variation of wind power density via profiles of higher moments of wind speed [15, 16].

2.1. Long-term probabilistic wind speed profile

As a statistical extension of Monin-Obukhov similarity theory, [12] derived the profile of long-term dimensionless mean wind speed, based on their universal form for observed dimensionless stability distributions. The latter was derived as a two-sided probability density of inverse Obukhov length \( L^{-1} \) for stable and unstable conditions, which we denote by subscripts “+” (\( L^{-1} > 0 \)) and “−” (\( L^{-1} < 0 \)) respectively:

\[
P(L^{-1}) = \frac{n \pm C_{\pm}}{\sigma_{\pm}} \exp \left[ -\frac{(C_{\pm} L^{-1})/\sigma_{\pm}^{2/3}}{\Gamma[5/2]} \right].
\]

The (inverse) Obukhov length is defined here as \( L^{-1} = -\kappa (g/\theta_0) H_{dc}/u_{*0}^2 \), i.e. through the kinematic flux of virtual temperature \( H_{dc} = u_{*0}^2 \), (i.e. heat flux per mass, accounting for humidity), friction velocity \( u_{*0} \equiv [(\vec{u} \vec{w})^2 + (\vec{v} \vec{w})^2]^{1/4} \), and temperature \( \theta_0 \) in the surface layer,\(^1\) with \( g = 9.8 \text{ m s}^{-2} \) the gravitational acceleration and \( \kappa = 0.4 \) the von Kármán constant. Here \( \{n_+, n_-\} \) are the respective fractions of occurrence of stable and unstable conditions, which together with \( C_{\pm}/\sigma_{\pm} \Gamma^{(5/2)} / 2 \) in (3) ensure that \( \int_{-\infty}^{\infty} P(L^{-1}) dL^{-1} = 1 \). The scale parameters \( \{\sigma_+, \sigma_-\} \) are representative magnitudes

\(^1\)These quantities can be measured using e.g. a three-dimensional (or 2-D) sonic anemometer within the atmospheric surface layer (below 1/10th of the atmospheric boundary layer depth, which can be shallower than 200 m at night), typically taken from a height of 10 m and averaged over 10–30 minutes. Averaging times shorter than 10 minutes should not be used (c.f. [17]).
of variations in $L^{-1}$ observed in the atmospheric surface layer over the course of years (here ‘observed’ connotes time averages generally taken over 10–30 minutes), for each side of the distribution (i.e. stability regime); they are proportional to the width of the $L^{-1}$ distribution for stable and unstable regimes, respectively. The $\sigma_\pm$ are found as in [12] via the long-term standard deviation of the heat fluxes $\{H_+, H_-\}$ and mean friction velocity $\langle u_{*0}\rangle$, for stable and unstable conditions, respectively:

$$\sigma_\pm \equiv \sqrt{\frac{g}{\rho_c T_0}} \langle \left( \frac{H_+ - \langle H_+ \rangle}{L}\right)_\pm^2 \rangle^{1/2} \langle u_{*0}\rangle_{\pm},$$

(4)

where $\langle \rangle_\pm$ denotes an average taken (separately) over either stable or unstable conditions. The stability variability parameters $\sigma_\pm$ can thus be calculated from (4) via observed mean friction velocity and r.m.s. heat flux under stable and unstable conditions, respectively.\(^1\) We note that temperature gradients cannot be relied upon for this purpose, as they do not reliably give universal stability distributions (c.f. [12]); among other issues, temperature signals measured at significantly different heights tend to fall within the surface layer for different proportions of time over the long term, and have statistically different behavior (see e.g. [18]). We also re-iterate the finding of [12] that it is the *widths* of the stable and unstable distributions which moderate the long-term wind profile, thus attempting to define some ‘mean stability’ ($1/\langle L \rangle$ for example) is not *useful* without more information.

[12] further generalized and adapted the ‘tall’ wind profile of [13]—which includes the effect of atmospheric boundary-layer depth $h$ through height-varying friction velocity $u_*(z/h)$ and geostrophic wind speed $G = U|_{z=h}$—to a climatological-mean form usable for wind resource estimates. The distribution (3) facilitated finding such a form for the mean wind profile. Using representative values $h_{\text{eff}}$ and geostrophic wind speed $G_{\text{eff}}$ (as well as a mean roughness $z_{0m}$, which is done implicitly but without acknowledgement in most wind resource estimates), then integrating the product of $P(L^{-1})$ and the wind profile over $L^{-1}$ approximates the ensemble-mean of the “tall” wind profile. Integrating the product of $U/u_{*0}$ and $P(L^{-1})$ over $L^{-1}$ (using a generalized version of [13]’s profile for $U/u_{*0}$) produces [12]

$$\langle kU \rangle (z; z_{0m}, \sigma_\pm, h_{\text{eff}}, G_{\text{eff}}) = \ln \left( \frac{z}{z_{0m}} \right) - \langle \psi \rangle (z) - \frac{z_{0m}}{h_{\text{eff}}} \left[ \psi (z) - \langle \psi \rangle (z) \right] + h_{\text{eff}} \frac{G_{\text{eff}}}{\kappa G} \left[ 1 - \left( 1 - \frac{z}{h_{\text{eff}}} \right)^2 \right] + \frac{z}{s_{\text{eff}}} \left( s_{\text{eff}} - 1 \right)$$

(5)

where $u_{*0}$ is the surface-layer friction velocity, $\langle \psi \rangle \equiv \int z \langle \psi \rangle (z) dz$ is the long-term stability correction averaged up to height $z$, and $s_{\text{eff}} \equiv kh_{\text{eff}}/\langle u_{*0} \rangle dG/dz$ is the mean dimensionless ABL shear.\(^3\) To get (5) we have defined the effective climatological (mean) geostrophic wind speed $G_{\text{eff}}$ as the corresponding mean wind speed evaluated at the effective ABL depth, $G_{\text{eff}} \equiv \langle U \rangle |_{z=h_{\text{eff}}}$. Essentially $h_{\text{eff}}^{-1}$ is the long-term mean inverse ABL depth, which is somewhat biased by stable conditions so that $h_{\text{eff}} \sim 300–500$ m, consistent with the distributions $P(h)$ given by [20] (c.f. also [15]).

Compared with a logarithmic profile, the mean profile (5) consists of the Monin-Obukhov ("M-O") profile [21] in climatological form (log-law with mean stability correction $\langle \psi \rangle$), plus terms representing the combined effect of stability and height-dependent friction velocity $u_*(z/h)$, a ‘matching’ term which helps to drive $\langle U \rangle$, towards $G_{\text{eff}}$ as $z \to h_{\text{eff}}$, and a geostrophic shear term (which can be due to e.g. large-scale horizontal temperature gradients, i.e. thermal wind). The matching coefficient $h_{\text{eff}}/2\kappa G_{\text{eff}}$ is the difference between $\kappa G_{\text{eff}}/u_{*0}$ and the vertically averaged dimensionless M-O profile evaluated at $h_{\text{eff}}$, minus the geostrophic shear contribution [12]:

$$h_{\text{eff}} \frac{G_{\text{eff}}}{\kappa G} \frac{2\kappa G_{\text{eff}}}{u_{*0}} = \left[ \ln(h_{\text{eff}}/z_{0m}) - h_{\text{eff}}^{-1} \int_{z_{0m}}^{h_{\text{eff}}} \langle \psi \rangle (z) dz \right] - (s_{\text{eff}} - 1).$$

(6)

For simplicity and consistency with the value implicit in the original ‘tall’ profile form of [13], we take the effective long-term dimensionless geostrophic shear to be unity, $s_{\text{eff}} = 1$; thus the last term of (5) vanishes, as does the corresponding $s_{\text{eff}}$ contribution to $\kappa G$ in (6).

