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Optimizing Yb concentration of fiber amplifiers in the presence of transverse modal instabilities and photodarkening

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The Yb concentration of double-clad optical fiber amplifiers is numerically optimized with respect to maximizing the transverse modal instability threshold in the presence of absorption arising from photodarkening. The pump cladding area is scaled with the Yb concentration to approximately maintain the pump absorption in operation. It is found that approximate analytical expressions can predict the optimized concentration levels found in numerical simulations with sufficient accuracy to be useful in fiber design.

OCIS codes: (140.3510) Lasers, Fiber; (140.6810) Lasers, Thermal effects; (060.2320) Fiber optics amplifiers and oscillators; (160.5690) Rare-earth-doped materials.

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1. Introduction

Transverse modal instabilities (TMI) are currently the main limiting factor for average-power scaling of Yb-doped fiber amplifier systems with large cores [1, 2]. It is by now broadly accepted that thermo-optic nonlinear coupling between the fundamental mode (FM) and first higher-order mode (HOM) of the fiber is the main cause of TMI, and models on various levels of sophistication have been presented [2–7]. An additional issue is the occurrence of induced signal absorption, so-called photodarkening (PD), upon prolonged laser operation [8]. Since extra absorption implies an additional heat load, the onset of PD is found to reduce the TMI power threshold [7, 9].

It has been shown by several authors that the TMI induced by the heat load from a given optical gain is reduced by inversion saturation [10, 11]. Therefore, this source of TMI can be diminished by increasing the Yb concentration, \( N_{Yb} \), while maintaining the value of the signal gain. The latter is important because a short amplifier length is often desired to mitigate nonlinear effects, such as self-phase modulation or Raman scattering. On the other hand, PD effects approximately increase with \( N_{Yb}^2 \) [7, 12], so \( N_{Yb} \) may be expected to have an optimum value with respect to maximization of the TMI threshold.

The purpose of this work is to numerically study the optimization of Yb doping concentrations in a double-clad step-index fiber amplifier. Analytical expressions for the optimal concentration are derived from simplified assumptions and compared to results of the numerical simulations. The expressions are found to give useful predictions for optimal \( N_{Yb} \) values when taking the variation of TMI threshold with \( N_{Yb} \) into account.

The paper is organized as follows: In section 2, the numerical model for calculating the TMI threshold is laid out, and an approximate analytical estimation of the optimal Yb concentration is derived. In section 3, numerical results are presented for optimal Yb concentrations, and the corresponding threshold powers, and comparison is made to the analytical estimate. Section 4 summarizes the conclusions.

2. Formal theory

The simplified model of TMI proposed in refs. [11, 13] is adopted, in an undepleted-pump approximation, for the numerical simulations. The power evolution in the fundamental and higher or-
der modes, $P_1(z)$, $P_2(z)$, is in this model given by
\[
\frac{dP_1}{dz} = \Gamma_1(z) P_1(z) 
\]  
\[
\frac{dP_2}{dz} = \left[ \Gamma_2(z) + \chi_2(z, \Omega) P_1(z) \right] P_2(z, \Omega) 
\]  
\[
\Gamma_1(z) = 2\pi \int_0^{r_d} dr R_i^2(r) \left[ \frac{g_0(z)}{1 + \int_0^{r_d} \frac{\Gamma(z)}{I_\text{sat}(z)}} - \gamma_{PD}(r, z) \right] 
\]  
\[
g_0(z) = N_Yb \left( \frac{P_p(z) \sigma_{as} - \sigma_{as} \sigma_{ep}}{I_p(z) \sigma_{ap} + \sigma_{ep}} + P_\tau \sigma_{as} \right) 
\]  
\[
I_s(z) = \frac{\omega_s}{\omega_p} \left( \sigma_{ap} + \sigma_{ep} \right) + P_\tau 
\]  
\[
I_s(r, z) = P_1(z) R_i^2(r); \quad P_\tau = \frac{h\omega_p}{\tau}; \quad I_p(z) = \frac{P_p(z)}{A_p} 
\]  
Here $\sigma_{as}$, $\sigma_{ap}$, $\sigma_{es}$, $\sigma_{ep}$ are absorption and emission cross sections for signal and pump respectively, $\omega_p$, $\omega_s$ are the pump and signal frequencies respectively, $\tau$ the Yb upper-state lifetime, and $\Omega$ the frequency detuning between the signals in the fundamental and higher-order mode, typically in the kHz range. The doping radius of the core is denoted $r_d$, and a uniform Yb-distribution inside this radius is assumed. In the present work, the core radius is taken to be equal to $r_d$, but this is not a necessary requirement. $A_p$ is the area of the pump core, or inner cladding. The radial guided-mode profiles $R_i$ are normalized so that
\[
2\pi \int_0^{r_d} dr R_i^2(r) = 1. 
\]
Assuming backward pumping, the pump power $P_p(z)$ obeys the evolution equation
\[
\frac{dP_p}{dz} = \frac{2\pi N_Yb}{A_p} \int_0^{r_d} dr \left[ \sigma_{ap} - n_2(r, z)(\sigma_{ap} + \sigma_{ep}) \right] 
\]
with $n_2(r, z)$ given by
\[
n_2(r, z) = \frac{g_0(z)}{N_Yb(\sigma_{as} + \sigma_{es}) \left( 1 + \frac{\int_0^{r_d} \Gamma(z)}{I_\text{sat}(z)} \right)} + \frac{\sigma_{as} + \sigma_{es}}{\sigma_{as} + \sigma_{es}} 
\]  
In writing Eq. (1) it has been assumed that $P_2 \ll P_1$ holds everywhere, so that the depletion of $P_1$ by the coupling term in Eq. (2) can be neglected. By the same assumption, the signal intensity $I_s$ is calculated from $P_1$ only.

