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Integrating robust timetabling in line plan optimization for railway systems

S. Burggraeve, S.H. Bull, P. Vansteenwegen, R.M. Lusby

Abstract

We propose a heuristic algorithm to build a railway line plan from scratch that minimizes passenger travel time and operator cost and for which a feasible and robust timetable exists. A line planning module and a timetabling module work iteratively and interactively. The line planning module creates an initial line plan. The timetabling module evaluates the line plan and identifies a critical line based on minimum buffer times between train pairs. The line planning module proposes a new line plan in which the time length of the critical line is modified in order to provide more flexibility in the schedule. This flexibility is used during timetabling to improve the robustness of the railway system. The algorithm is validated on a high frequency railway system with little shunt capacity. While the operator and passenger cost remain close to those of the initially and (for these costs) optimally built line plan, the timetable corresponding to the finally developed robust line plan significantly improves the minimum buffer time, and thus the robustness, in eight out of ten studied cases.

Keywords: railway line planning; timetabling; robustness; mixed integer linear programming.

1 Introduction

Railway line planning is the problem of constructing a set of lines in a railway network that meet some particular requirements. A line is often taken to be a route in a high-level infrastructure graph ignoring precise details of platforms, junctions, etc. In our case, a line is a route in the network together with a stopping pattern for the stations along that route, as a line may either stop at or bypass a station on its route (which saves time for bypassing passengers). We define a line plan as a set of such routes, each with a stopping pattern and frequency, which together must meet certain targets such as providing a minimal service at every station.

Timetabling is the problem of assigning precise utilization times for infrastructure resources to every train in the rail system. These times must ensure that trains can follow their routes in the network, stop at appropriate stations where necessary, and avoid any conflicts with other trains. A conflict rises where two trains want to reserve the same part of the infrastructure at the same time, for example at a switch, platform or turning track. If timetabling is performed separately from line planning, the line plan specifies the lines and the number of hourly trains operating on each line but not the exact times for those trains and not the precise resources that a train on a line will utilize. Those timings and utilizations are decided as part of the timetabling.

Traditionally, a railway line plan is constructed before a timetable is made. However, an optimal line plan does not guarantee an optimal or even a feasible timetable (Kaspi and Raviv, 2013). An integrated approach can overcome this problem. Nevertheless, since line planning and timetabling are both separately already very complex problems for large railway networks (Michaelis and Schöbel, 2009; Goerigk et al., 2013), solving the resulting integrated problem is in most cases not computationally possible (Schöbel, 2015). We propose a heuristic algorithm that constructs a line plan for which a feasible timetable exists. We call a line plan *timetable-feasible* if there exists a conflict-free timetable for that line plan. Moreover the algorithm improves the robustness of the line plan by making well chosen changes in the stopping patterns of the lines while the existence of a feasible timetable remains assured.

There are different interpretations of robustness in railway research. According to Dewilde et al. (2011), a railway planning is *passenger robust* if the total travel time in practice of all passengers is minimized in case of frequently occurring small delays. The focus of this definition is twofold, as both short and reliable travel times have to be provided by the planning. Passenger robustness is also what we want to strive for with our approach.

The context of this research is a high frequency network where trains are forced to turn on their platform in their terminal stations due to a lack of shunting area. The proposed integrated approach originates from insights on why some line plans do not allow feasible timetables and why some line plans allow more robust timetables. A first insight is that a line can be infeasible on its own, which we call *line infeasibility.* A second insight is that line combinations can be infeasible due to their frequencies. We call this *frequency combination infeasibility.* In Section 3 we explain these insights. Furthermore, we present a technique to develop a line plan that guarantees a feasible timetable. We introduce a timetabling model based on the Periodic Event Scheduling Problem (PESP) to create passenger robust timetables. We illustrate with a case study that a smart and targeted interaction of both techniques develops a line plan from scratch which guarantees a feasible and passenger robust timetable. Moreover, the integrated approach can also be used to improve the robustness of an existing line plan. The line planning and timetabling technique and the integrated approach are explained in Section 4.

Related work is discussed initially in Section 2. The case study is described in more detail in Section 5. In Section 6 the results of the case study are presented and examined and the integrated approach is illustrated in an example. The paper is concluded and ideas for future research are suggested in Section 7.

2 State of the art

The planning of a railway system consists of several decisions on different planning horizons (Lusby et al., 2011). The construction of railway infrastructure and a line planning are long term decisions. A timetable, a routing plan, a rolling stock schedule and a crew schedule are made several months up to a couple of years in advance. Decisions on handling delays and obstructions in daily operation are made in real time. Each of these decisions affects the performance of the other decisions. Ideally, a model that optimizes all these decisions simultaneously is preferred. Each of the separate decision problems, however, is NP-hard for realistic networks (Schöbel, 2015). In practice these planning decisions are usually made one after the other, although the solution from a previous decision level problem does not even guarantee that a feasible solution exists for the next level problem (Schöbel, 2015). In the case that the output of the previous decision level leads to infeasibility at the next planning step, there are several possible approaches for looking for a feasible solution to both planning levels together. First, the outcome of the previous level can be replaced by a second best outcome in the hope that a feasible solution for the next level exists. Secondly, the outcome of the previous level can be specifically oriented towards making a feasible solution for the next level possible, by using case dependent restrictions specifically for this goal. Thirdly, the constraints on the outcome of the next level can be loosened. These approaches increase the possibility of finding a feasible solution for the next level, but not necessarily guarantee a good outcome for both levels together. A few integrated approaches for two or three of the typical decision problems in railway research are described in the literature and clearly outperform the hierarchical approach (Goerigk et al., 2013). Most of these solution algorithms are heuristics to overcome the high computation times of an exact approach for a realistic railway network. As in this paper we propose an algorithm towards the integration of line planning and timetabling, we elaborate on existing integrated approaches for these two planning problems in the first part of this literature review. We also explain the place of the individual timetabling and line planning modules that are used in our integrated approach within existing literature on timetabling and line planning.

2.1 Integration of line planning and timetabling

This paper is not the first attempt towards an integration of line planning and timetabling in railway scheduling. In Liebchen and Möhring (2007), some line planning decisions are included in the timetabling process. They assume that, for some parts (sequence of tracks) of the network, the number of lines serving each part is known beforehand. On these track sections they put an artificial station in the middle. Every line along this track section is then partitioned into two line segments, before and after the artificial station. They use the Periodic Event Scheduling Problem (PESP) which was introduced by Serafini and Ukovich (1989) to model the timetabling problem in which they add constraints such that a perfect matching between the arriving and the departing line segments is forced. This is achieved by matching arrival and departure times of the line segments in the artificial station which are assigned by this same model. Here one line corresponds to one train. This approach is deficient if, for some network parts, the number of passing trains is not known beforehand. This is often the case in real world networks. Furthermore, this approach can lead to a set of lines in which each line is a different combination of track sections which is not transparent for the passengers.

Kaspi and Raviv (2013) present a genetic algorithm that builds a line plan and timetable from scratch. They start from a given line pool and per line a fixed number of potential trains. A solution consists of three characteristics for each train: the value zero or one, which indicates if the train should be scheduled or not, an earliest start time and a stopping pattern. A member of the initial population is constructed by drawing values for each characteristic from separate Bernoulli distributions. The timetable and line plan are constructed by scheduling trains with value one for the first characteristic according to a fixed priority rule. If a train cannot be scheduled without one or more conflicts with other already scheduled trains, this train is omitted from the solution. For the resulting timetable, the passenger travel time and the operator cost are calculated. These performance results affect the distribution parameters of the Bernoulli distributions from which the next generation will be drawn. This approach uses the performance of the timetable as input for the line planning of the next iteration. This interaction between line planning and timetabling is also the case in our approach. But in contrast to the stochastic approach of Kaspi and Raviv (2013), we use information of the timetable to make some deterministic and tactical changes to the line planning. Also in Goerigk et al. (2013) timetable performance is used to evaluate line plans. However, they do not iterate between the construction phase of the line planning and the timetabling, and they do not use this information to improve the line planning. They only use it to compare different ways to construct a line plan.

Michaelis and Schöbel (2009) offer an integrated approach in which they reorder the classic sequence of line planning, timetabling and vehicle scheduling for bus planning. The different planning steps are, however, performed one after each other such that the approach is still sequential. Vehicle scheduling or rolling stock scheduling are not integrated in our approach, but we take turn restrictions in the end stations into account which significantly simplify the rolling stock scheduling. Taking turn restrictions into account is useful if terminal stations are not equipped with enough shunting space for efficient turning during daily operation. In fact, neglecting turn restrictions can lead to infeasible timetables. To the best of our knowledge, no other integrated approach for timetabling and line planning takes turn restrictions during daily operation into account. This is explained in the next section.

Very recently, Schöbel (2015) published a mixed integer linear program (MILP) in which line planning and timetabling are integrated for railway planning. This model is based on the PESP of Serafini and Ukovich (1989). In the model, binary variables are introduced to indicate if a certain line is added to the line plan. There are also big M-constraints added to the PESP model in which these binary variables are used to push event times of lines which are not in the line plan to zero and also to switch off lower bounds of activities for unassigned lines. The objective function minimizes the planned travel time of the passengers. Transfer penalties are not taken into account, but they can easily be introduced as a weight in the objective function. No performance results of this model are presented yet.

An added value of our approach is that robustness is taken into account when constructing a line plan (and timetable). With our approach we want to shift the focus in current research on integration of line planning and timetabling to the creation of passenger robust line plans (and timetables). The algorithm that we propose constructs a line plan that minimizes planned passenger travel time and operator costs but also prevents unreliable travel times during daily operation in order to provide a short travel time in practice for all passengers. As mentioned in the introduction, a *passenger robust* plan minimizes this total travel time in practice. In order to obtain short travel times in practice, the propagation of delays from one train to another train has to be avoided. This can be achieved if the line plan allows a timetable in which the buffer times between trains are above a certain threshold. Also in Kroon et al. (2008); Caimi et al. (2012); Salido et al. (2012); Dewilde et al. (2013); Sels et al. (2016) and Vansteenwegen et al. (2016) the (minimum) buffer times between train pairs are lengthened in order to reduce the propagation of delays.

Another added value of our approach is that trains with the same route are equally spread over the period of the cyclic timetable. Making the reasonable assumption that passengers arrive uniformly in a station of a high frequency network, a constant time interval between two trains with the same route minimizes the average waiting time of the passengers before boarding.