The use of (3) also led to a probabilistic long-term mean stability correction in (5), of the form

$$\langle \psi \rangle (z) = -n_{+} \frac{3\sigma_{+}}{C_+} A^{+} z + n_{-} f_{-} \left( \frac{\sigma_{-}}{C_-} \right) B^{+} z,$$

(7)

\(^1\) Over land one can use $\langle u_{*0} \rangle_T \approx \langle u_{*0} \rangle$ without breaking it into separate means for stable and unstable conditions, because doing so has a minor effect on $\sigma_\pm$ in most situations—as noted by [12], who also state that the $\sigma_\pm$ are defined as in (4) avoiding parameters based on $L^{-1}$ (e.g. moments of $L^{-1}$), because the latter can be relatively biased by the tails of the $L^{-1}$ distribution (where M-O similarity fails to apply) and thus reduce the applicability of (4).

\(^3\) Note that [12] contains a typographical error in the definition of $s$, which was limited there to baroclinic shear; the error was introduced after article proofing. The ABL shear ($s$ or $s_{\text{eff}}$) can contain a baroclinic contribution (due to large-scale horizontal temperature gradients), as well as contributions from e.g. surface inhomogeneities or terrain (see e.g. [19]).
Here $b' \equiv b/\Gamma (5/2) \approx 0.75 b$ and $b \approx 4.7$ is the conventional M-O stability correction coefficient for stable conditions, and the mean unstable correction function $f_\infty$ is derived from the classical form $\psi(z/L)$ (via integration over all unstable states $L < 0$, see [12]).

For practical use, because of the dominance of the stable-contributions, for $n_+ > 0.1$ (i.e. stable conditions occurring at least 10% of the time, which is rarely ever violated) we recommend approximating the vertical-mean correction in (5) by

$$\langle \psi \rangle_{z} = z^{-1} \int_{z_0}^{z} \langle \psi \rangle(z')dz' \approx \langle \psi \rangle(z/2). \quad (8)$$

We also suggest using a ‘default’ value of $h_{\text{eff}} = h_{\text{eff}}/2$, consistent with the findings reported in [13] and noting further that the term with this coefficient has the weakest $z$-dependence of the three correction terms in (5); furthermore, this weak $z$-dependence can also change character if the friction velocity profile $u_*(z/h)$ is prescribed differently (e.g. if $u_*$ is made to be linear in $z/h$). We also use the approximation for the mean unstable correction noted in the Appendix of [12]:

$$f_\infty\left(\frac{\sigma_+}{\sigma_-} \frac{\beta z}{2} \right) \approx \psi_-\left(\frac{z}{L_{\text{equiv}}}\right), \quad L_{\text{equiv}} \equiv (-0.4 \sigma_-)^{-1} \quad (9)$$

where

$$\psi_-(\xi) = \frac{\pi}{\sqrt{3}} + \frac{3}{2} \ln \left\{\frac{1}{3} \left[1 + x^{1/3} + x^{2/3}\right]\right\} - \sqrt{3} \tan^{-1}\left[\frac{1 + 2x^{1/3}}{\sqrt{3}}\right], \quad x \equiv (1 - \beta \xi) \quad (10)$$

is the Monin-Obukhov stability correction function that gives the appropriate behavior in the limit of free convection, with $\beta \approx 12$ (as in e.g. Carl et al. [22]).

Using (8) and the approximations above including a ‘default’ $h_{\text{eff}} = h_{\text{eff}}/2$, a practical (‘user-friendly’) form of the long-term dimensionless wind profile (5) can be written as

$$\left(\frac{u_*}{\sqrt{u_{\text{rms}}}}\right)(z) \simeq \ln \left(\frac{z}{z_{\text{off}}}\right) - \langle \psi \rangle(z) - \frac{z}{h_{\text{off}}} \left[\langle \psi \rangle(z/2) - \langle \psi \rangle(z)\right] + \frac{z}{h_{\text{off}}} \left(2 - \frac{z}{h_{\text{off}}}\right)\psi_-(\frac{z}{h_{\text{off}}} - 0.4 \sigma_-) \quad (11)$$

where from (7) and (9) the corresponding climatological stability function is approximately

$$\langle \psi \rangle(z) \simeq -10.6 n_+ z \sigma_+ + (1 - n_+) \psi_-(-0.4 z \sigma_-) \quad (12)$$

with $\psi_-$ again given by (10). The simplified long-term stability correction (12) tends to be dominated by the stable-side correction, which is proportional to height $z$, $\sigma_+$ (variability of $L_{\text{equiv}}$ in stable conditions), and the fraction of conditions at a site which are stable, $n_+$. The unstable component $\psi_-$ is positive and increases monotonically with height, but weaker than linear in $z$. Thus in the wind profile (e.g. Eqn. 11) the stability correction tends to have a stable contribution which increases the shear and wind with height, plus a weaker unstable contribution which opposes this; basically the end result is a climatological wind profile which has higher shear than the log-law over most of the ABL, but which also has decreasing shear as the effective (climatological) ABL ‘top’ ($h_{\text{eff}}$) is approached. The climatological stability correction and dimensionless wind profile are discussed and shown in more detail in [12].

2.2. European Wind Atlas Method

The European Wind Atlas methodology [14] exploits the assumption that observation and prediction sites share the same geostrophic wind $G$ (forcing, in a nonlocal statistical-mean sense), and models the effects of geostrophic-scale surface heat flux through perturbation theory. That is, the EWA method does large-scale (non-local, geostrophic) perturbation of (locally) observed wind statistics in its treatment of stability and vertical extrapolation. The height ($z_m$) of minimum stability-induced wind deviations is determined from geostrophic theory (see Eqn. A.10 in Appendix), and the Monin-Obukhov (M-O) wind profile—which is originally valid on a local, microscale level—is perturbed around $z_m$. The perturbations have essentially two kinds of contributions. First, the M-O form for wind speed is evaluated at $z_m$, and has two components in its climatological stability correction: one due to fluctuations in surface heat flux (Obukhov length via rms value $H_{\text{rms}}$, and an “offset” piece due to stable conditions having a mean dominating effect on the wind shear (via $L_{\text{off}} \approx H_{\text{off}}$). The second type of contribution to the perturbed M-O form consists of geostrophic-scale stability-induced perturbations of friction velocity, for a given forcing (derived from surface heat flux perturbations, see Appendix); these also include both an rms and offset component.