The absorption from photodarkening (PD) is described by $\gamma_{PD}$, whose asymptotic value in long-term use of the amplifier at a specific inversion level is assumed to be
\[
\gamma_{PD}(r, z) = \alpha_{PD} N_Yb^2 n_2(r, z) 
\]  
While this model is in reasonable, though not perfect, accord with various published data, the magnitude of reported PD effects vary widely across the literature. For this reason, $\alpha_{PD}$ will in this work be taken as a variable parameter, whose order of magnitude will be discussed in section 3.

The crucial nonlinear coupling parameter $\chi_2(z, \Omega)$ is given by
\[
\chi_2(z, \Omega) = 2\pi \frac{\eta_{dP}}{cN_0} \int_0^{r_d} dr R_1(r) R_2(r) 
\]
\[
\int_0^{r_d} dr' q(z, r') \text{Im} \left[ g_1(r, r', \Omega) \right] R_1(r') R_2(r') 
\]
\[
q(z, r') = \frac{g_0(z) \left( \frac{\omega}{\omega_p} - 1 \right)}{\left( 1 + \frac{\int_0^{r_d} \Gamma(z)}{I_\text{sat}(z)} \right)} + \gamma_{PD}(r, z) 
\]
where $g_1(r, r', \Omega)$ is the $m=1$ component of the Greens function that solves the steady-periodic heat transfer problem at the frequency $\Omega$, as given in [3].

The above equations are solved numerically to estimate the TMI threshold in a given amplifier layout. The threshold is determined by requiring that the total HOM power, integrated over the $\Omega$ range where $\chi_2$ is appreciable, is below 10 per cent of the FM power when seeding the HOM with quantum noise (one photon per frequency bin). The threshold power is only weakly dependent on the exact choice of this criterion. As discussed in other works, the threshold is reduced if the HOM is seeded by amplitude noise [6], but this is of little consequence for the $N_{Yb}$ optimization which is the central purpose of this paper.

To obtain an analytical estimate for the optimal $N_{Yb}$, the following approximations are made: i) The transverse variation of the guided mode profile is neglected, so the signal intensity becomes $I_s(z) = I_s(z) = \frac{I_0}{A_p}$, where $I_0$ is the overlap integral of the FM with the doped area $A_d$. ii) $P_\tau \approx 0$ is used to simplify Eqs. (4), writing $g_0 \approx N_Yb(\sigma_{ap} \sigma_{es} - \sigma_{as} \sigma_{ep})/(\sigma_{ap} + \sigma_{ep})$, iii) The saturated gain, $g_s = g_0 / (1 + P_1/P_{sat})$ is approximated to be constant along the amplifier, with $P_{sat} = \frac{I_0 A_d}{\Omega_1}$. Under these assumptions, one finds for $n_2$ and the total gain for the HOM going through the amplifier,
In the present work, input parameters, using Eq. (16) to determine the change it induces in the TMI gain.

Using Eq. (13), this assumption yields:

\[ G_2 = \frac{g_s + N_Yb \sigma_{as}}{N_Yb (\sigma_{as} + \sigma_{es})} \quad (13) \]

\[ G_2(\Omega) = g_s O_2 L + \chi_2(\Omega) \left\{ \left( \frac{\omega_p}{\omega_s} - 1 \right) \frac{g_0}{g_0} \left( 1 + \frac{P_B}{P_{sat}} \right)^2 + \alpha_{PD} N_Yb \right\} \]

\[ \int_0^L dz P_1(z) = \]