In our heuristic approach, a line planning and timetable module alternate, where each consists of an exact optimization model. We now motivate our choice for the timetable and line planning models that are used and briefly discuss related literature.

2.2 Timetabling

The goal of the timetabling module is to construct a passenger robust timetable. This avoids propagation of delays in case of small delays during daily operation in order to provide reliable travel times to the passengers and is achieved by maximizing the (minimum) buffer times between train pairs. Parbo et al. (2015) give an extensive overview of passenger perspectives in railway timetabling. The PESP model of Serafini and Ukovich (1989) is the foundation of many timetable models (Schrijver and Steenbeek, 1993; Nachtigall, 1996; Liebchen, 2006; Peeters, 2003) and (Großmann, 2011) and is also the framework of our timetabling model. The PESP model schedules events in a period of the cyclic timetable and takes precedence constraints and relations between events into account. Arrivals and departures of trains at stations or reservations and releases of track sections are events. If two events are related or can affect each other they form an activity. Examples of activities are the arrival and departure of the same train in a station or the reservation times of a shared switch or platform by two different trains. The PESP model constrains each activity time, which is the time between the two events that define the activity. The PESP is originally defined without an objective function, but several objective functions for PESP can be found in the literature. We add an objective function that maximizes the (minimum) buffer times between trains using the same part of the infrastructure, in order to achieve robustness. In our timetabling model, we also have 'turning' and 'providing buffer time' as activities between events besides the usual driving, waiting and transferring activities. Furthermore, we include extra constraints such that trains can be equally spread over the hour. A recent and elaborate discussion on timetable literature in general and PESP in specific can be found in Sels et al. (2016).

2.3 Line planning

Railway line planning is, generally, the construction of a set of lines to operate in a rail network. There are parallels to line planning problems in bus network design and network design for liner shipping. Line planning for rail takes the physical rail network as a fixed input, and provides a fixed input to subsequent timetabling and rolling stock planning. So when creating the line plan, assumptions can potentially be made about the form or characteristics of timetables, rolling stock and rolling stock planning. Schöbel (2012) gives an overview of different approaches to model and solve the line planning problem, broadly categorizing line planning approaches that are (operator) cost-oriented, and those that are passenger-oriented.

Goossens et al. (2006) focus on minimizing operator cost, for the less-studied case of line planning where lines may not stop at every station. Also in our research the stopping pattern of a line is decided in the line planning problem. The advantage of allowing lines to skip stations is the potential to combine fast lines which only stop at the stations with high demand and slow lines which also stop in stations with low demand (with the classification of stations not specified but decided during line planning). Using fast lines shortens the travel time of a lot of passengers and the slow lines assure a service in every station.

With a passenger focus, a common objective function is to maximize the number of direct travelers, i.e. the number of passengers who have a route from their origin to destination that does not require transfers. The simplest interpretation of this objective is to count the number of passengers for which there exists a line in the solution visiting both their origin and destination. This does not actually find passenger routes and does not guarantee that all counted passengers can actually *use* the line, as there may be insufficient capacity on some lines. Using this objective also has the risk in some networks that the passengers with no direct route may be faced with many transfers. Another disadvantage is that maximizing the number of direct travelers encourages long train lines and, critically in our case, does not favour skipped stations. Bussieck et al. (1997) is one example which uses this direct traveler objective, while ensuring that direct lines also have sufficient capacity to accommodate the passengers.

Another objective function with passenger focus is a travel time objective that takes into account the passenger's time traveling in trains and a penalty for switching trains (transfers). The calculation of this objective requires knowledge on the routing of passengers in the network taking into account travel time and switching. This routing of the passengers can be modelled in a graph, which can become very large due to the large number of passenger routes required (potentially one for every pair of stations). Schöbel and Scholl (2006) and Borndörfer et al. (2007) are examples where passengers between a pair of stations are routed by minimizing the sum of the travel time costs of the used paths. This passenger routing objective could be used as part of a weighted sum objective along with some operator cost (Borndörfer et al., 2007), or used alone but with an additional operator cost budget constraint (Schöbel and Scholl, 2006). In some practical problems the inclusion of a budget can be very important when combined with a passenger-oriented objective, as without it, solutions can contain many lines to individually satisfy every type of passenger. Our line plan model uses also the passenger's travel time objective. In our case study, however, there are tight rate limits on the maximum number of trains turning at a terminal station and on the use of certain infrastructure. Thus even without an operator budget consideration we do not risk solutions having particularly many lines.

Operator focused or passenger focused is a first partitioning that can be made. Another partitioning is that a line planning model may be based on a predetermined set of lines (a line pool), or it may find new lines dynamically. An advantage of a predetermined line pool is that all lines in the pool can be guaranteed to be feasible in terms of line planning requirements, or advantageously for our case, in terms of timetabling requirements. This latter is explained in the next section. A predetermined pool also has the advantage of limiting the problem size in a useful and dynamic way (because the pool can be limited to be as diverse or as focused as desired). However, it has the disadvantage that the full set of possible lines may be very large and so enumerating them all could be intractable, while taking only a subset of all possible lines risks missing good solutions. Schöbel and Scholl (2006) present a model that takes as input a predetermined pool of lines. In contrast, Borndörfer et al. (2007) present a method where lines are generated dynamically in an infrastructure network as a pricing problem, finding maximum-weighted paths to introduce as lines to a restricted master problem. However, the master problem itself is formulated in terms of a known line pool.

With respect to decision variables, many approaches are similar in using either a binary decision for the presences of each line, or a non-negative or integral decision for the frequency of each line, where a frequency of zero means that the line is not in the solution. In our approach we may only select one of a set of frequencies defined individually for every line, so our model uses a binary decision variable indicating the presence of a (line, frequency)-pair.

Specifically related to the S-tog problem at DSB (which we will address as a case study), Rezanova (2015) solves the line planning problem with an operator focus, considering train driving time and a particular competing objective related to new regulations for drivers. The author notes the problem of finding line plan solutions that are not feasible for timetabling, and suggests that an integrated approach would be valuable.

3 Timetable-infeasibility

In this section we explain how limited shunt capacity and frequency combinations of lines that share a part of the network can lead to timetable-infeasibility of line plans. Our integrated approach uses these insights to construct line plans that allow conflict-free and passenger robust timetables.

3.1 Line infeasibility

Consider Figure 1, showing a single train line operating at six times per hour. The black dots on the time-axis show the scheduled departures from the beginning station for this line, which is once every ten minutes. We illustrate the first two time-distance graphs; the first departing from the beginning station at minute zero (solid blue line), and the subsequent train following with a departure at minute ten (red line). In this example, the

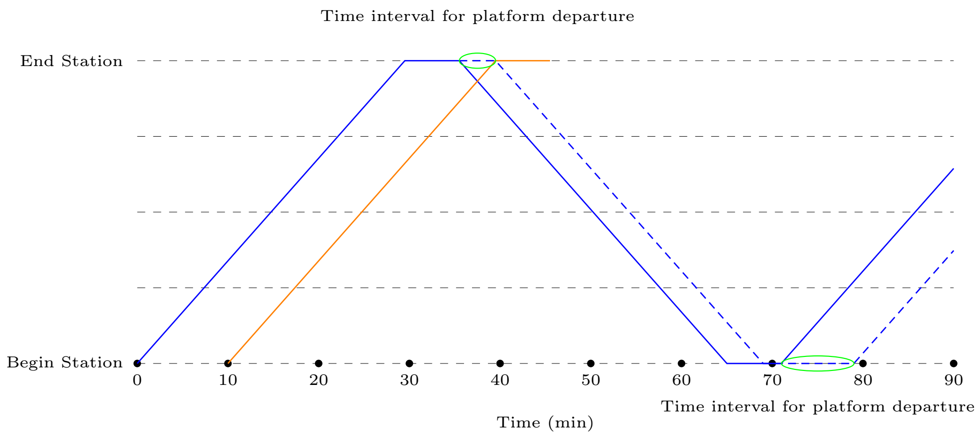


Figure 1: A line can be infeasible on its own

travel time between the beginning and ending stations for the line is 29 minutes and we assume that the train has to turn on its platform due to restricted shunt capacity in its terminal stations. We define a minimum dwell time of seven minutes before a train can leave its platform in the other direction. This dwell time is the time required by passengers to leave the train, by the driver to change to the driver position at the other end of the train and by new passengers to board the train before a departure returning to the beginning station. Note that the subsequent train that departed ten minutes later is therefore arriving at the ending station ten minutes later as well, so the first train has a well-defined latest departure which is marked as a dashed blue line. The train then drives for 29 minutes back to its beginning station, arriving there between 65 minutes and 68 minutes after its first departure at minute zero. It can leave for the next round trip at 72 minutes after minute zero at the earliest (minute 65 arrival with seven minutes minimum required turn time) and 78 minutes at the latest (68 minute arrival with a maximum of ten minutes dwell time, assuming that the next train arrives ten minutes later on the same platform). However, no train is planned to leave the begin station in the interval of 72 to 78 minutes, which can be seen in Figure 1 as no black dot falls in the interval indicated with the green line. Therefore no feasible timetable can be found for this line. We will call this *line infeasibility.*

This insight can be mathematically formulated as: *In case of restricted shunt capacity in its terminal stations, a line is infeasible on its own if there exists no* $k \in \mathbb{Z}^+$ *such that*

$$2 * dr_{l,s_{l,0},s_{l,e}} + \text{ntt}_{s_{l,0}} + \text{ntt}_{s_{l,e}} \leq \frac{P}{f_l} * k \leq 2 * dr_{l,s_{l,0},s_{l,e}} + 2 * \frac{P}{f_l},$$

where $dr_{l,s_{l,0},s_{l,e}}$ *is the planned travel time between the start station* $s_{l,0}$ *and the end station* $s_{l,e}$ *of line* l*. Further* $\text{ntt}_{s_{l,0}}$ *and* $\text{ntt}_{s_{l,e}}$ *are respectively the necessary turn time for line* l *in its start and end station,* f_l *is the frequency of line* l*,* P *is the period of the cyclic timetable and trains of the same line are equally spread over the period and use the same platform in the terminal stations for passenger convenience.*

3.2 Frequency combination infeasibility

Suppose that different lines share a part of the network and that trains of the same line are equally spread in the cyclic timetable. A second insight is that the frequencies of these lines affect the potential buffer time between these lines. Let $f_{l_1} \leq f_{l_2}$ be the frequencies of two lines l_1 and l_2 respectively. It is straightforward that the higher the frequencies the smaller the potential buffer time between trains of these lines. But we also claim that the buffer time between a line at a higher frequency and a lower frequency is no greater than between two lines at the higher frequency. For example, assume $f_{l_1} = 4$ and $f_{l_2} = 5$, then in one cycle of the timetable, there will be, at some point, two trains of line l_2 which are planned between two succeeding trains of l_1. The time between two trains of l_1 is 15 minutes and between two trains of l_2 is 12 minutes. This would lead to the situation in Figure 2. Because a is strictly smaller than three, the smallest buffer between a train of line l_1 and line l_2 is smaller than or equal to one-and-a-half minutes.