The EWA’s total stability contribution to (perturbation of) the logarithmic wind profile for a given site and height $z$ above the surface can be written simply as

$$\left(\frac{z}{z_m}\right) \left[\Delta u_{\text{rms}} \ln \left(\frac{z_m}{z_0}\right) - \psi \left(\frac{z_m}{L_{\text{off}}}\right) - \psi \left(\frac{z_m}{H_{\text{rms}}}\right)\right] = \Delta u_{\text{rms}} \ln \left(\frac{z}{z_0}\right). \quad (13)$$
i.e. the mean wind at height $z$ is comprised of the sum of (13) and the logarithmic ‘base’ profile $U_0(z) \equiv u_0 \kappa^{-1} \ln(z/z_0)$. We also point out that for $z_0$ the EWA uses a geostrophic-scale roughness, which for a given direction is calculated upward via weighted averages [14]. Normalizing (13) by $U_0$ for a given site, we obtain the dimensionless perturbation $p'$, which expresses the relative effect of stability on the mean wind. A mean wind “correction factor” $c\, f_1$ can be defined as the ratio of scaling factors at target (‘receiver’) and measurement (‘source’) sites, allowing calculation of the mean wind at height $z_{rec}$ over roughness $z_{0\text{rec}}$ from the wind measured at $z_{src}$ over $z_{0\text{src}}$, i.e. via $\langle U_{\text{rec}}/U_{\text{src}} \rangle = c\, f_1 \langle U_{\text{src}}/U_{\text{src}} \rangle$; this is indeed e.g. how the industry-standard software WASP finds $\langle U(z_{\text{rec}}, z_{0\text{rec}}) \rangle$ from $\langle U(z_{\text{src}}, z_{0\text{src}}) \rangle$ (after accounting for variations in roughness and terrain elevation). The correction factor for wind can thus be expressed as

$$c\, f_1 = \frac{1 + p'(z_{\text{src}}, z_{0\text{src}}, H_{\text{off}}, H_{\text{rms}}, G)}{1 + p'(z_{\text{src}}, z_{0\text{src}}, H_{\text{off}}, H_{\text{rms}}, G)}.$$

(14)

The EWA [14] used a perturbation of the geostrophic drag law around its basic form to first order in $(u_*/fL)$ to obtain a relationship between perturbations $du_*$ and $dH$ (see A.6 for full derivation); it then separates heat flux contributions into a mean ‘offset’ component $H_{\text{off}}$ and a fluctuating component $H_{\text{rms}}$, which subsequently give a ${\Delta}u_{\text{off}}$ and $\Delta u_{\text{rms}}$. Using the expressions for $du_*$ developed in the Appendix to write $\{\Delta u_{\text{off}}, \Delta u_{\text{rms}}\}$ in terms of $\{H_{\text{off}}, H_{\text{rms}}\}$ and dividing (13) by $U_0$, the dimensionless stability perturbation can be written compactly as

$$p' = \frac{\kappa}{\ln(z/z_0)} \left[ a_G \frac{C_{\text{rms}} H_{\text{rms}}}{f G^2} \ln\left(\frac{z_m}{z_0}\right) - \psi_W(\zeta_0, H_{\text{off}}, H_{\text{rms}}, G, f) \right] + a_G \frac{H_{\text{off}}}{f G^2}. $$

(15)

Here we abbreviate $\psi_W \equiv \psi(z_m/L_{\text{off}}) + \psi(\zeta_m/L_{\text{rms}})$ as the effective stability function, and remind that $z_m$ actually depends on $\{\zeta_0, G, f\}$. The fluctuating contribution is modeled by the EWA through an unstable correction function $\psi(\zeta_m/L_{\text{rms}})$, where $L_{\text{rms}}$ is the Obukhov length corresponding to $C_{\text{rms}} H_{\text{rms}}$ (since $H_{\text{rms}} > 0$), and $C_{\text{rms}}$ is a constant prescribed by [14] to be 0.6. The offset (mean) component of $\psi_W$ depends on $H_{\text{off}}$ through $L_{\text{off}}$, and the EWA-recommended (WasP default) over-land value of $H_{\text{off}} = -40$ W m$^{-2}$ leads to a stable contribution $\psi_+(\zeta_m/L_{\text{off}})$ to $\psi_W$ (if $H_{\text{off}} > 0$ is chosen, then the unstable form $\psi_-(\zeta_m/L_{\text{off}})$ is used in Eqs. 13 and 15). The $\psi$s are calculated using Monin-Obukhov similarity functions [23], with Obukhov length $L$ defined using the respective offset and rms heat fluxes, along with the geostrophic-scale friction velocity. The latter is approximated by the EWA through the wind speed ($U_{\text{impd}} \equiv A(1 + 2/k)^{1/2}$) corresponding to maximum available power density from observed (input) Weibull parameters, along with an assumed logarithmic profile for this statistic ($u_* = \kappa U_{\text{impd}} / \ln(z_{\text{obs}}/z_0)$), again using the geostrophic (averaged upward) roughness $z_0$; see [14] for more details. The geostrophic wind speed can then be obtained from the geostrophic drag law

$$\frac{\kappa G}{u_*} = \sqrt{\ln\left(\frac{u_*/f}{z_0}\right) - A_0^2} + B_0^2,$$

(16)

where $A_0$ and $B_0$ are the neutral barotropic geostrophic drag coefficients, taken to be 1.8 and 4.5, respectively [14].

The last term in the EWA non-dimensional wind perturbation (15) is independent of height, and represents the perturbation $\Delta u_*/u_*$ in near-surface geostrophic-scale friction velocity ($u_*$) due to surface-based stability contributions to the geostrophic balance, with $a_G \equiv 2.5g/(\rho_c T_0)$ arising from the definition of $L$. Note also that in (15) we have absorbed the functional dependences of $z_m(\zeta_0, G, f)$ and $u_*(\zeta_0, G, f)$ into $\psi_W$, to show with (13) that the EWA stability model gives only a linear height dependence of dimensional stability contributions to mean wind speed—the height $z$ appears only in front of the bracketed term of (15). One can also see from (15) that $H_{\text{off}}$ has a larger influence on the EWA stability correction than $H_{\text{rms}}$, particularly over land where $H_{\text{off}}$ is negative; this parameter tends to be the primary one dictating (the shear in) WasP’s vertical extrapolation.

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* The European Wind Atlas (EWA) finds the geostrophic roughness upward by geometric average (equivalent to averaging the logarithms of roughnesses), with weighting function $\exp(-r/\ell_r)$ for distance $r$ upward. The $\ell_r$-folding distance $\ell_r$ is suggested to be 10 km [14] (which is the default value prescribed in e.g. WasP).

** The factor $C_{\text{rms}}$ leads to an effective EWA-recommended rms heat flux of 60 W m$^{-2}$, from $C_{\text{rms}} H_{\text{rms}} = 0.6 \times 100$ W m$^{-2}$. The EWA-recommended (WasP default) value of $H_{\text{rms}} = 100$ W m$^{-2}$ appears directly only in the perturbation of wind-speed variance, which is used for calculation of the Weibull- $k$ profile; see [13]. Further, implementations of the EWA (e.g. WasP 9-10) also include a damping coefficient $\exp(-z/h_b)$ multiplying the bracketed term in eqns. (13) and (15); however, the damping height $h_b$ is large enough that $c\, f_1$ is not significantly affected, even at heights $z$ up to 250 m.