\[ g_s O_2 L + \chi_2(\Omega) \left\{ \left( \frac{\omega_p}{\omega_s} - 1 \right) \frac{g_0^2}{g_0} + \frac{\alpha_{PD} N_Yb g_s}{(\sigma_{as} + \sigma_{es})} \left( 1 + \frac{N_Yb \sigma_{as}}{g_s} \right) \frac{\Delta P_1}{g_s O_1} \right\} \]

\[ \chi_2(\Omega) = 2 \pi \frac{\hbar \omega_p}{c n_0} \int_0^\infty \int_0^{r_d} dr' \text{Im} \left\{ g_s (r, r', \Omega) R_1(r') R_2(r') \right\} \quad (15) \]

where \( O_2 \) is the HOM overlap with the doped area and \( L \) is the amplifier length. \( \Delta P_1 \) is the difference between output and input signal power, and it was utilized in evaluating the \( z \)-integral that the constant FM signal gain is given by \( g_s O_1 \). The direct contribution of PD loss to the gain has been omitted, as it is expected to be negligible compared to the change it induces in the TMI gain.

Since the magnitude of the signal gain is not known \textit{a priori}, a final approximation is made: iv) \( g_s O_1 = \alpha_p \), where \( \alpha_p \) is the pump absorption in operation. Using Eq. (13), this assumption yields:

\[ \alpha_p = N_Yb (\sigma_{ap} - n_2 (\sigma_{ap} + \sigma_{ep})) \quad (16) \]

\[ \frac{A_d}{A_p} = \frac{\alpha_p (\sigma_{as} + \sigma_{es})}{N_Yb (\sigma_{as} + \sigma_{es}) (\sigma_{ap} + \sigma_{ep})} \quad (17) \]

\[ \frac{\alpha_{PD} N_Yb \alpha_p}{(\sigma_{as} + \sigma_{es}) O_1} \left( 1 + \frac{O_1 N_Yb \sigma_{as}}{\alpha_p} \right) \frac{\Delta P_1}{\alpha_p} + \frac{\alpha_p O_2 L}{(\sigma_{as} + \sigma_{es}) O_1} \quad (18) \]

This third-order equation is readily solved for \( N_Yb \). If \( \frac{2 \sigma_{as} O_1}{\alpha_p} N_Yb \ll 1 \) one has the explicit expression:

\[ N_Yb = \sqrt{\frac{\alpha_p \left( \frac{\omega_p}{\omega_s} - 1 \right) (\sigma_{ap} + \sigma_{ep}) (\sigma_{as} + \sigma_{es})}{\alpha_{PD} (\sigma_{ap} \sigma_{as} - \sigma_{as} \sigma_{ep}) O_1}} \quad (19) \]

This approximation is, however, not always accurate, as will be shown in section 3.

3. Numerical results and discussion

In the simulations, an amplifier having \( L=1 \) m, a core radius of 20 \( \mu \)m, \( \alpha_p=10 \) dB/m, pump wavelength of 976 nm, signal wavelength \( \lambda_s=1030 \) nm, a \( V \)-parameter of 4, and input signal power of either 10 or 40 W is used as a reference system. Variations in either \( \alpha_p \), signal wavelength or \( \alpha_{PD} \) around their reference values are considered, in order to determine the optimal Yb concentration, \( N_{opt} \), and test the accuracy of Eqs. (18), (19).

The choice of a range for \( \alpha_{PD} \) merits some discussion. A parametrization corresponding to \( \alpha_{PD} \approx \)
$2 \cdot 10^{-15} \mu m^6 dB/m$ was recently proposed [7], based on earlier experimental work [12, 14]. On the other hand, Mattsson reported PD loss measurements suggesting an $\alpha_{PD} < 10^{-16} \mu m^6 dB/m$ for a uniform core doping [15]. Other authors have published PD loss measurements in between these extremes [16, 17]. For the calculations in the present work an $\alpha_{PD}$ range of $10^{-15} - 10^{-16} \mu m^6 dB/m$ is therefore considered.

In figure 1, the results of Eqs. (18), (19) are compared to optimal Yb concentrations derived from the numerical simulations, when varying individual parameters as discussed above. Clearly, Eq. (18) is only accurate to typically within 10-30%, with some deviations exceeding 50%. However, as illustrated by the final plot of selected TMI thresholds versus the deviation of $N_{Yb}$ from its optimum value $N_{opt}$, these inaccuracies are of minor consequence for the resulting threshold. For instance, a 30% deviation from the optimum $N_{Yb}$ typically leads to a $\sim$5% deviation in TMI threshold. It may also be noted that the dependence of $N_{opt}$ on the seed power, which does not enter into Eqs. (18), (19), is relatively small. Thus, the calculations confirm that the proposed equations are useful for estimating optimal Yb concentrations.