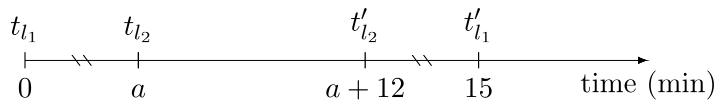


Figure 2: If lines l_1 and l_2 have frequencies $f_{l_1} = 4$ and $f_{l_2} = 5$ respectively, then once in 60 minutes two trains (t_{l_2} and t'_{l_2}) of line l_2 will pass in between two trains (t_{l_1} and t'_{l_1}) of line l_1. Without loss of generality we can assume that this happens in the first quarter. Here $a \in \mathbb{R}$ and $0 < a < 3$.

This can be mathematically generalized and formulated as: *The minimum time between a passage of a train of line l_1 and line l_2 with frequencies $f_{l_1} \leq f_{l_2}$ respectively is smaller than ($\leq$)*

$$\frac{\frac{P}{f_{l_1}} - (\lceil \frac{f_{l_2}}{f_{l_1}} \rceil - 1)\frac{P}{f_{l_2}}}{2} \tag{1}$$

where P is the period of the cyclic timetable and $\lceil a \rceil$ is the smallest integer b with $b \geq a$.

If this minimum buffer time is strictly smaller than the minimum necessary buffer time according to safety regulations in the network, then l_1 with frequency f_{l_1} and l_2 with frequency f_{l_2} are not feasible together. In the example, if the minimum necessary buffer time according to safety regulations is two minutes, then these lines l_1 and l_2 cannot be combined at frequencies $f_{l_1} = 4$ and $f_{l_2} = 5$. Note that formula (1) is maximal in case f_{l_2} equals f_{l_1} or is a multiple of f_{l_1} and then reduces to $\frac{P}{2f_{l_2}}$ which proves our claim.

4 Methodology

In this section, we propose an integrated approach that constructs a line plan from scratch that minimizes a weighted sum of operator and passenger cost and allows a feasible and robust timetable. First a timetable-feasible line plan is constructed. Then, iteratively and interactively, a line planning module produces a line plan, and for the plan, a timetable module produces a timetable that maximizes the (minimum) buffer times between train pairs. Each loop an analysis of the timetable indicates how the line plan could be adapted in order to allow a more robust timetable. This adaptation increases the flexibility of the line plan which is used, in the timetabling module, to increase the minimum buffer times between train pairs. The line plan module then calculates the new line plan that includes this adaptation while minimizing the weighted sum of operator and passenger costs. This feedback loop stops when there is no further improvement possible or if there is no improvement during a fixed number of iterations for the minimum buffer times between train pairs. We first discuss the line planning module and the timetabling module separately and then the integration of both. Both the timetable and the line planning module consist of an exact optimization model, though our combined approach, and the fact that we do not always solve the models to optimality, result in an overall heuristic method.

4.1 Line planning module

Constructing a line plan consists of selecting a set of lines which meet certain requirements from a pool of predetermined lines. The line pool is not exhaustive; there are many more possible lines than those considered, but the set is reduced to those that meet certain criteria as discussed with the rail operator. This also keeps the problem size small. The model performs three functions: (i) selecting the lines and frequencies and creating a valid plan, (ii) routing passengers between origin and destination stations and (iii) relating passenger routes to line selections.

Let us first define the set of all lines available: $\mathcal{L}$. For every line $l \in \mathcal{L}$ we define a set of valid frequencies for the line: $\mathcal{F}_l$. The operator must meet certain obligations for any valid line plan and must not exceed certain operational limits. These restrictions are referred to as service constraints. We define these all in terms of a set of resources $\mathcal{R}$, and define all limitations as either a minimum (rmin_r) or maximum (rmax_r) number of trains using that resource $r \in \mathcal{R}$ every hour. The subset of lines that make use of resource $r \in \mathcal{R}$ is defined as $\mathcal{L}_r$. Let $c_{l,f}$ be the cost to the operator for operating line l at frequency f.

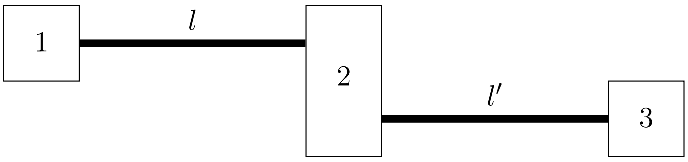
The line planning module starts from a known origin-destination (OD) matrix containing the passenger demand for travel between every origin and destination, where origins and destinations are simply stations in the rail network. Let $\mathcal{S}$ be the set of stations. For two stations $s_1, s_2 \in \mathcal{S}$ we know the demand d_{s_1,s_2}. We model passengers as a flow from each origin station to every relevant destination station in a graph constructed for the network and all lines in the pool. This graph captures the passenger cost in terms of travel time on lines and estimated waiting time between lines (estimated by frequency) in case a transfer is required. We refer to this graph as the passenger graph. The graph contains a *(line,* *frequency, station)* vertex for every line, frequency, and every station visited by that line. The edges of this graph represent travel possibilities, with the edge cost being the known train driving time or the estimated transfer time. Additionally, this graph contains source *(r_s)* and sink *(t_s)* vertices for every station s where passengers originate from or terminate their travel. These vertices are connected to the appropriate *(line, frequency, station)* vertices with edges representing boarding or alighting from a line. These edges have zero cost. Finally, for every station s we have a platform vertex *(p_s)* with edges from and to every *(line, frequency, s)* vertex, where the costs correspond to an estimate of perceived waiting time which consists of a fixed penalty component and a frequency-dependent component.

Let $\mathcal{V}$ and $\mathcal{E}$ be the set of all vertices and edges of this graph, respectively, and t_e be the cost to a single passenger of using edge $e \in \mathcal{E}$. In total there are five types of edges.

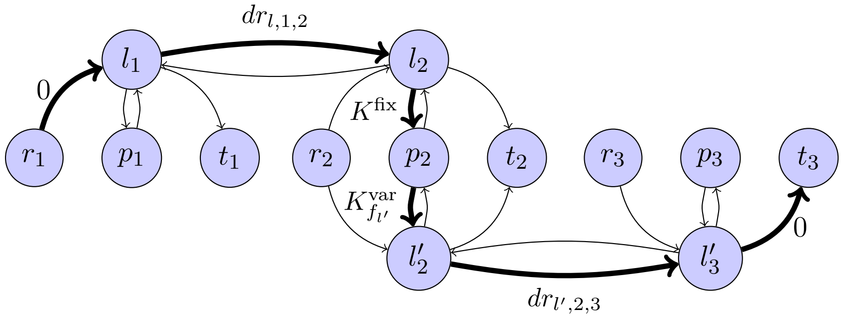
- Type 1. From (l, f, s) to (l, f, s') for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and s and s' two succeeding stations visited by line l.
- Type 2. From (l, f, s) to (p_s) for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and s a station visited by line l and (p_s) the platform vertex of station s.
- Type 3. From (p_s) to (l, f, s) for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and s a station visited by line l and (p_s) the platform vertex of station s.
- Type 4. From (r_s) to (l, f, s) for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and s a station visited by line l and (r_s) the source vertex of station s.
- Type 5. From (l, f, s) to (t_s) for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and s a station visited by line l and (t_s) the sink vertex of station s.

See Figure 3 for an example of a simple network with three stations, 1, 2 and 3, and two lines l and l' visiting two of the stations each (3a) and corresponding passenger graph (3b).

This graph is similar to the *change&go* graph of Schöbel and Scholl (2006), but distinguishes between line transfers that in our case happen to lines with discrete frequencies, with a frequency-dependent cost.



(a) Line structure (two lines, three stations)



(b) Corresponding passenger graph structure

Figure 3: The upper figure shows a simple network with three stations 1, 2 and 3, and two lines l and l'. Line l visits stations 1 and 2, and line l' visits stations 2 and 3. Each line operates at just a single frequency. The lower figure shows the subsequent passenger graph structure used for this network. Costs are labelled on the edges for a passenger travelling from station 1 to station 3, transferring lines at station 2, with used edges in bold. The costs to the passenger are $dr_{l,1,2}$, travelling (**dr**iving) on line l from station 1 to 2; fixed cost K^{fix} for a transfer and an additional $K^{\text{var}}_{f_{l'}}$ frequency dependent cost for transferring to line l'; and $dr_{l',2,3}$ travelling on line l' from station 2 to 3.

Let l_e be the line that edge e is related to and f_e be the frequency of the line that e is related to. This line and frequency of an edge are uniquely defined as the two vertices connected by edge e are either both related to the same line and frequency or only one of them is related to a line and frequency.

Let a_v^s be the demand for passengers originating from station s at vertex v in the passenger graph. For vertex types *(line, frequency, station)*, $a_v^s = 0$. For a given station s_1, there is a single source vertex v corresponding to s_1. For this vertex, and for passengers originating from that station, $a_v^{s_1} = -\sum_{s_2 \in \mathcal{S}} d_{s_1,s_2}$. For a given station s_2, there is also a single sink vertex corresponding to s_2. For every sink vertex v the demand of passengers to that vertex from station s_1 is the demand of passengers from s_1 to s_2: $a_v^{s_1} = d_{s_1,s_2}$.

For relating passengers to lines, let C_f be the passenger capacity of any line operating at frequency f. We are therefore assuming the same rolling stock unit type and sequence for every line, but a higher frequency provides more seats than a lower frequency. We require that no more passengers use a line as the line capacity permits for the frequency the line is operating at.