*** If $(H_{\text{off}}, H_{\text{rms}})$ are given in W m$^{-2}$, an additional factor $\rho_c p / \rho_c p_0$ appears; $\rho$ and $p_0$ are the surface-layer air-density and constant-pressure heat capacity.
2.3. Adaptation of statistical ‘tall’ profile to EWA framework

The long-term dimensionless profile (5) can be translated into an effective profile correction factor, i.e. expressed as perturbation-like form, by normalizing it with the neutral ‘uncorrected’ log-profile:

\[
f_1(z_{src}, z_{rec}) = \left\{\frac{\langle kU(z, h, L, z_0)\rangle_{L,h,z_0,G}}{\langle kU(z, h, L, z_0)\rangle_{L,h,z_0,G}} \right\}_{z_{rec}}^{z_{src}} \ln(\frac{z}{z_{rec}}),
\]

\[
\left\{\frac{\langle kU(z, h, L, z_0)\rangle_{L,h,z_0,G}}{\langle kU(z, h, L, z_0)\rangle_{L,h,z_0,G}} \right\}_{z_{src}}^{z_{rec}} \ln(\frac{z}{z_{src}})
\]

which is again a ratio taken from observation to prediction sites that share a wind climate (G), with the ‘tall’ long-term stability model for normalized deviations from the logarithmic profile (i.e. from eq. 11) denoted by \(p_T\). The dependence of \(U\) upon \(z_{src}, n_+, h_{eff}, \sigma_{G}\) is suppressed hereafter for notational simplicity.

2.3.1. Adaptation for use with geostrophic drag-law.

In order to use the probabilistic profile theory within the context of wind resource assessment, however, one must connect the statistics from a given observation site and the site of application, while accounting for the relevant differences between sites. Thus again the stability-induced long-term mean shift in geostrophic friction velocity needs to be treated (i.e. stable conditions affect \(u_s\) for a given \(G\), meaning the normalized dimensionless profile \(\langle kU(z)\rangle/[u_s \ln(z/z_0)]\) will need to be multiplied by a factor \((1 + \Delta u_{eff})\) since in practice the perturbation \(a_G H_{eff}/f G^2\), as seen in eq.15) is much smaller than 1, and because the other normalized perturbations are relatively small, for the new ‘tall’ profile we approximate by adding \(a_G H_{eff}/f G^2\) to \(p_T\) in (17). Figure 1 shows the correction factor (17) as a function of target height, i.e. the dimensionless wind profile, for the case of observations taken at 40 m height, with stability statistics typical of those found from measurements at flat mid-latitude sites [12]: \(\sigma_s = 0.007 m^{-1}\), \(n_s = 0.6\) (i.e. negative heat flux or stable conditions 60% of the time), \(\sigma_s = 0.004 m^{-1}\), and effective ABL depth \(h_{eff} = 400 m\) over surface roughness length \(z_{0,rec} = 3 cm\). One can see from the figure that these values in effect correspond to the empirically determined values used by the EWA formulation, including the geostrophic ‘offset’ heat flux of \(H_{eff}=40 W m^{-2}\). One can see from Figure 1 how the new tall model diverges from the EWA model above ~150 m; this is due to the ‘tall profile’ accounting for the effect of the ABL depth, and it prevents the new model from over-predicting winds far above the surface layer.

2.3.2. Adaptation for application over different roughnesses; consistency with geostrophic drag law.

Since the probabilistic dimensionless profile in (11) (and [12]) was not derived with regard to application within the context of the geostrophic drag law, its mean stability function lacks the effect of a roughness-dependent geostrophic friction velocity, as in (16). Simply using the profile correction factor (17), based on the probabilistic profile form (11), will result in an improper scaling of wind speed when using observation and target locations with different roughnesses; in fact, the \(z_0\)-dependence of \(f_1\) will be dominated by \(\ln(z/z_0)\) − 1, giving the opposite trend than observed in reality. Invoking a scale-separation argument (the geostrophic scales are much larger than the footprint of surface heat fluxes), the ‘missing’ geostrophic friction velocity dependence can be put into \(\langle \psi \rangle\), knowing that \(L^{-1} \propto u_z^{-3}\). Because the stable side dominates the stability correction, a simple model is to multiply \(L^{-1}\) by \(\langle \psi \rangle\) — thus \(\langle \psi \rangle\) by \(\langle \psi \rangle \langle z_0 \rangle / \langle z_0 \rangle \langle z_0 \rangle\rangle^{-3}\), where \(u_z(z_0)\) is simply a reference friction velocity, equal to the (minimum) geostrophic friction velocity (i.e. \(G(z_0)\) via Eq. 16) occurring over sea roughness \(z_0\). Accounting for this, the new model profile can be expressed as \(p_T\) and cast in a way to allow comparison with the accepted EWA form (15):

\[
p_T = \frac{z/h_{eff}}{\ln(z/z_0)} \left[ -\frac{\langle \psi \rangle G(z)}{z/h_{eff}} + \frac{\langle \psi \rangle G(z)}{z/h_{eff}} - \frac{\langle \psi \rangle G(z/2)}{2/h_{eff}} \right] + a_G \frac{H_{eff}}{f G^2}.
\]
Here in (18) using (7) and (9–10) with $C_+\pm$ from [12], we have

$$
\langle \psi \rangle_G \equiv -10.6n_+\sigma_+z \left[ \frac{u_*(z_0)}{u_*(2z_0)} \right]^{-3} + n_-\psi_-(-0.4\sigma_-z) \tag{19}
$$

denoting the probabilistic stability function modified to account for having different roughness lengths at observation and prediction sites, when relating via $G_{src} = G_{rec}$. The factor (18) is normalized as in the perturbation-form stability corrections employed by WAsP [14]. Figure 2 shows the effective roughness dependence of the adapted tall-profile treatment, along with the EWA behaviour. One can see from the figure that the model incorporating the geostrophic-scale roughness effect on friction velocity into the new stability treatment (19) produces a very similar $z_0$-dependence in predicted mean wind speed compared to the EWA. Neglecting such a dependence would in effect render the new statistical profile treatment incompatible with application of the geostrophic drag law at two sites; this is shown by the black dotted lines in Figure 2, which show $c_f(z_0)$ using (12) for $\langle \psi \rangle$, without geostrophic adaptation.

We further see that for the profile to be consistent with the geostrophic drag law, then $G/u_*$ from (16) should be consistent with the coefficient of the geostrophic-matching term,

$$
\frac{h_{eff}}{2\ell_{eff}} \simeq \frac{KG}{u_*} - \ln(h_{eff}/z_0) + h_{eff}^2\int_{z_0}^{h_{eff}} \langle \psi \rangle (z) dz, \tag{20}
$$

which is observed to be of order 1 (e.g. [12, 13]). Such a value—and consistency with the geostrophic drag law—is only possible when setting $\langle \psi \rangle = \langle \psi \rangle_G$, i.e. including the $u_*$ dependence as in (19). Otherwise, unreasonably large magnitudes of the $h_{eff}/2\ell_{eff}$ ‘matching’ term ensue.