In figure 2, the threshold powers obtained at the optimal values of $N_{Yb}$ are shown for the various cases. It is important to stress that all thresholds plotted are for the optimal value of $N_{Yb}$ as shown in Fig. 1, and with the pump cladding area $A_p$ scaled according to Eq. (16). Therefore the figures cannot be interpreted as showing the effect of varying only a single parameter, such as the signal wavelength. Clearly, a variation of $\alpha_p$, i.e. the relation between $N_{Yb}$ and pump cladding area, has relatively small influence on both the optimal $N_{Yb}$ value, and the resulting threshold. On the other hand, there is a considerable variation with $\lambda_s$, with a value around the pump absorption maximum at 1030 nm or even lower being optimal. In addition, as expected, minimization of $\alpha_{PD}$ is crucial for improving the TMI threshold. Also shown in Fig. 2 is a scatter plot displaying all calculated threshold
powers, each normalized to the threshold power at the optimal value of $N_{Yb}$, $N_{opt}$, as a function of $N_{Yb}/N_{opt}$. The plot shows that a factor-of-two deviation from the optimal value of $N_{Yb}$ will imply a reduction of the threshold power to 70-90% of its maximal value. The sensitivity is found to increase with decreasing $\alpha_p$, $\alpha_{PD}$ and $\lambda_s$, i.e. fibers with a high optimized threshold power will also be the most sensitive to variations in $N_{Yb}$.

It is clear from the simulation results that the material parameter $\alpha_{PD}$ is critical for the calculation. A simple method for estimating this parameter is to compare the TMI threshold of a fresh fiber with that of one in which PD is fully developed under the desired operating conditions. $\alpha_{PD}$ can then be extracted by numerical modelling. An analytical estimate may also be obtained in the spirit of the above derivations. Assuming that the TMI threshold corresponds to a certain level of nonlinear gain, Eq. (14) yields

$$\left(\frac{\omega_p}{\omega_s} - 1\right)\frac{g_1}{g_0O_1^2}\Delta P_{11} = \left(\frac{\omega_p}{\omega_s} - 1\right)\frac{g_2}{g_0O_1^2} + \frac{\alpha_{PD}N_{Yb}}{(\sigma_{as} + \sigma_{es})O_1} \left(1 + \frac{N_{Yb}\sigma_{as}O_1}{g_2}\right)\Delta P_{12}$$  \hspace{1cm} (20)

omitting the linear gain term for simplicity. Here $g_1$, $g_2$ are the average signal gains (recall that FM signal gain is $g_0O_1$) before and after PD saturation (which in this experiment may be determined from the signal input and output levels), and $\Delta P_{11}$, $\Delta P_{12}$ are the $\Delta P_1$ levels before and after PD. Eq. (20) can be rearranged to yield

$$\alpha_{PD} = \left(\frac{\omega_p}{\omega_s} - 1\right)\left(\sigma_{ap} + \sigma_{ep}\right)\left(\sigma_{as} + \sigma_{es}\right)\frac{g_1}{O_1N_{Yb}^2\left(\sigma_{ap}\sigma_{as} - \sigma_{as}\sigma_{ep}\right)\left(1 + \frac{N_{Yb}\sigma_{as}O_1}{g_2}\right)\times g_1\Delta P_{11} - g_2\Delta P_{12}}{\Delta P_{12}}$$  \hspace{1cm} (21)

Fig. 3 shows values of $\alpha_{PD}$ calculated from results of numerical simulations using Eq. (21) rel-
four different cases. \( \alpha \) as a function of \( N \) for an accurate determination of the level of 10-30\%. Thus Eq. (21) is useful for a simplified formula and the numerical results is on the value in a fresh fiber. The deviation between the simulations was found for a step-index amplifier with a \( \alpha_{PD} \) relative to optimum \( N_{Yb}=N_{opt} \) for four different cases. \( \alpha_{est} \) is calculated from Eq. (21) based on output from the numerical simulations.

4. conclusion
In conclusion, numerical estimates for the optimal Yb concentration with respect to TMI mitigation in the presence of photodarkening have been presented as a function of PD magnitude, signal wavelength and pump absorption. Simplified equations for predicting optimal Yb concentrations and photodarkening strength have been derived and shown to agree with numerical simulations within \( \sim 50\% \). The main assumptions behind the calculations are the dependence of photodarkening strength on Yb concentration and inversion level, Eq. (10), and the validity of the simplified theoretical model. It is also important to note that the simulations were done for a step-index amplifier with a \( V \)-parameter of 4, meaning that the LP\(_{11} \) mode is reasonably well confined, and not overly sensitive to thermal perturbations. If one considers fibers which are closer to the single-mode regime, thermally induced changes in the mode profiles [18] could lead to a considerable variation with propagation distance of the overlap integrals determining the thermo-optic nonlinear coupling. In such a case, one may need to consider a more comprehensive numerical and analytical model.

References