We use two classes of decision variables: $x_{l,f} \in \{0,1\}$ is a binary decision variable indicating whether or not line l is selected at frequency f, and y_s^e decides the number of passengers from origin station s that use edge e in the passenger graph.

The line planning model is:

$$\text{Minimize} \quad \lambda \sum_{l \in \mathcal{L}} \sum_{f \in \mathcal{F}_l} c_{l,f} x_{l,f} + (1-\lambda) \sum_{e \in \mathcal{E}} \sum_{s \in \mathcal{S}} t_e y_s^e \tag{2}$$

$$\text{s.t.} \quad \sum_{f \in \mathcal{F}_l} x_{l,f} \leq 1 \qquad \forall l \in \mathcal{L} \tag{3}$$

$$\sum_{l \in \mathcal{L}_r} \sum_{f \in \mathcal{F}_l} f x_{l,f} \leq \text{rmin}_r \qquad \forall r \in \mathcal{R} \tag{4}$$

$$\sum_{l \in \mathcal{L}_r} \sum_{f \in \mathcal{F}_l} f x_{l,f} \geq \text{rmax}_r \qquad \forall r \in \mathcal{R} \tag{5}$$

$$\sum_{(u,v) \in \mathcal{E}} y_s^{(u,v)} - \sum_{(v,w) \in \mathcal{E}} y_s^{(v,w)} = a_v^s \qquad \forall s \in \mathcal{S}, \forall v \in \mathcal{V} \tag{6}$$

$$\sum_{s \in \mathcal{S}} y_s^e \leq C_f x_{l_e,f_e} \qquad \forall e \in \mathcal{E} \tag{7}$$

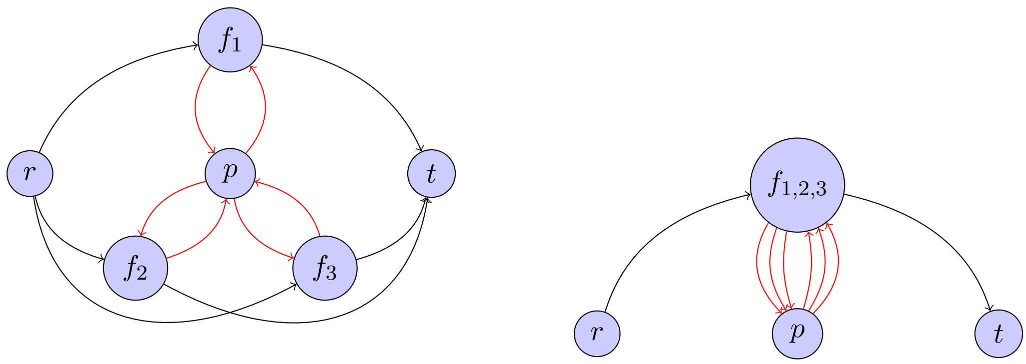
$$x_{l,f} \in \{0,1\} \qquad \forall l \in \mathcal{L}, f \in \mathcal{F}_l \tag{8}$$

$$y_s^e \in \mathbb{R}^+ \qquad \forall s \in \mathcal{S}, e \in \mathcal{E} \tag{9}$$

The objective function (2) is a weighted sum of the operator cost and the passenger travel time (ride time and wait time), using a parameter $\lambda \in [0,1]$ to determine the importance of one component over the other.

Constraints (3) ensure that a line is chosen with at most one frequency (i.e. combinations of frequencies are not permitted, as if valid a discrete frequency would be present in the frequency set $\mathcal{F}_l$ for the line). Constraints (4) and (5) ensure that the obligatory and operational requirements are met for the line plan. Constraints (6) consist of the flow conservation constraints. The number of passengers leaving from an origin station must flow from that station with the appropriate number arriving at every destination station, such that flow is conserved. Constraints (7) link the flows of passengers to the line decisions. The presence of a positive passenger flow on an edge in the graph is dependent on some line being present in the plan. The maximum flow on that edge depends on the passenger capacity of the corresponding line at the appropriate frequency. Finally, constraints (8) and (9) restrict the line variables and flow variables to be binary variables and positive otherwise unrestricted variables, respectively.

The presented model is very large, as the passenger graph we construct is very large and there are therefore a large number of flow variables and corresponding constraints. However, we observe that many of the vertices and edges in the graph are very similar and differ only in line frequency. For lines with many possible frequencies there is significant duplication. For the edges related to switching lines at a station, frequency is required to determine the cost to the passenger. However for all other edges the frequency information is redundant. Indeed, the cost of travelling on a line between stations does not depend on the frequency of that line. A first simplification of the model is that for each line and its frequencies, we replace the edges (and vertices) which do not depend on frequency with an edge (and vertex) related only to *line* and *station* instead of *line*, *frequency* and *station*. This is shown in Figure 4. The capacity of the replacement edge (and resulting right hand side of constraints (7)), is given by $\sum_{f \in \mathcal{F}_l} C_f x_{l,f}$.



(a) Original graph structure

(b) Reduced graph structure

Figure 4: The full and reduced graph structure for a single line with three frequencies at a single station.

Figure 4 shows the graph structure for a single station and a single line with three frequencies as originally described (Figure 4a) and with the explained reductions (Figure 4b). Nodes r and t are respectively the station source and sink vertices for passengers and p is the platform vertex for that station. The vertices f_1, f_2, f_3 are the *(line, frequency, station)* vertices for the three considered frequencies of the line, in that station. The red edges are the line switching edges (though no other lines are shown). Edges connecting these vertices f_1, f_2, and f_3 to corresponding vertices at other stations are not shown. Vertex $f_{1,2,3}$ is the combination of the vertices f_1, f_2, f_3. The edge between s and $f_{1,2,3}$, and between $f_{1,2,3}$ and t, is the combination of the edges between s and f_1, f_2 and f_3 in (Figure 4a), and f_1, f_2 and f_3 and t, respectively.

A second simplification of the model is that we consider line-switching edges only at a minimal set of switching stations. This set of stations is fixed beforehand and suffices to facilitate all optimal passenger flows, when every passenger origin-destination pair is considered individually. Any solution that is feasible for this restricted problem is feasible if switching is allowed at any station, but some solutions that are feasible if switching is permitted anywhere may not be feasible with the restriction (although we have not observed this). At stations where we do not permit switching we do not include switching edges, and this reduces the total number of edges in the graph by between 23% and 34% when tested for a range of line pools. Finally, we can determine that only a subset of all edges should be used for the flows from a given origin station; generally it is never true that in an optimal solution passengers will be assigned an edge that travels “towards” the station they originate from. This is a third measure to simplify the model.

By making these three alterations we find that the line planning problem is solvable directly as a MILP, though not to optimality in the time frame we require. For our tests, finding line plans with no other restriction, we use a time limit of one hour, or until a gap between the solution and best lower bound is below 0.5% (in most cases the gap limit is reached, but for some weightings of objectives, one hour is insufficient). However, for a reduced line pool that we use in the integrated approach described later, the problem becomes easier and is solvable to optimality in an acceptable time frame.

The formulation (2)–(9) defines the basic line planning model, ignoring some minor considerations not given here. However, when searching for line plans that only differ a little from a give line plan we may impose some additional restrictions. The simplest types are the following:

$$\sum_{l\in\mathcal{L}}\sum_{f\in\mathcal{F}_l} f\cdot x_{l,f} \geq k_1 \tag{10}$$

$$\sum_{l\in\mathcal{L}}\sum_{f\in\mathcal{F}_l} f\cdot x_{l,f} \leq k_2 \tag{11}$$

That is, we require that the total number of (one-directional) trains running in the network per hour is between some upper and lower bound. This may be, for example, to find solutions that do not differ too much from some original solution. We use this because, from the point of view of the timetable module, two solutions that differ only in line frequency but not in line routes can be very different. Without such constraints, when seeking a line plan that is similar but different to a given plan, a change of frequency would not maintain the similarities in timetabling that we seek. Now, suppose we are given a line plan or a partial line plan, in the form $\mathcal{X} = \{(l_1, f_1), (l_2, f_2), (l_3, f_3), ...\}$ where every (l, f) in $\mathcal{X}$ is a valid line and frequency combination, and that this (partial) line plan should not be in the solution. Then we may impose the following constraint for every such line plan:

$$\sum_{(l,f)\in\mathcal{X}} x_{l,f} \leq |\mathcal{X}| - 1. \tag{12}$$

Such constraints are used to forbid solutions we have already discovered and do not wish to find again, and also to forbid partial solutions which we already know are problematic for timetabling, i.e. they lead to timetable-infeasibility. Finally, and similarly, we may have some given line plan $\mathcal{X}$ and desire that the solution line plan contains at least k lines from the plan:

$$\sum_{(l,f)\in\mathcal{X}} x_{l,f} \geq k. \tag{13}$$

Such constraints ensure that a discovered line plan is similar to some previous line plan, while differing by some number of (unspecified) lines. If instead the lines that may differ are specified, we can fix the variables of the lines that may not differ and only permit those variables corresponding to the specified lines that may differ to change (along with variables corresponding to lines not in the plan). These extra restrictions are used in the integrated approach when looking for a similar line plan that is more flexible, i.e. allows a more robust timetable.

4.2 Timetabling module

The timetable problem is modelled as a PESP. We indicate our event-activity network as $(\mathcal{E}, \mathcal{A})$. The set of trains is indicated as T, the set of lines in the line plan (output of the line planning module) as $\mathcal{X}$, the line operated by train t is indicated as l_t, the set of stations is $\mathcal{S}$ and the set of stations on a line l is indicated as $\mathcal{S}_l$. As we assume a railway network with limited shunt capacity, our model assumes that all the trains can and must turn on their platform at end stations. The set T_{turn} contains the train couples (t, t') for which it holds that t becomes train t' after turning on the platform in its end station. Trains t and t' share the same rolling stock. Line l_t and $l_{t'}$ contain the same stations but in opposite direction. The set $T_{line\ spread}$ contains the train couples (t, t') where t and t' are two succeeding trains of the same line, i.e. no other train operating on the same line drives in between them.

The event set $\mathcal{E}$ of the event-activity network consists of the following events.