3. VALIDATION AND RESULTS

Self-predictions of wind speed employing both the EWA model and the adapted new model were made at a total of 8 onshore and 5 offshore sites, with each site having observations at numerous measurement heights. Four of the offshore sites are located around the North Sea and use LIDAR (Siri, TAQA platform, Utsira, Schooner, see [24]), with one offshore site (Stora Middelgrund) located between Sweden and Denmark; the microscale winds of all offshore sites are unaffected by coasts. One land site, Østerild, uses LIDAR data; the LIDAR is located in an extended clearing within a limited forest, but we use only measurements higher than twice the mean tree height in order to avoid the forest-induced roughness sublayer and also related distortion of the LIDAR measurements [25]. All of the land sites are in relatively flat terrain;
Høvsøre, Ferrel, and Risø are affected by a coastline††, while the others have relatively insignificant roughness changes. The Hamburg data is for the sectors in a suburban environment, as discussed in [12]. An integer number of years of data, consisting of 10-minute mean winds and directions, were used from each site; each dataset has a recovery rate greater than 90%, with only randomly distributed gaps. The sites used in this study are listed in Table I.

### Table I. Sites and measurement heights used for vertical extrapolation study.

<table>
<thead>
<tr>
<th>Site</th>
<th>Type</th>
<th>Measurement heights (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Høvsøre</td>
<td>coastal</td>
<td>10, 60, 100, 116.5</td>
</tr>
<tr>
<td>Hamburg</td>
<td>suburban</td>
<td>10, 50, 110, 175, 250</td>
</tr>
<tr>
<td>Cabauw</td>
<td>flat land</td>
<td>20, 40, 80, 140, 200</td>
</tr>
<tr>
<td>Ferrel</td>
<td>flat land</td>
<td>10, 30, 100</td>
</tr>
<tr>
<td>Risø</td>
<td>flat land</td>
<td>27, 43, 76, 117</td>
</tr>
<tr>
<td>Sprogø</td>
<td>offshore</td>
<td>18, 55, 68</td>
</tr>
<tr>
<td>Kivenlahti</td>
<td>uniform forest</td>
<td>21, 92, 224</td>
</tr>
<tr>
<td>Østerild</td>
<td>forest (lidar)</td>
<td>45, 60, 80, 100, 120, 140, 170, 200</td>
</tr>
<tr>
<td>Schooner</td>
<td>offshore (lidar)</td>
<td>76, 92, 99, 102, 107, 116, 126, 152, 182, 216</td>
</tr>
<tr>
<td>Siri</td>
<td>offshore (lidar)</td>
<td>85, 105, 125, 145, 161, 175, 205</td>
</tr>
<tr>
<td>Stora Middelgrund</td>
<td>offshore</td>
<td>20, 60, 92, 117</td>
</tr>
<tr>
<td>TAQA</td>
<td>offshore (lidar)</td>
<td>70, 90, 110, 130, 150, 170, 190, 210</td>
</tr>
<tr>
<td>Utsira</td>
<td>offshore (lidar)</td>
<td>40, 53, 73, 93, 113, 133, 153, 173</td>
</tr>
</tbody>
</table>

3.1. Extrapolation without flux observations

Results of our vertical extrapolation calculations are given in Figure 3, which shows the mean absolute error for upward extrapolations as a function of relative extrapolation distance \( \ln(z_{\text{rec}}/z_{\text{src}}) \); the error, i.e. difference between predictions and measurements at heights \( z_{\text{rec}} \), is given for both the EWA model (blue, using standard WAsP settings for its stability treatment, as below Eq.15) as well as the new model (red). The mean is calculated for ‘bins’ of \( \ln(z_{\text{rec}}/z_{\text{src}}) \), where all of the sites’ data have been aggregated together. The ranges of \( \ln(z_{\text{rec}}/z_{\text{src}}) \) have been chosen so that each bin contains approximately the same number of results (~10), in order to avoid introducing an artificial \( z_{\text{rec}}/z_{\text{src}} \) dependence in the extrapolation error. The standard deviation per bin is also indicated by the vertical bars in the plot. For this first validation comparison, in the new model we used values of the stability variability parameters \( \sigma_\pm \) and \( n_\pm \) that correspond to the

†† Note at Risø there is a narrow, shallow fjord affecting the mast, but only for a narrow range of wind directions.
recommended EWA (default WASP) parameter values of \( \{ H_{\text{off}}, H_{\text{rms}} \} \) (as in Figs. 1–2), and with effective ABL depths of 400 m and 250 m (for terrestrial and off-shore sites, respectively).\(^{11}\) This was done because it is the ‘default’ method, i.e. the settings most likely employed by a wind assessment engineer lacking any stability information (below we treat cases using measured stability statistics). Note the predictions shown in Fig. 3 include minor terrain effects due to the inclusion of the EWA (WASP) models for roughness and terrain elevation changes; this was done to evaluate the models’ performance in the most realistic manner possible (again, the ‘default’ way that the model would be used), and to facilitate comparison to observed results. The sites considered have minimal elevation changes, and with the exception of Høvsøre (located 1.5 km from the coast), have only minor roughness changes that do not significantly influence the modelled or measured winds.

From Figure 3 one can see that the new model reduces the absolute error for nearly all extrapolations. The figure also shows enhanced improvement for more drastic extrapolations, though these are less common in practice (as well as in the measurements), as evidenced by the increased bin widths; for example a value of \( \ln(z_{\text{rec}}/z_{\text{src}}) \) equal to 2.2 corresponds to extrapolation to 9 times the observation height (which amounts to extrapolating measurements from around 10–15 m to typical hub heights). While there is improvement shown by the new ‘tall’ model, one can also see that the variability for a given extrapolation distance is appreciably larger than the improvement afforded by the new model.

Results for representative extrapolations at a number of onshore mast sites are shown in Table II, where the ‘source’ and ‘receiver’ (target) heights were chosen as those most resembling typical measurement and hub heights at each site. For this first comparison, in the new model we used values of \( \{ \sigma_{z}, n_{z} \} \), corresponding to the recommended EWA (default WASP) parameter values of \( \{ H_{\text{off}}, H_{\text{rms}} \} \) over land, as in Figs. 1–2. From the table one sees that the probabilistic model, implemented as described above, gives predictions which tend to be better than those of the current EWA/WASP method. However, we note that such predictions will be changed when the values of \( \sigma_{z} \) are calculated (or adjusted to be representative) for the given sites, giving overall improvement of the new model’s predictions as shown in section 3.2.

<table>
<thead>
<tr>
<th>Site</th>
<th>( z_{\text{src}} ) (m)</th>
<th>( z_{\text{rec}} ) (m)</th>
<th>( U_{\text{src}} ) (m/s)</th>
<th>( U_{\text{rec}} ) (m/s)</th>
<th>( U_{\text{EWA}}^{\text{rec}} ) (m/s)</th>
<th>( U_{\text{EWA}}^{\text{new}} ) (m/s)</th>
<th>error_{EWA} (%)</th>
<th>error_{new} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Høvsøre</td>
<td>60, 116.5</td>
<td>8.67</td>
<td>9.69</td>
<td>9.56</td>
<td>9.63</td>
<td>-1.4</td>
<td>-0.7</td>
<td></td>
</tr>
<tr>
<td>Hamburg</td>
<td>50, 175</td>
<td>4.62</td>
<td>6.99</td>
<td>6.61</td>
<td>6.82</td>
<td>-5.5</td>
<td>-2.3</td>
<td></td>
</tr>
<tr>
<td>Cabauw</td>
<td>40, 140</td>
<td>5.80</td>
<td>8.03</td>
<td>7.86</td>
<td>7.94</td>
<td>-2.1</td>
<td>-1.1</td>
<td></td>
</tr>
<tr>
<td>Risø</td>
<td>43, 117</td>
<td>6.63</td>
<td>7.95</td>
<td>8.17</td>
<td>8.22</td>
<td>2.7</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>Kivenlahti</td>
<td>92, 224</td>
<td>5.37</td>
<td>7.35</td>
<td>7.25</td>
<td>7.41</td>
<td>-1.3</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

Table II. Extrapolation error at various sites, using available heights \( \{ z_{\text{src}}, z_{\text{rec}} \} \) closest to typical measurement and prediction levels.