- The reservation of a station environment s (and a platform in this station) by a train t is a reservation event (t, s, res). We define $\mathcal{E}^{\text{res}}$ as $\{(t, s, \text{res}) \,|\, \forall t \in T, s \in S_{l_t}\}$.
- The release of a station environment s (and a platform in this station) by a train t is a release event (t, s, rel). We define $\mathcal{E}^{\text{rel}}$ as $\{(t, s, \text{rel}) \,|\, \forall t \in T, s \in S_{l_t}\}$.
- The reservation of a platform p_{s_e} in a terminal station s_e by a train t in order to turn is a platform reservation event $(t, p_{s_e,t}, \text{res})$. We define $\mathcal{E}^{\text{res},p}$ as $\{(t, p_{s_e,t}) \,|\, \forall t \in T\}$.
- The release of a platform p_{s_e} in a terminal station s_e by a train t in order to turn is a platform release event $(t, p_{s_e,t}, \text{rel})$. We define $\mathcal{E}^{\text{rel},p}$ as $\{(t, p_{s_e,t}) \,|\, \forall t \in T\}$.

The following inclusions hold $\mathcal{E}^{\text{res},p} \subset \mathcal{E}^{res} \subset \mathcal{E}$ and $\mathcal{E}^{\text{rel},p} \subset \mathcal{E}^{\text{rel}} \subset \mathcal{E}$ and $\mathcal{E} = \mathcal{E}^{\text{res}} \cup \mathcal{E}^{\text{rel}}$. So platform $p_{s_e,t}$ of train t in its terminal station can be interpreted as an extra station where the train arrives after arriving in its terminal station s_e.

The activity set $\mathcal{A}$ contains:

- *driving activities* between the release of a train in a station and the reservation of this train of the next station on its line. Let $\mathcal{A}^{drive} = \{((t, s, \text{rel}), (t, s', \text{res})) \in \mathcal{E}^{\text{rel}} \times \mathcal{E}^{\text{res}} \,|\, \forall t \in T$ and s and s' succeeding stations of $l_t\}$;

- *waiting activities* between the reservation and the release of a train in a station on its line. Let $\mathcal{A}^{wait} = \{((t, s, \text{res}), (t, s, \text{rel})) \in \mathcal{E}^{\text{res}} \times \mathcal{E}^{\text{rel}} \,|\, \forall t \in T, s \in S_{l_t}\}$;

- *buffer activities* between the release of one train and the reservation of another train on the same platform in the same station. Let $\mathcal{A}^{buffer} = \{((t, s, \text{rel}), (t', s, \text{res})) \in \mathcal{E}^{\text{rel}} \times \mathcal{E}^{\text{res}} \,|\, \forall t, t' \in T : t \neq t', s \in S_{l_t} \cap S_{l'_t}, p_{s,t} = p_{s,t'}\}$;

- *line spreading activities* between the reservations of two succeeding trains on the same line in the stations on their line. Let $\mathcal{A}^{line\ spread} = \{((t, s, \text{res}), (t', s, \text{res})) \in \mathcal{E}^{\text{res}} \times \mathcal{E}^{\text{res}} \,|\, \forall t, t' \in T : (t, t') \in T_{line\ spread}, s \in S_{l_t}\}$;

- *turning activities* between the release of a train of the platform in its end station and the release of the next train of the opposite line that leaves from that terminal station. Let $\mathcal{A}^{turn} = \{((t, p_{s_e,t}, \text{rel}), (t', s_e, \text{rel})) \in \mathcal{E}^{\text{rel},p_{s_e,t}} \times \mathcal{E}^{\text{rel}} \,|\, \forall t, t' \in T : (t, t') \in T_{turn}\}$. This next train is the same physical train.

As mentioned in Section 2, we want to maximize the minimum buffer times between train pairs. In terms of the event-activity graph, we want to maximize the minimum activity time of the buffer activities which is the time between the release of an infrastructure part by one train and the reservation of that same infrastructure part by another train. Mathematically we have

$$\max \min_{a=(i,j)\in\mathcal{A}^{buffer}} (\pi_i - \pi_j + k_a P), \tag{14}$$

where π_i and π_j are the event times of event i and j respectively which define together a buffer activity. However, this objective function is not linear, but as it is a max-min objective function, it can easily be linearized. Therefore, we introduce an auxiliary variable $y \in [0, P]$, where P is the period of the cyclic timetable. We add the constraints

$$y \leq \pi_i - \pi_j + k_a P \qquad \forall a = (i, j) \in \mathcal{A}^{buffer} \tag{15}$$

and we change the objective function to the maximization of y: $\max y$. The complete model is then the following.

$$\max \quad y \tag{16}$$
$$y \leq \pi_i - \pi_j + k_a P \qquad \forall a = (i, j) \in \mathcal{A}^{buffer}$$
$$L_a \leq \pi_i - \pi_j + k_a P \leq U_a \qquad \forall a = (i, j) \in \mathcal{A} \tag{17}$$
$$0 \leq \pi_i \leq P \qquad \forall i \in \mathcal{E} \tag{18}$$
$$k_a \in \{0, 1\} \qquad \forall a = (i, j) \in \mathcal{A} \tag{19}$$

Constraints (17) bound all activity times from below and above. The term $k_a P$ avoids negative activity times. To ensure a unique value for k_a, the value of U_a has to be smaller than the period P. The specific values of U_a and L_a are listed in Table 1 for all activities $a \in \mathcal{A}$. The driving activity times are bounded by the time that a train of line l needs between the release of a station s and the reservation of the next station s', indicated as $dr_{l,s,s'}$. The driving time between the terminal station of a train and the platform in its terminal station is zero minutes. The waiting activity times are bounded by the time that is necessary and provided for a line l to occupy a station s, indicated as $dw_{l,s}$. This is the time between the reservation and release time of that station. The bounds are different for waiting activities on platforms in terminal stations. Trains have to stay there for at least the necessary turn time in the terminal station s, which is indicated as ntt_s. Trains have to leave the platform before the next train arrives. This is ensured if they all get the same maximum time that they can stay on the platform which is equal to the period of the cyclic timetable divided by the number of trains that turn on platform p. The number of trains that turn on platform p is indicated as f_p. The buffer activities have to be positive and smaller than $P - dw_{l_{t'},s} - \epsilon$ to ensure that the timetable is feasible, i.e. occupation intervals may not overlap, independently of the order of both trains that will be assigned. On platforms in terminal stations the upper bound is smaller because trains occupy the platform for a longer time, i.e. the upper bound in our model is $P - \frac{P}{f_{p_{s_e,t}}} - \epsilon$. Before initializing the timetable module, a check is necessary to determine if too many trains are scheduled on one platform, i.e. if $\frac{P}{f_p} \leq ntt_s$. If so, the trains do not have enough time for turning and consequently the timetable will be infeasible. The value of ϵ depends on the time discretization. We use 0.1 minutes. In this model we equally distribute trains of a line over the period, and therefore the line spread activity times have to be equal to the period divided by the line frequency. The frequency of a line l is indicated as f_l. The turn activity times have to be equal to zero, ensuring that the 'turning' platform is freed if the next train leaves in the opposite direction.

Activity	L_a	U_a
$((t, s, \text{rel}), (t, s', \text{res})) \in A^{drive}$	$dr_{l_t,s,s'}$	$dr_{l_t,s,s'}$
$((t, s, \text{res}), (t, s, \text{rel})) \in A^{wait} : s \neq p_{s_e,t}$	$dw_{l_t,s}$	$dw_{l_t,s}$
$((t, s, \text{res}), (t, s, \text{rel})) \in A^{wait} : s = p_{s_e,t}$	ntt_s	$\frac{P}{f_{p_{s_e,t}}}$
$((t, s, \text{rel}), (t', s, \text{res})) \in A^{buffer}$	0	$P - dw_{l_{t'},s} - \epsilon$
$((t, s, \text{rel}), (t', s, \text{res})) \in A^{buffer} : s = p_{s_e,t} = p_{s_e,t'}$	0	$P - \frac{P}{f_{p_{s_e,t}}} - \epsilon$
$((t, s, \text{res}), (t', s, \text{res})) \in A^{line\ spread}$	$\frac{P}{f_l}$	$\frac{P}{f_l}$
$((t, p_{s_e,t}, \text{rel}), (t', s_e, \text{rel})) \in A^{turn}$	0	0

Table 1: Lower and upper bounds for the PESP constraints (17)

4.3 Integrated approach

Here, we explain how the line planning and timetabling module can be integrated to construct a line plan and timetable that induce a low passenger and operator cost and maximize the buffer times between train pairs in order to provide a passenger robust railway schedule. The line planning and timetabling module work iteratively and interactively. The line planning module creates an initial line plan which is evaluated by the timetabling module. Based on the minimum buffer times between line pairs, a critical line in the line plan is identified. The line planning module then creates a new line plan with at least one different line, i.e. the time length of this critical line is changed. The goal is to create more flexibility in the line plan. This flexibility will be used by the timetabling module to improve its robustness. This heuristic approach which is divided into two parts is now further explained. In Figure 5, a visual overview of the algorithm is presented and in Section 6.2 we apply the approach to an example.

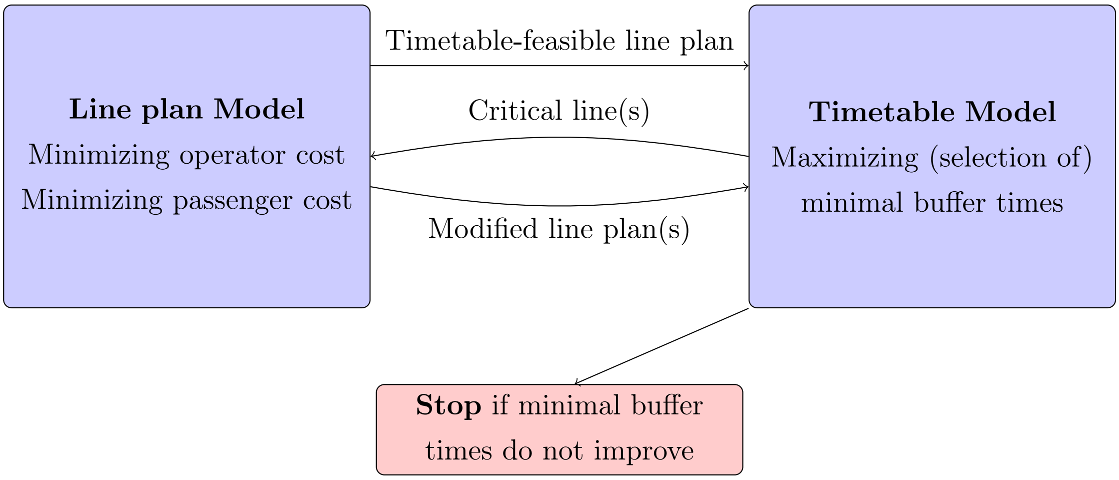


Figure 5: Overview of the integrated approach

Part 1: Initialization

Step 1: Construct an initial line plan

We construct a line plan that satisfies service constraints and optimizes a weighted sum of the passenger and operator cost with the line planning module. Beforehand, we check for infeasible lines in the line pool as discussed in Section 3. We check with the timetable module if a feasible timetable can be constructed for this line plan. A feasible timetable is a timetable in which no occupation intervals of trains overlap: if a station or platform is occupied by one train, no other train can occupy this station or platform until the first train leaves it. In case the constructed line plan is not timetable-feasible, different strategies can be applied. A straightforward strategy is to take the second best line plan for the weighted sum of the passenger and operator cost and if the second best is not timetable-feasible then the third best and so on. The disadvantage of this strategy is that it is possible that a lot of line plans are to be tested before a timetable-feasible line plan is found, because no insight in the problem is used. We propose another more effective strategy for a network with restricted shunt capacity as is assumed in this research. Due to the restricted shunt capacity in the terminal stations, the occupation of the terminal stations is critical in finding a timetable-feasible line plan. So an effective strategy for looking for a timetable-feasible line plan with a close to optimal objective value is by restricting the number of lines that share a terminal station. If a line using a shared terminal station also passes a different station that may be a terminal station, a close to optimal solution is a line plan in which this line is replaced by one that ends at this alternative terminal station. This decreases the number of lines sharing an end station and in some cases has minimal impact on operator and passenger costs. This new line plan is only feasible in case all service constraints remain fulfilled.

Part 2: Iterative steps

Step 2: Evaluate the line plan

Construct a timetable with the timetable module that maximizes the minimum buffer times between a selection, or between all the train pairs in the line plan. Calculate the minimum buffer times between all line pairs in the line plan, and the overall minimum buffer time. Refer to Section 6.2 for a concrete example. Test the following stop criteria:

- STOP if the minimum buffer time is closer than 5% to the desired minimum buffer time. The *desired minimum buffer time* can be found by identifying the station or track section which has the highest ratio of occupation time over free time and dividing the free time by the number of trains that pass by this section or station.
- STOP if the minimum buffer times do not improve the best encountered value during three succeeding iterations.

Otherwise, select the most critical line from the list. The most critical line is the line that is responsible for the highest number of buffers in the category of smallest buffers in the list. This is illustrated in the example in Section 6.2. In case of a tie, look at the next category of buffer times to identify the most critical line. If there is still a tie, let the decision be made by the line planning module in the next Step, based on the objective values there. The threshold to categorize the buffer times depends on the signaling system and the appearance of single tracks in the network. Go to Step 3.

Step 3: Adapt the line plan by changing the stopping pattern

Make a new line plan that alters the time length of the critical line by adding or removing a stop in a station on that line, such that this line becomes more flexible. This flexibility will be used to improve the buffer times in the timetabling module. This effect can be seen in the results and the example presented in Section 6. There are three important considerations. Firstly, changing the time length can also make a line infeasible as discussed in Section 3 which has to be avoided. Secondly, an extra stop cannot be added to a line in cases where there are no skipped stations on the line. Thirdly, some stations cannot be skipped due to service constraints.

We potentially solve the line plan problem with three different line pools, sequentially, to attempt to find a feasible solution. If a feasible line plan is found, the line plan problem does not need to be solved for the other line pools in the sequence. The three line (and frequency) pools are as follows.

i. All lines of the solution of the previous iteration are fixed, including their frequency, except that of the critical line. We add all lines that differ by one stop from the critical line. For those lines we only allow the frequency of the critical line.

ii. All lines of the solution of the previous iteration are fixed, including their frequency, except that of the critical line. We add lines to the line pool that differ by one stop from the critical line, which we now allow at any frequency.

iii. Solution lines that share no stations with the critical line are fixed. We introduce lines that differ by one stop from the critical line and lines that differ from other non-fixed non-critical lines by one station, at any frequency.

Because the number of lines in the line pool and the number of feasible solutions is much more restricted, the run time for the line planning module is now much shorter. The objective function is the same as in Step 1. For the first line pool, if feasible, the best alternative line will be selected, i.e. the line that provides the lowest passenger and operator costs. For the second line pool, if feasible, one or more of these new lines will be selected, often with a frequency combination that sums to the frequency of the critical line. For the third line pool, one or more lines similar to the critical line will be selected, and other solution lines from the previous iteration may be replaced with one or more similar lines. A simple example of solution from the third line pool is where a stop at a certain station is shifted from the critical line to a line that first skipped this station. The time length of the critical line changed by removing a stop and the station that is now skipped by the critical line is still served, but by another line. Note that in this example, the length of the non-critical line is also changed. A composition resulting from the second line pool is captured in the example in Section 6.2.

In the case that a feasible solution is found, return to Step 2. In the case that no feasible solution is found, and if there is a second most critical line, solve the three line plan problems for the second most critical. Otherwise STOP.

End

The intuition behind the integrated approach is the following. Changing the number of stops of a line changes the time length of the line. This time length of a line affects the flexibility of that line. So we alter the stop pattern of a line to make the line more flexible in order to improve the spreading in the whole network. The station where the stop pattern is changed is decided by the line planning module, which takes a weighted sum of passenger and operator cost into account. These costs are not taken into account during timetable construction. We note here that in general we do not require that the lines created to modify a line plan are all in the original pool of lines specified for the original problem. This explains why the adapted line plan can have a better weighted sum of passenger and operator cost than the original one. For the second stop criterion we take three non-improving iterations, to both restrict the run time while still allowing improvements that require multiple lines to change before a resultant improvement in minimal buffer time is observed.

5 Case study

The railway system on which the approach is tested is the S-tog network in Copenhagen operated by Danish railways operator DSB. This is a cyclic high-frequency network with a one hour period of repetition and which transports 30 000 to 40 000 passengers per hour at peak times between 84 stations. The network is visualized in Figure 6. It contains a central corridor, indicated in red; five 'fingers', indicated in blue; and a circle track, indicated in yellow. During peak hour on weekdays, there is a service requirement of 30 trains per hour through the central corridor in each direction. Each train occupies a station area on its route, in the direction that the train is driving, for one minute. The minimum *desired buffer time* in the DSB S-tog network is therefore one minute, which is (60 min - 30 min)/30, where 60 minutes is the period of the cyclic timetable and 30 trains occupy a station in the central corridor each for one minute. With almost no exception, there are at least two tracks in between every two adjacent stations. Furthermore, given that the network is designed with bridges and tunnels, there are very few locations where trains in opposite directions have to cross each other during normal conditions. In this research we assume that trains in opposite directions only interact with each other in terminal stations. One requirement specified by the operator is that only lines at frequency three, six, nine or twelve are allowed in the weekday line plan. This restriction decreases the probability of frequency combination infeasibility (though that is not necessarily the intention for the requirement). In order to enable and maintain this high frequency in the central corridor, the spreading of the trains in this part of the network is crucial. Therefore the timetable module will be sequentially used twice with two different objective functions. First, the minimum buffer times in the central corridor are optimized. In a second optimization round the minimum buffer in the rest of the network is optimized while bounding the buffer times in the central corridor by the value found in the first optimization. We also considered one combined weighted objective function, but this proved computationally worse in our experiments, i.e. the run times were significantly higher.

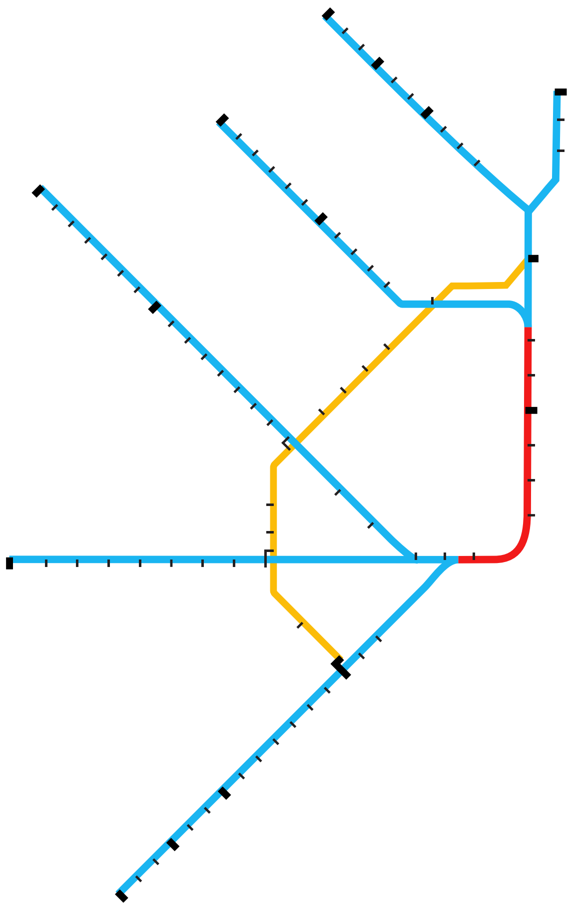


Figure 6: DSB S-tog network of Copenhagen

We test our approach on ten line plans for this network. The approach can be applied to a pre-existing line plan, or applied to first create and then improve a line plan. The full approach is tested for five line plans created as described in Step 1 of the integrated approach, while the other five line plans come from the operator or are created by hand. The first two line plans (1-2) were recently in use for the S-tog network in Copenhagen. We have not considered the current line plan as it is only temporarily active and specifically developed for implementing the new signaling system in the central corridor of the network. The third line plan (3) is a night line plan for weekdays. As the demand during night time is lower, the frequencies of the lines in this line plan are also lower. All other line plans are line plans that are planned with the requirements for use during daytime on weekdays. So, the setting of this third line plan is different from the other ones. This third line plan is also not the current plan in the S-tog network as at the present time a temporary plan is in use. The fourth up to the eighth line plan (4-8) are created within our algorithm by solving the weighted sum line planning module, using a range of weights that give distinct line plans. For each of these weights, we solve the line planning model with a one hour time limit and to a 0.5% relative gap limit, and terminate when either is reached. We initially solve the line planning module finding distinct solutions with no consideration for the feasibility of timetables except for infeasible lines as explained in Section 3. Then we test whether or not these are timetable-feasible. We find for these considered weights that only a single line plan (4) is feasible for timetabling. This endorses the statement that the output of a previous level in railway planning is not necessarily adequate for the next planning level (Schöbel, 2015). For those that are not feasible, we introduce restrictions on the use of terminal platforms, requiring only one line terminating at each terminal platform a station has. This is described in Step 3 of the integrated approach in Section 4. This is sometimes too conservative, since it can be possible for more than one line to share a single terminal platform. Conversely this alone does not guarantee that a feasible timetable is present for a line plan, but we observe that it is often a sufficient restriction. Applying this restriction we find four other distinct line plans (5-8). We note that, when considering the two line plan objectives of operator cost and passenger cost, none of the final four plans dominates any other. The ninth and the tenth line plan (9-10) are two special 'manually created' line plans, which are each based on one of the weighted-objective line plans (5 and 8 respectively). These paired plans only differ in stopping pattern from the plan they are manually adapted from, as we force every line to stop in every station it passes, while the original line plans contain many skipped stations. We want to investigate if each pair (5 and 9, 8 and 10) converges to a final line plan of similar quality when we modify stopping patterns of lines.