### 3.2. Extrapolation including measured flux statistics

For several test sites (Cabauw, Hamburg, Høvsøre), sonic anemometers were part of the instrumentation, giving velocity component, temperature and flux statistics (see e.g. [12] for more details). For these sites, we also did extrapolations using (12) in (16) and (17) with the momentum flux and heat flux statistics, via the stability variability parameters \( \sigma_{z} \) calculated as in (4) from 10 m-height sonic anemometer measurements at each of these sites. Figure 4 shows the mean absolute error per relative extrapolation \( \ln(z_{\text{rec}}/z_{\text{src}}) \), for the new model both using the default values (as in Fig. 3) and using observed flux statistics, compared to the EWA/WASP model. Note that the bin widths are wider in Fig. 4, because there were less data points in the dataset (3 sites) where \( \sigma_{z} \) could be calculated; again the bins were chosen such that each contained the same number of samples.

From Figure 4 one can see that using the stability measurements leads to further improvement of the extrapolations compared to the EWA method, with systematically smaller absolute error per extrapolation distance. There is a minor exception to the general improvement, for extreme extrapolations to at least 10 times measurement height. This is not unexpected, and does not significantly impact use of the new model with flux statistics—addition of surface-layer flux information is not expected to improve extrapolation of surface-layer wind data to heights which may sometimes lie above the ABL and are more directly affected by the capping inversion, and such extrapolation (e.g. from 10–15 m to 100–250 m) is not recommended. For extrapolations of \( \ln(z_{\text{rec}}/z_{\text{src}}) \approx 0.4 \) (about 50% above measurement height), the new model including flux information gives mean absolute errors slightly larger than the EWA model. But, as shown in Figure 5, for extrapolations of \( \sim 50\% \) the mean error from the new model including fluxes is a bit smaller than those from the EWA. Overall, Fig. 5 shows that the new model using default settings gives mean extrapolation error closer to zero

\(^{11}\) The ‘default’ values of \( h_{\text{eff}} \) over land and sea are chosen to be consistent with the distributions aggregated by Liu and Liang [20], and also give mean wind profile shapes consistent with those observed e.g. by the offshore LIDAR in this study.
than the EWA model, and that using flux observations tends to reduce this error further. We note however, that while an improvement can be seen using the new model, the differences between new and old (EWA/WAsP) tend to be of the same magnitude (or smaller) than the standard deviation per log-extrapolation distance (represented by the bars in Figs. 3– 5).

![Figure 4. Mean of absolute error in wind speed extrapolation per relative extrapolation bin in \( \ln(z_{rec}/z_{src}) \), also using measured flux statistics (yellow triangles) instead of the ‘default tall’ parameters (red squares).](image1)

![Figure 5. Mean error in wind speed extrapolation per relative extrapolation bin in \( \ln(z_{rec}/z_{src}) \), also using measured flux statistics (yellow triangles) instead of the ‘default tall’ parameters (red squares).](image2)

The new model with default settings gives smaller standard deviations of absolute error per extrapolation distance than the EWA, as shown in Figure 6 (and by the error bars in the preceding plots); this is also generally true of the new model using observed fluxes, though the added flux information translates into more variability at some extrapolation distances. From the plots using the limited number of sites that include sonic anemometer flux measurements, i.e. Figures 4–5, one might expect little improvement from the new model for more ‘conservative’ extrapolations to heights less than double the observation height \( \ln(z_{rec}/z_{src}) \leq 0.7 \); however, the reader is reminded that the full data set here, as seen in Figure 3, shows improvement of the new model over the EWA for such ‘typical’ extrapolations.
3.3. Uncertainty in vertical extrapolation

In the previous section we evaluated the performance of the original EW A and new ‘tall’ vertical extrapolation modelling, but this can also be extended to gauge the uncertainty in the models. Looking at the figures of the previous section, one can see direct correspondence between absolute error in wind speed and relative vertical extrapolation \((z_{\text{rec}}/z_{\text{src}})\). Considering all the data in this work, i.e. Figure 3, along with the fact that the EW A’s perturbation form (13) is dominated by contributions that involve \(\ln(z/z_0)\) and \(z/z_m\) and this is applied as source-receiver ratios as in Eq. 14, one may expect the extrapolation error to be proportional to some combination of \(\ln(z_{\text{rec}}/z_{\text{src}})\) and \((z_{\text{rec}}/z_{\text{src}})\). This is also expected when using the adapted ‘tall’ model as well, due to the logarithmic and linear \(z\)-dependences in the climatological profile (11) and associated stability correction (12). Indeed, for extrapolations up to twice a given measurement height, as seen by the linear trend in Figure 3, the absolute error has a behavior of roughly

\[
|\%\text{error}|_{z_{\text{rec}}<2z_{\text{src}}} \approx 0.6\% + c_{\varepsilon 0} \ln \left( \frac{z_{\text{rec}}}{z_{\text{src}}} \right) .
\]  

We find a proportionality constant \(c_{\varepsilon 0}\) of approximately 2% for the EW A model (14–15). For the new tall profile model (11–12) adapted to the EW A perturbation framework (i.e. Eqs. 17–19), the extrapolation error and thus the coefficient in (21) is smaller, \(c_{\varepsilon 0} \approx 1.3\%\), reflecting improved performance and presumably lower uncertainty. Using the EW A (WAsP) framework with either model, one may take (21) as an estimate for the uncertainty for ‘typical’ extrapolations, i.e. not significantly beyond twice the observation height. For more extreme extrapolations, the mean error deviates from logarithmic. For extrapolations beyond twice the measurement height, the uncertainty could be estimated using a form involving both dependences; e.g. a log-linear form \(\sqrt{c_{\varepsilon} \ln(z_{\text{rec}}/z_{\text{src}}) + c_{\text{lin}}(z_{\text{rec}}/z_{\text{src}})}\), where \(\{c_{\varepsilon}, c_{\text{lin}}\}\) would be roughly \(\{4, 0.5\}\) % for the EW A/WAsP model and \(\{2.5, 0.3\}\) % for the new tall model.

We re-iterate that our uncertainty estimates (and thus Eq. 21) are based on fewer than 15 sites, and that the bin-wise standard deviation (as seen in Fig. 6) is nearly as large as the mean absolute error.