6 Results and discussion

In this section we show the results of the integrated approach for all ten line plans described in Section 5. Furthermore, we demonstrate the integrated approach for line plan 2.

6.1 Results for ten line plans

A first performance indicator is the estimated operator cost of a line plan. This cost is calculated by the line planning module. The total cost of a line plan is simply the sum of estimated operator costs for each line, which we take as given by the rail operator, here DSB. Each line in the pool has an operating cost associated with each frequency at which it could operate, and in calculating the total cost there are no additional considerations given to the combinations of lines.

A second performance indicator is the estimated passenger cost of a line plan. This cost is calculated by the line planning module. It is the sum of travel time of all passengers in the OD matrix. Because a timetable is not known (by the line planning module), the transfer time is estimated based on the frequency of the line as half of the time between two trains of that commuter line. For each passenger transfer an additional penalty of six minutes is added to the estimated passenger cost as transfers are perceived to be worse than direct connections.

The third performance indicator is the minimum buffer time between train pairs in the central corridor of the DSB S-tog network, optimized by the timetable module. The fourth performance indicator is the minimum buffer between train pairs everywhere in the network, while bounding the minimum buffer time in the central corridor first.

This fourth performance indicator is also optimized by the timetable module. The focus on the minimum buffer time first in the central corridor of the network and thereafter on the minimum buffer time overall is in consultation with DSB S-tog.

A fifth performance indicator is the sum of the inverse of the minimum buffer times between train pairs in each station that they have in common (and pass by in the same direction). We take the inverse minimum buffer times in order to give smaller buffers a higher weight than large buffers. As in Dewilde et al. (2013) a buffer time smaller than the time discretization ϵ (here 0.1 minute) has a contribution of 15 to the sum of the inverse buffer times. So the lower the sum of the inverse buffer times the better, because this means generally larger buffer times. The results are summarized in Table 2, Table 3 and Table 4.

In Table 2 we observe that there is a significant improvement in the buffer times for eight out of the ten line plans. For three out of the ten line plans, the desired minimum buffer time is reached both in the central corridor and in the rest of the network. For three other line plans the desired minimum buffer time is reached in the central corridor but not in the rest of the network. Furthermore, we see that the sum of the inverse buffer times

Line plan		Min buffer in central corridor (min)			Min buffer overall (min)			Sum of inverse buffer times (1/min)		
		initial	final	impro	initial	final	impro	initial	final	impro
1	real	0.63	1.00	+58%	0.00	1.00	+∞%	2 639	2 189	-17%
2	real	0.73	1.00	+36%	0.00	1.00	+∞%	2 348	2 212	-6%
3	real	3.00	3.00	+0%	0.70	2.55	+264%	482	382	-21%
4	random	0.33	0.64	+93%	0.00	0.05	+∞%	3 293	3 323	+1%
5	random	0.17	0.83	+400%	0.00	0.20	+∞%	3 840	2 365	-38%
6	random	0.37	0.99	+170%	0.00	0.01	+∞%	3 211	2 929	-9%
7	random	0.23	0.23	+0%	0.00	0.00	+0%	4 324	4 324	-0%
8	random	0.23	0.23	+0%	0.00	0.00	+0%	4 357	4 348	-0%
9	special	1.00	1.00	+0%	0.70	1.00	+43%	2 318	2 203	-5%
10	special	0.92	1.00	+8%	0.00	0.00	+0%	3 179	3 362	+6%

Table 2: The integrated approach significantly improves the buffer times in eight out of ten of the studied line plans.

between train pairs in every station they have in common decreases, which means that the buffer times themselves increase as desired. Moreover, the results on the sum of the inverse buffer times are very similar to the minimum buffer time results in the central corridor and in the overall network. We note that a big absolute improvement of the minimum buffer time in the central corridor (or of the minimum buffer time overall) corresponds to a big improvement in the sum of the inverse buffer times, and vice versa. Unfortunately, for two out of the ten line plans (7 and 8) no improvement in minimum buffer time is achieved. To identify the critical line in Step 2 of the integrated algorithm, we categorize the buffers as zero, smaller than 30 seconds, smaller than one minute and bigger than one minute. We observe that for the two timetables corresponding to the initial line plans almost half of the minimum buffer times between line pairs are smaller than 30 seconds, while for the other line plans this is at most one third of the minimum buffer times. As a possible explanation, we note that for these line plans almost every line has a pairwise minimum buffer time below half a minute with some other line, and we may therefore expect that multiple lines must be modified to see an improvement. We typically change a single line in every iteration and in such cases, it may take more than three non-improving iterations before seeing an improvement, given that every line plan we consider has between six and ten lines.

The buffer times in Table 2 appear to be small. However, as discussed earlier, the minimum desired buffer time in a daytime week line planning is one minute and this is thus the value of the stopping criterion in Step 2 of the integrated approach. The maximal minimum buffer time everywhere in the network is also restricted by this maximal minimum buffer time in the central corridor. Moreover, even if the buffer time between two trains is zero the timetable is still feasible. A zero buffer time only means that the second train reserves and occupies a track section between two signals immediately after the first train leaves and releases this track section. However, a zero buffer time is undesirable and any delay of the first train is immediately propagated to the second. In the case study, a line plan performs best if it allows the desired buffer time of at least one minute between every two trains. As an exception, for line plan 3 the desired buffer time is three minutes. This desired buffer time is for example achieved for line plan 9.

One explanation for not reaching the desired value for some line plans could be no further improvement was made, because at each iteration the same line was identified as being critical. Either changing this line was no longer feasible or changing this line was feasible, but did not result in acceptable solutions. If changing the critical line is not feasible, then the second most critical line of the last found line plan is chosen. In the current algorithm, however, if the critical line itself does not give rise to good results, there is no backtracking to a previous iteration to try the second most critical line.

		Operator cost ($\times 10^5$)			Passenger cost ($\times 10^7$)		
Line plan		initial	final	change	initial	final	change
1	real	6.79	6.84	+0.74%	4.17	4.23	+1.47%
2	real	6.84	7.21	+5.40%	4.22	4.21	-0.12%
3	real	3.40	3.43	+0.64%	1.05	1.06	+1.08%
4	random	6.25	6.64	+6.23%	4.24	4.27	+0.87%
5	random	6.48	6.80	+4.94%	4.27	4.29	+0.36%
6	random	6.66	6.74	+1.13%	4.12	4.14	+0.51%
7	random	7.02	7.02	+0.00%	4.09	4.09	+0.00%
8	random	8.27	8.32	+0.71%	4.05	4.04	-0.22%
9	special	7.15	7.14	-0.17%	4.43	4.44	+0.32%
10	special	9.00	9.01	+0.20%	4.35	4.30	-1.06%

Table 3: The operator cost and passenger cost differ only slightly when applying the integrated approach for most plans.

In Table 3 the operator cost and the passenger cost for the initial and final line plan are presented. We observe that they differ only slightly for most plans. This is important because it illustrates that our improvement in terms of expected delay propagation, by modifying the line planning and timetable, does not necessarily require an increase in operator and passenger cost. Note in fact that in Table 3 some plans do improve for one measure but become worse for another, and although it is possible for both to improve (as we are in fact allowing lines that were not in the original line pool), we do not observe this here. Note also that for line plans 4 and 5 we do see a relatively large increase in operator cost (6.23% and 4.94%), combined with an increase in passenger cost which may be a relatively large cost to pay for timetable improvement. In contrast, for line plan 2 though we see a similarly large increase in operator cost but a reduction in passenger cost. Here the impact must be judged by the perceived relative importance of the two measures.

Line plan		Stop criterion	# iterations	# out-of-pool lines	Average run time timetabling (min)
1	real	DES	4	5	183.40
2	real	DES	3	5	4.88
3	real	BFV	2	3	0.50
4	random	BFV	6	5	75.71
5	random	BFV	7	7	385.563
6	random	BFV	7	5	167.19
7	random	BFV	3	0	126.75
8	random	BFV	3	1	9.25
9	special	DES	1	2	47.5
10	special	BFV	5	3	346.83

Table 4: Characteristics of the integrated approach

In Table 4 some characteristics of the integrated approach are presented together. We indicate there under which stop criterion the algorithm was terminated. If the algorithm terminated because the minimum buffer time between the selected train pairs is closer than 5% to the desired minimum buffer time, we indicate this as 'DES' from desired. If the algorithm ended because the minimum buffer time between the selected train pairs did not improve the best found value in three consecutive iterations, we indicate this as 'BFV', i.e. best found value. We see that three out of the ten line plans were within the desired minimum buffer time and for the remaining seven the algorithm ended with the best found value. The table also reports how many iterations the integrated approach passed through before a stop criterion was achieved. This value ranges between one and seven. We report the number of out-of-pool lines which are in the final solution, referring to lines that are in the final solution but do not come from the original, restricted, line pool but instead are similar to a line in the pool but with a modified stop pattern. We observe that the five line plans with the highest number of out-of-pool lines (line plan 1, 2, 4, 5 and 6) have the greatest relative improvement of the minimum buffer time in the central corridor; have the greatest increase in operator cost; and (with one exception) have the highest increase in passenger cost. Therefore, including new lines in the line pool has the potential to improve the minimum buffer times significantly, but may have a negative effect on passenger and operator costs.

The final characteristic in Table 4 is the run time for timetabling. The total run time for timetabling consists of the creation of the optimal timetable in Step 2 of the integrated approach for each iteration and for determining the initial timetable. As described in Section 5 the timetable is solved sequentially with two objective functions at each iteration. Firstly, the buffer time in the central corridor is maximized, and secondly the buffer time in the rest of the network is increased with a bound on the buffer time in the central corridor fixed by the first step.

For example the algorithm stops after three iterations for line plan 2. This means that eight timetables are calculated: two initially (for the two optimization criteria) and two at each of the three iteration steps. The average run time for timetabling for optimizing line plan 2 is 4.88 minutes, which means that the run time for each calculated timetable in the integrated approach on average is 4.88 minutes. All timetables are calculated with CPLEX 12.6 on an Intel Core i7-5600U CPU @ 2.60 GHz. We observe that there is a high variability in the average run times for the different line plans. Moreover, a high computation time may occur in case where there is both a big improvement (line plans 1 and 5) and also where there is either no improvement (line plan 7) or only a small improvement (line plan 10). Furthermore, the run time for timetabling can differ significantly from one iteration to the next. Even if two line plans are not dissimilar one can be intrinsically more difficult to solve. An explanation could be that due to changes in the stopping pattern, trains of different lines are more or less susceptible to catching up with each other in the fingers of the S-tog network, resulting in it being more complex to spread the trains optimally. The timetable module runs to optimality (relative gap smaller than 0.05%) for about 85% of the timetables. The average run time per timetable optimized within the time limit of 12 hours is 3801 seconds. For the other optimizations a time limit of 12 hours is imposed. The line planning module for the selected line pool determined by the critical line runs to optimality in all instances, taking at most up to ten minutes for cases where many lines are to be changed.

Finally, from Tables 2, 3 and 4, we deduce that line plans 5 and 9 did not converge to the same final line plan and line plans 8 and 10 did not either. We see that the final line planning and timetable for line plan 9 and 10 score better on robustness, i.e. minimum buffer time between line pairs is larger, while line plan 5 and 8 score better on operator and passenger cost. Based on these results, the final decision on which line plan is preferred, rests with the operator. In our opinion, the optimized version of line plan 1 will be the most passenger robust, without adding expense for the operator or passengers.

6.2 Illustration

In order to illustrate the integrated approach, we apply it to line plan 2. As this is an existing line plan, we skip Part 1 of the algorithm and only look at the iterative steps in Part 2. The estimated operator cost of this line plan is 6.84×10^5, the estimated total passenger travel time is 4.22×10^7. The optimal value for the minimum buffer time for this line plan in the central corridor of the network is 0.73 minutes. The optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 0.73 minutes is zero minutes. The minimum buffer times between the line pairs are present in minutes in Table 5. The smallest buffer time between line i and j is the same as the smallest buffer time between line j and i, so Table 5 is in fact symmetric, but we omitted here the superfluous information. If two lines do not share a part of the network, the minimum buffer time between these lines is indicated as 60 minutes, which is the period of the cyclic timetable. The smallest buffer time between two lines is zero minutes. This buffer time is between line 1 and itself. This means that the turn platform of line 1 in one of its terminal stations is permanently occupied by a train of the first line. Obviously, the critical line is line 1.

line	0	1	2	3	4	5	6	7
0	1.73	0.73	0.73	2.47	2.88	3.13	2.47	60
1	-	**0.00**	2.47	0.73	1.15	2.57	0.73	60
2	-	-	2.2	1.30	1.47	1.30	3.50	60
3	-	-	-	7.17	0.88	1.40	6.70	60
4	-	-	-	-	4.45	7.58	1.03	60
5	-	-	-	-	-	2.87	0.80	60
6	-	-	-	-	-	-	6.97	60
7	-	-	-	-	-	-	-	0.57

Table 5: The minimum buffer time overall is zero minutes, if the minimum buffer time in the central corridor is bounded below by 0.73 minutes

The line planning module adds a stop to line 1 by considering only the line pool that contains alternatives for line 1 of the same frequency. The new estimated operator cost increases to 6.99×10^5 and the new estimated total passenger travel time slightly increases to 4.23×10^7. The optimal value for the minimum buffer time for this line plan in the central corridor of the network is 1.00 minute. The optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 1.00 minute is still zero minutes. The minimum buffer times between the line pairs of the first modification of line plan 2 are present in minutes in Table 6. The smallest buffer time between two lines is still zero minutes. This buffer time is now only associated with line 6. Again, this means that the turn platform of line 6 in one of its terminal stations is permanently occupied by a train of line 6. The critical line is line 6.

line	0	1	2	3	4	5	6	7
0	6.17	1.01	2.99	60	3.01	1.00	3.99	2.99
1	-	2.14	2.99	60	1.00	1.99	1.00	1.00
2	-	-	0.29	60	1.00	1.00	1.01	1.00
3	-	-	-	0.67	60	60	60	60
4	-	-	-	-	1.82	2.99	2.99	2.99
5	-	-	-	-	-	7.23	6.99	6.99
6	-	-	-	-	-	-	**0.00**	1.00
7	-	-	-	-	-	-	-	3.02

Table 6: The minimum buffer time overall is zero minutes, if the minimum buffer time in the central corridor is bounded below by 1.00 minute

In Step 2, the line planning module first considers line pools that contain only alternatives for line 6 of the same frequency and of different frequencies, but they do not lead to a feasible line plan. We then consider the line pool that also contains alternative lines for different frequencies for the lines that share a part of the network with line 6. The result is a feasible line plan that does not include original line 6 and 7, each of frequency three, but contains a new line of frequency 6. The original line 6 stops at the same stations as the original line 7, but has some additional stops at one end of the line. The new line is similar to the original line 6 but skips one stop of that line. The new estimated operator cost is 7.22×10^5 and the new estimated total passenger travel time is 4.20×10^7. The optimal value for the minimum buffer time of this line plan in the central corridor of the network remains 1.00 minute. However, the optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 1.00 minute has now increased to 0.70 minutes. The minimum buffer times between the line pairs of the second modification of line plan 2 are present in minutes in Table 7. The smallest buffer time between two lines is now 0.7 minutes. This buffer time is now only associated with line 3, so the new critical line is line 3.

line	0	1	2	3	4	5	6
0	10.16	2.99	1.00	60	2.99	8.98	1.00
1	-	2.10	2.99	60	1.00	2.01	1.00
2	-	-	1.15	60	1.00	1.01	3.01
3	-	-	-	**0.70**	60	60	60
4	-	-	-	-	2.06	3.01	2.99
5	-	-	-	-	-	8.05	1.00
6	-	-	-	-	-	-	1.69

Table 7: The minimum buffer time overall is 0.70 minutes, if the minimum buffer time in the central corridor is bounded below by 1.00 minute

The line planning module skips a stop of line 3. The new estimated operator cost is 7.21×10^5 and the new estimated total passenger travel time is 4.21×10^7. The optimal value for the minimum buffer time of this line plan in the central corridor of the network is still 1.00 minute. The optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 1.00 minute is now 1.00 minute. The minimum buffer times between the line pairs of the second modification of line plan 2 are present in minutes in Table 8. This minimum buffer time overall is closer than five percent to the minimum desired buffer time of one minute, so this is the last iteration of the algorithm.

line	0	1	2	3	4	5	6
0	10.20	3.01	1.00	60	2.99	8.98	1.00
1	-	2.15	2.99	60	1.00	1.99	1.00
2	-	-	1.09	60	1.00	1.00	2.99
3	-	-	-	1.00	60	60	60
4	-	-	-	-	2.11	2.99	2.99
5	-	-	-	-	-	8.02	1.00
6	-	-	-	-	-	-	1.66

Table 8: The minimum buffer time overall is 1.00 minute, if the minimum buffer time in the central corridor is bounded below by 1.00 minute

7 Conclusion and further research

This paper presents a heuristic algorithm that builds a line plan from scratch resulting in a feasible and robust timetable. Our method iterates interactively, alternating between a line planning module and a timetabling module, improving the robustness of an initially built line plan. Both modules consist of an exact optimization model. The line planning module optimizes a weighted sum of passenger and operator costs, while the timetabling module focuses on improving minimum buffer times between line pairs. Appropriate and sufficiently large buffer times between train pairs are needed to reduce the risk of delays being propagated from one train to the next, thereby obtaining a robust railway schedule. The timetable module identifies a critical line based on the minimum buffer times between line pairs. The line planning module creates a new line plan in which the time length of the critical line is changed. Changing the time length of a line may create more flexibility in the schedule, which may result in improvements in robustness. The approach was tested for ten different line plans on the DSB S-tog network in Copenhagen. This is a high-frequency railway network with 84 stations, currently nine lines and restricted shunt capacity in the terminal stations. For eight out of ten initial line plans the robustness could be significantly improved, while the changes to the line plan generally did not result in significant changes in the weighted sum of operator and passenger cost. Ultimately the operator makes the final decision on the preferred criterion, considering the measures we have presented and others we have not captured.

An initial idea for future research is a smart extension of the integrated approach to overcome the situation where a certain line remains critical in each iteration, while keeping the computation time restricted. Another extension would be to allow different shunt characteristics in different terminal stations. In the presented research we had the very strict requirement present in the DSB S-tog system to have a schedule in which no train uses shunt capacity in a terminal station during daily operation. Furthermore, the development of a single integrated exact model that combines line planning and robust timetabling which is solvable in a reasonable amount of time for other real networks (similar to the DSB S-tog network) would be a next noteworthy step. A further idea for future research is to remove the requirement that trains of a line must operate exactly evenly timed (e.g. once every ten minutes for a six-per-hour line). Currently, this requirement is consistent with operation and ensures a regular service for customers. However it is potentially severely restrictive for the timetable given the tight spacing of trains in the central corridor. Relaxing this requirement could increase the complexity of the timetable model, both by expanding the solution space and by requiring new constraints and possibly an objective measure for the evenness of train timings.

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