4. DISCUSSION AND SUMMARY

Here we have adapted the probabilistic ‘tall’ dimensionless profile theory of [12] for use within the framework of the European Wind Atlas (EWA) methodology [14], i.e. to be consistent with site-to-site application via the geostrophic drag law. The dimensionless climatological wind profile (5) is a natural choice for extension of the EWA methodology, since it is expressed simply as a logarithmic piece plus terms for the effects of stability and ABL-depth. Using \(\langle U/\nu_0 \rangle\) facilitates treatment of ‘direct’ (e.g. Monin-Obukhov) stability corrections separately from (geostrophic-scale) surface friction velocity perturbations, as in the EWA. We have made a simplification by defining the mean dimensionless wind through integration over the stability distribution \(P(L^{-1})dL^{-1}\), effectively simplifying the joint behavior of \(\nu_0\) and \(L^{-1}\). We have also implicitly assumed \(\nu_0\) to be Weibull-distributed—which is an inherent consequence of the form of the first-order EWA treatment as well. The applicability of the stability-based simplification becomes weaker over water, where the...
air-sea temperature difference $\Delta T_{aw}$ becomes the relevant quantity (as opposed to the near-surface heat flux and $L$), and
where $u_{*}$ and $L^{-1}$ have a more complicated relationship that may demand 'two-dimensional' treatment, i.e. consideration
of the joint probability density $P(\Delta T_{aw}, u_{*})$. This is reflected (and practically compensated for) by the $\sigma_n$ used over sea
in practical implementation (WAsP), which to match the successful EWA surface-layer mean-wind modeling, is set to be
smaller than the average $\sigma_n$ found from over-sea measurements and mesoscale modelling made thus far (e.g. [26]). Details
of the latter are beyond the scope of this work, but progress continues in this area.

To make the dimensionless wind profile amenable for use with the geostrophic drag law (16) applied in the EWA,
we retain the mean geostrophic-scale ('offset') heat flux contribution perturbing the (mean) friction velocity, as in the
perturbation form of the EWA. This is necessary to account for the effect of heat flux upon the geostrophic balance,
whereby the integrated atmospheric boundary-layer momentum transfer (in effect from geostrophic level to the ground)
is different than for neutral conditions. For a given $G$, the neutral value of $u_{*}$ is in effect perturbed (to first order, see
Appendix) by the geostrophic-scale mean heat flux $H_{\text{offset}}$, which leads to a height-independent “offset” of the implied
long-term mean wind profile. We have normalized the new 'tall' form for dimensionless profile $(U/u_{*0})$ by its logarithmic
component, then included the (geostrophic) $\Delta u_{*0}$ “offset” term in the profile. This normalization, consistent with the EWA
form, results in a perturbative mean wind profile expression $(1 + \Delta U/U_0)$ which, while containing terms nonlinear in the
height $z$ above ground, does not vary through the ABL as much as the EWA form. That is, the new model has decreasing
shear as the climatological effective ABL ‘top’ is approached, as the mean wind approaches the mean geostrophic value
(and the mean direction changes as well, not explicitly represented in the new model). The new adapted wind profile model
can be viewed like a perturbation around $z \approx 0$ (actually $z_{zo}$), whereas the EWA version is a linear perturbation around
$z = z_m$; thus we allow concerns with the new model retaining a geostrophic piece, i.e. the geostrophic-scale “offset” heat
flux being affected by the surface-layer shear $dU/dz$ (below $z_m$). However, the offset term $\Delta u_{*0}$ (and thus $H_{\text{offset}}$) might be
expected to depend slightly on the newly-utilized stability statistics (primarily the effective width $\sim n_{z}, \sigma_n$ of the stability
distribution during stable conditions), though this small effect is neglected in the current treatment. Further, to adapt the
probabilistically-derived mean wind profile for application from one surface to another, the geostrophic-scale effect of
roughness was included in the stability treatment, modeled in a way consistent with the EWA treatment. Doing so involves
a low-order model with uncertainty, on par with application of the first-order EWA methodology.

The performance of both the new ‘tall’ model and the WAsP/EWA ‘standard’ model for vertical extrapolation was
investigated at a significant number of ideal (relatively homogeneous, mostly flat) sites on land and offshore, over a
large number of extrapolation distances. Without including measured heat flux statistics, the tall model performed a bit
better than the EWA model, though the improvement was not large compared to the variability in extrapolation errors.
Guided by the form of the EWA extrapolation model and the extrapolation error results, we suggest a basic form (21)
for estimating the uncertainty in vertical extrapolation using the original EWA formulation. Similarly we arrive at the
same form for the adapted ‘tall’ model, with a slightly different constant reflecting the smaller expected error in the new
model. We also consider extrapolations at a small number of sites with sonic anemometers measuring fluxes of heat and
momentum, using the flux statistics in the new model; inclusion of this information improves the results further over the
EWA method, generally compensating (perhaps over-compensating) for the negative bias seen using the EWA method with
default settings.

**Implications and ongoing work**

While the ‘tall profile’ adaptation and implementation modestly improved results for upward extrapolation of long-term
mean wind speed, we point out that for prediction of annual energy production (AEP), one must also use an appropriately
modified model for vertical extrapolation profile of the Weibull-$k$ parameter or long-term second-moment of wind speed.
The EWA framework includes such a model for Weibull-$k$ and long-term $(U^2)$, while a successful ‘tall’ implementation
is currently under development and testing [15].

Continuing work also includes universal characterization of the connection between the distributions of stability
$P(L^{-1})$ and shear $dU/dz$ in the surface layer, particularly under stable conditions, to produce useful estimates of
the primary profile-impacting parameter $\sigma_n$ from measured wind shear statistics. Further concurrent work includes
incorporation of expected ABL depth distributions in the theory, including correlation with stability, for improvement of
Weibull-$k$ profiles obtained via profiles of long-term higher (e.g. second) moments of the wind speed along with the mean
wind. Also under development is a model of the turning of the wind with height (veer), consistent with the probabilistic
theory and geostrophic drag-law application.
A. APPENDIX: STABILITY PERTURBATION OF GEOSTROPHIC DRAG LAW AND WIND PROFILE

We provide an updated derivation of geostrophic-scale stability-induced perturbation of the friction velocity (and subsequently logarithmic wind profile around the height of minimum stability-induced deviations, $z_m$). The derivation is rooted in perturbation of the geostrophic drag law, and leads (with some assumptions) to the stability model of the European Wind Atlas [14]. The derivation is extended to update the parameter relating $z_m$ to the geostrophic wind, latitude, and roughness.

The EWA [14] in effect estimates the geostrophic-scale perturbation $d u_*$ due to first-order perturbations $dH$ in geostrophic-scale heat flux (from neutral, i.e. $H = 0$), arising from the stability-dependence of the barotropic geostrophic resistance-law constants $A_0$ and $B_0$. We begin by employing the stability parameter $\mu \equiv \kappa(u_*/f)/L$, so to first order in $\mu$ we have $dA = d\mu(dA_0/d\mu)$ and $dB = d\mu(dB_0/d\mu)$. Because $u_*/L = -(\kappa g/T_0)H/u_*^2$, one may write

$$\frac{d\mu}{\mu} = \frac{dH}{H} - 2\frac{du_*}{u_*},$$

i.e. $d\ln \mu = (d\ln H - 2d\ln u_*)$. For a given forcing $G$ and ignoring $dz_0$, the differential of the geostrophic drag law

$$G = \frac{u_0}{\kappa} \sqrt{\left[ \ln \left( \frac{u_*/f}{z_0} \right) - A_0 \right]^2 + B_0^2},$$

provides

$$0 = dG = G \frac{d \mu}{u_*} + \frac{u_*/\kappa}{\ln(u_*/fz_0) - A_0^2} \left\{ \left( \frac{d \mu}{u_*} - dA_0 \right) \left[ \ln \left( \frac{u_*/f}{z_0} \right) - A_0 \right] + B_0 dB \right\}.$$  \hspace{1cm} (A.3)

now using $dB_0/d\mu = 0.2 = -dA_0/d\mu$ as in [14], then dividing by $G$ we get

$$0 = d \ln G = d \ln u_* + \left( \frac{u_*/\kappa G}{\ln(u_*/fz_0) - A_0} \right)^2 \left\{ (d \ln u_* + 0.2d\mu) \left[ \ln \left( \frac{u_*/f}{z_0} \right) - A_0 \right] + 0.2B_0 d\mu \right\}. \hspace{1cm} (A.4)$$

Exploiting Eq. A.1 (e.g. $d\mu = \mu d\ln H$), we can rearrange Eq. A.4 to give a relation between first-order perturbations in geostrophic heat flux and friction velocity,

$$d \ln u_* = \frac{-0.2 u_*/\kappa G}{1 + (u_*/\kappa G)^2} \left\{ B_0 + \left[ \ln \left( \frac{u_*/f}{z_0} \right) - A_0 \right] \right\} \left( \mu d \ln H \right). \hspace{1cm} (A.5)$$

But $\mu d \ln H = -(\kappa v^2/u_* fT_0) dH$, and we are perturbing around a neutral state, so that $\mu = 0$ and (A.5) becomes

$$d \ln u_* = 0.2 \left\{ \ln \left( \frac{u_*/f}{z_0} \right) + B_0 - A_0 \right\} - \frac{g}{fT_0 G^2} dH. \hspace{1cm} (A.6)$$

A form equivalent to $d \ln u_* = \kappa c_G (g/\rho c_f T_0 f G^2) dH$ is given by the EWA [14] (where the factor $\rho c_f$ is for $dH$ given in W/m$^2$), which is equal to (A.6) but with $0.2\left[ \ln(u_*/fz_0) + B_0 - A_0 \right]$ replaced by a constant $c_G$ equal to 2.5. This value falls within the range of effective $c_G$ found via (A.6), for the ranges of $u_*$, $f$, and $z_0$ encountered in practice. For smoother surfaces ($z_0$ of several cm or less) and appreciable friction velocities the choice of $c_G = 2.5$ gives low $d u_*$ compared to (A.6), while rougher surfaces and low friction velocities lead to the opposite result.

One can also interpret the perturbation theory here in terms of the geostrophic-to-surface wind turning (cross-isobaric) angle $\phi_G$; for the ideal case of barotropic boundary layers (i.e. insignificant horizontal gradients of surface temperature) then the conventional relation $\tan \phi_G = -B_0/\ln(u_*/fz_0) - A_0$ applies [27, 28]. The lack of $\phi_G$ dependence can be interpreted as a sensible choice in the EWA model, given that the $\tan(\phi_G)$ dependence is not proper for latitudes approaching the equator [29], and from the EWA’s implicit assumption that (in the stability treatment) variations in $dH$ would dominate those do to geostrophic turning. We also remind that near the equator the geostrophic wind becomes ill-defined (the boundary-layer depth implied by $\ln(u_*/fz_0)$ becomes unphysically large), due to the dominant ABL balance arising not from the Coriolis force but from other mechanisms. However, the geostrophic drag law form is still used with limited success in the tropics, due to the Coriolis parameter being limited in WAsP in such a way that the implied time scale corresponds to that of dominant (diurnal) forcings of the ABL.
A.1. Height of minimum stability-induced perturbations

The scale $z_m$ is defined by the EWA [14] to be where first-order effects of surface heat-flux variations vanish, at a height where the differential of the Monin-Obukhov wind profile is zero:

$$ \frac{d}{dz} \left[ \ln \left( \frac{z}{z_0} \right) - \psi (z L^{-1}) \right] \bigg|_{z=z_m} = 0. $$

Thus

$$ \ln \left( \frac{z_m}{z_0} \right) - \psi (z_m L^{-1}) du_* - u_* L^{-1} \frac{d \psi}{dL} \left[ d \ln H - 3 d \ln u_* \right] = 0. \quad (A.7) $$

Since the stable-side correction function is $\psi_s = -bzL^{-1}$ and consequently $d \psi / dL \rightarrow -bz$ in the neutral limit ($dL^{-1} \rightarrow 0$), then employing the (dominant) stable-side form for $\psi$ and the definition of Obukhov length $L$, and from (A.6) we arrive at

$$ \frac{z_m}{\ln(z_m/z_0)} = \frac{c_G}{b} \frac{u_*^2}{\kappa f G z_0}. \quad (A.8) $$

Again from (A.5–A.6) we see that $c_G \approx 0.2 \ln(u_*/f/z_0) - A_0 + B_0$, with the EWA [14] making a practical choice setting $c_G = 2.5$. The EWA continues by using the reduced geostrophic drag law [23]

$$ u_{*G} = \frac{0.485 G}{\ln(G/f z_0) - A_0} \quad (A.9) $$

to replace $(u_*/G)^3$ in (A.8) with $0.5^3/[\ln(u_*/f/z_0) - A_0]^3$, giving $0.1(G/f)/[\ln(u_*/f/z_0) - A_0]^3$ for the right-hand side of (A.8); finally the EWA approximates the expression for $z_m / \ln(z_m/z_0)$ by using a power law to obtain

$$ z_m = \alpha_m z_0 \left( \frac{G}{f z_0} \right)^{0.9} \quad (A.10) $$

where $\alpha_m |_{\text{EWA}} = 0.002$. But we note that such a choice for $\alpha_m$ implies $b = 8$, because the constant of 0.1 derives as an effective approximation of $0.5^3 c_G / (\kappa b)$; however, this should be equal to 0.16 for $b = 4.7$ and $c_G = 2.5$. Thus one should have $\alpha_m = 0.003$ for this value of $c_G$, which corresponds to observed reversal heights for Weibull-k profiles [15], whereas using the EWA value of 0.002 gives $z_m$ between 65–80 m. On the other hand, the value of $c_G > 2.5$ implied by (A.6) for small to moderate roughness lengths (i.e. not forest or urban areas) is more consistent with the EWA value of $\alpha_m$. The difference in reversal height due to $\alpha_m$ has a minor impact on the wind profile, and can in some cases affect annual energy production calculations more than the mean wind profile, due to its role in extrapolation of Weibull-k (discussed in [15]).

Note if $c_G$ is changed, then one would need to adjust the values of $H_{\text{off}}$ and $H_{\text{rms}}$ used in the EWA framework. For mid-latitude, simple sites (e.g. those with moderately small roughnesses considered in [12]), we find that the actual $H_{\text{rms}}$ is roughly 30–50% of its EWA-recommended value of 100 Wm$^{-2}$, consistent with increasing $c_G$ by a factor of 2–3 and also (A.6). This points to the difficulty of assigning a physical meaning to the geostrophic-scale perturbation heat flux $H_{\text{rms}}$; any change of $c_G$ would also demand a change of the ‘offset’ heat flux $H_{\text{off}}$ in WASP as well.

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REFERENCES


